Enumerative Combinatorics, Volume 1

Richard Stanley's two-volume basic introduction to enumerative combinatorics has become the standard guide to the topic for students and experts alike. This thoroughly revised second edition of Volume 1 includes ten new sections and more than 350 new exercises, most with solutions, reflecting numerous new developments since the publication of the first edition in 1986.

The material in Volume 1 was chosen to cover those parts of enumerative combinatorics of greatest applicability and with the most important connections with other areas of mathematics. The four chapters are devoted to an introduction to enumeration (suitable for advanced undergraduates), sieve methods, partially ordered sets, and rational generating functions. Much of the material is related to generating functions, a fundamental tool in enumerative combinatorics.

In this new edition, the author brings the coverage up to date and includes a wide variety of additional applications and examples, as well as updated and expanded chapter bibliographies. Many of the less difficult new exercises have no solutions so that they can more easily be assigned to students. The material on P-partitions has been rearranged and generalized; the treatment of permutation statistics has been greatly enlarged; and there are also new sections on q-analogues of permutations, hyperplane arrangements, the cd-index, promotion and evacuation, and differential posets.

RICHARD P. STANLEY is a professor of applied mathematics at the Massachusetts Institute of Technology. He is universally recognized as a leading expert in the field of combinatorics and its applications to a variety of other mathematical disciplines. In addition to the seminal two-volume book *Enumerative Combinatorics*, he is the author of *Combinatorics and Commutative Algebra* (1983) and more than 100 research articles in mathematics. Among Stanley's many distinctions are membership in the National Academy of Sciences (elected in 1995), the 2001 Leroy P. Steele Prize for mathematical exposition, and the 2003 Schock Prize.

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Second Edition

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"Yes, wonderful things."
—Howard Carter when asked if he saw anything, upon his first glimpse into the tomb of Tutankhamun

Contents

Preface

Enumerative combinatorics has undergone enormous development since the publication of the first edition of this book in 1986. It has become more clear what the essential topics are, and many interesting new ancillary results have been discovered. This second edition is an attempt to bring the coverage of the first edition more up to date and to impart a wide variety of additional applications and examples.

The main difference between this edition and the first is the addition of ten new sections (six in Chapter 1 and four in Chapter 3) and more than 350 new exercises. In response to complaints about the difficulty of assigning homework problems whose solutions are included, I have added some relatively easy exercises without solutions, marked by an asterisk. There are also a few organizational changes, the most notable being the transfer of the section on P-partitions from Chapter 4 to Chapter 3, and extending this section to the theory of (P,ω)-partitions for any labeling ω. In addition, the old Section 4.6 has been split into Sections 4.5 and 4.6.

There will be no second edition of volume 2 nor a volume 3. Since the references in volume 2 to information in volume 1 are no longer valid for this second edition, I have included a table entitled "First Edition Numbering," which gives the conversion between the two editions for all numbered results (theorems, examples, exercises, etc., but not equations).

Exercise 4.12 has some sentimental meaning for me. This result, and related results connected to other linear recurrences with constant coefficients, is a product of my earliest research, done around the age of 17 when I was a student at Savannah High School.

I have written my work, not as an essay which is to win the applause of the moment, but as a possession for all time.

It is ridiculous to compare *Enumerative Combinatorics* with *History of the Peloponnesian War*, but I can appreciate the sentiment of Thucydides. I hope this book will bring enjoyment to many future generations of mathematicians and aspiring mathematicians as they are exposed to the beauties and pleasures of enumerative combinatorics.

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