

Index

- absolute temperature, 187
- acceleration, 4-vector, 194
- accessibility, 181, 182
- accumulation point, 106
- action, 152, 274, 524
 - euclidean, 551
 - first variation of, 154
 - group, 454
 - Hamilton's principle of stationary action, 154
 - Jacobi's principle of least action, 281
 - relativistic, 196
- Ad, 486
 - bundle, 487, 489
 - connection, 487
- adiabatic
 - distribution and leaf, 183
 - process, 180
- adjoint, 392, 632
 - group, 486
 - representation, 486
- admissible boundary form, 378
- admittance matrix, 637
- affine
 - connection, 242
 - group of the line $A(1)$, 394
 - parameter, 272
- Aharonov–Bohm effect, 447–8, 554
- Aharonov–Susskind and spinors, 517
- algebra homomorphism, 78
- Ampere–Maxwell law, 121, 163
- annihilator subspace, 167
- anticommutator, 478
- antiderivation, 89, 135
- antisymmetric, 66
- antiquark, 641
- associated bundle, 482
 - connection, 483–7
- Atiyah–Singer index theorem, 465
- atlas, 15
- Bernoulli's theorem, 234
- Berry phase, 468–72
 - equation, 472
- Bertrand–Puisseux and Diguët, 288
- Betti numbers, 157, 346
- Bianchi identities, 300, 489
- bi-invariant
 - connection on a Lie group, 580
 - forms on a Lie group, 561
 - Riemannian metric and their geodesics, 563
- binormal, 196
- Bochner's theorems, 374, 530
- Bonnet's theorem, 229
- boson, 650
- Bott's version of Morse theory, 665, 667
- boundary (of a manifold) = edge, 106
- boundary
 - group, 344
 - homomorphism, 338, 601
 - operator, 335
- boundary conditions
 - essential or imposed, 527
 - natural, 527

- bracket
 - anticommutator, 478
 - commutator, 408
 - Lagrange, 80, 100
 - Lie, 126, 402; of \mathfrak{g} -valued forms, 477
 - Poisson, 154
- Brillouin and the stress form, 627
- Brouwer degree, 210–13, 360
 - fixed point theorem, 217
- bump form, 107
- bundle
 - associated, 482
 - complex line, 433
 - cotangent, 52
 - determinant, 487
 - dual, 482
 - electromagnetic, 441
 - fiber, 415
 - frame, 453
 - gauge, 490
 - line, 433
 - local trivialization, 417
 - monopole, 444, 473
 - normal, 419
 - orientable, 611
 - principal, 454, 481
 - product, 418
 - projection, 415
 - pull back, 619
 - section, 50, 416, 466
 - space, 415
 - structure group, 433, 452
 - tangent, 48
 - transition functions, 24, 254, 414
 - trivial, 418
 - unit tangent, 51
 - vector, 413–17
 - volume, 488
- canonical form, 394
- canonical map, 149
- Caratheodory's
 - formulation of the second law of thermodynamics, 181
 - theorem, 182
- Cartan's
 - bi-invariant forms, 562
 - exterior covariant differential, 250, 430
 - method for computing curvature, 257
 - structural equations, 249
 - theorem $\pi_2(G) = 0$, 606
 - 3-form on a Lie group, 566
- H. Cartan's formula, 135
- Cauchy
 - equations of motion, 618
 - Green tensor, 82
 - Riemann equations, 158, 159
 - stress form, 617; Lie derivative of, 626
- center
 - of a Lie algebra, 580
 - of a Lie group, 565
- centralizer, 659
- chain complex, 628
- chain group, 337
 - integer, 336
 - simplicial, 343
 - singular, 333
- character, 657
- characteristic cohomology class, 616
- charge form, 118
- Chern's
 - forms and classes, 587–91; as obstructions, 608–16
 - integral, 612
 - proof of Gauss–Bonnet–Poincaré, 462–5, 553–7
 - theorem, 615
- Chern–Simons form, 586
- Chern–Weil theorem, 589
- Chow's theorem, 178, 187
- Christoffel symbols, 229
- circulation, 144, 377
- Clairaut's relation, 530
- classical
 - force, 195
 - momentum, 194
 - velocity, 193
- Clifford
 - algebra, 500
 - embedding, 262
 - numbers, 503
- closed
 - form, 156, 158
 - manifold, 120
 - set, 11
- closure, 106
- coboundary, 630
- cochain, 630
- coclosed, 370
- cocycle, 631
- Codazzi equation, 229, 302, 311–13, 320
- codifferential d^* , 364
- codimension, 6
- coefficient group, 337
 - field, 343

- cohomology H^p , 356
 - integral class, 615
- commutative diagram, 338
- commutator bracket of matrices, 408
- compact, 13
- completable relative cycle, 387
- complex
 - analytic map, 158, 214
 - line bundle, 433; connections, 434
 - manifold, 21
- composing rotations, 499
- configuration space, 9, 50
- conformally related metrics, 531
- conjugate point, 327
- conjugates, 659
- connected space, 347
- connection, 242
 - coefficients of, 243, 429
 - curvature of, 244
 - electromagnetic, 440
 - flat, 260
 - forms ω , 249, 256
 - forms ω^* in the frame bundle, 462, 480
 - induced, 309
 - Levi-Civita or Riemannian, 242, 245
 - on a Lie group, 580; flat, 581
 - on a vector bundle, 428–31
 - on the associated Ad bundle, 486
 - Simon, 472
 - spinor, 518–21
 - symmetric, 245
 - torsion of, 245
 - torsion-free, 245
- constraint
 - holonomic, 175
 - nonholonomic, 175
- continuous, 12
- continuum mechanics, 617–27; equilibrium
 - equations, 622
- contractible to a point, 161
- contraction, 89
- contravariant
 - tensor, 59
 - vector, 23
- coordinate
 - change of, 29
 - compatible, 15
 - frame, 243
 - homogeneous, 17
 - inertial, 192
 - local, 3, 4, 13
 - map, 20
 - patch, 20
- coset space G/H , 456
 - fundamental principle, 457
- cotangent space, 40
- coupling constant or charge, 539
- covariance, 430
- covariant
 - components of a tangent vector, 43
 - constant, 267
 - derivative ∇_X , 235, 241–4, 430; second, 301;
 - of a tensor, 298–9
 - differential ∇ , exterior, 248
 - tensor, 58
 - vector = covector, 41
- covector, 41
 - transformation law, 42
- covering space, 569–76
 - associated to a subgroup of π_1 , 575
 - orientable, 573
 - universal, 570; covering group, 575
- critical manifolds, 665
- critical points and values, 28, 382–7
 - homotopically, 382, 387
 - index, 384
 - inessential, 383
 - nondegenerate, 383
- (cross) section, 50, 416, 466
- curl, 93
- current
 - 2-form j , 118
 - 3-form \mathfrak{S} , 199
 - 3-vector \mathbf{J} , 119
 - 4-vector J , 199
 - convective, 119
 - electric, as a chain, 656
- curvature
 - of a connection, 243
 - extrinsic, 318
 - forms θ , 251, 256, 431; and the Ad bundle, 489;
 - of a surface, 257; θ^* on a frame bundle, 462;
 - θ^* on a principal bundle, 481
 - Gauss, 207
 - geodesic, 235
 - intrinsic, 318
 - mean, 207
 - and parallel displacement, 259–61
 - of the Poincaré metric, 258
 - principal, 207
 - Riemann sectional $K(X \wedge Y)$, 313
 - Riemann tensor, 244
 - of a space curve, 191
 - of a surface, 207
 - of a surface of revolution, 258

678

curvature (*continued*)
 total, 215
 transformation $R(X, Y)$, 244
 vector, 192, 194
 cycle
 absolute, 344
 completeable, 387
 group, 344
 relative, 379
 \mathcal{D} , 200
 d'Alembertian \square , 293, 371
 deformation
 retract, 406, 506
 tensor, 82
 theorem, 350
 degree of a map, *see* Brouwer degree
 de Rham's
 theorem, 355–60
 vector space \mathcal{R}^p , 356
 derivation, 134
 derivative
 covariant, 235
 exterior, 73
 intrinsic, 235
 normal, 364
 determinant line bundle, 487
 dictionary relating forms and vectors,
 94
 diffeomorphism, 27
 differentiable, 20
 differential
 exterior d , 73; covariant, 250
 of a function, 40,
 of a map F_* , 7, 27
 differential form, *see* form
 differentiation of integrals, 138–43
 Dirac
 adjoint or conjugate spinor, 532
 algebra, 509
 equation, 503
 Lagrangian, 531
 matrices, 510
 monopole, 444; quantization, 445
 operator, 511, 514, 521; in curved space,
 515–21
 program, 502
 representation ρ , 512
 (4-component) spinor, 513
 string, 162
 Dirichlet's principle, 373
 distance from a point to a hypersurface,
 579

INDEX

distribution (of subspaces), 166
 adiabatic, 183
 horizontal, 263
 integrable, 167
 divergence, 93, 136, 304
 exterior covariant, 545
 of a form, 365
 of a symmetric tensor, 300
 theorem, 139
 dual
 basis, 39
 bundle, 417, 482
 Hodge $*$, 362
 space, 39
 ϵ_J , 67
 eigenvalue of a quadratic form, 63, 209
 eight-fold way, 641
 Einstein
 equations, 296, 316, 317; Wheeler's version,
 318
 geodesic assumption, 292, 297
 tensor G , 315
 electric field \mathbf{E} , 119
 1-form \mathcal{E} , 120
 2-form $*\mathcal{E}$, 121
 and topology, 123, 378, 381
 electromagnetic
 bundle, 441
 connection, 440
 field strength F^2 , 197
 Lagrangian, 308
 stress-energy-momentum tensor, 308
 vector potential 1-form A^1 , 199
 electromagnetism and Maxwell's equations
 in curved space–time, 366–7
 existence and uniqueness, 378, 387
 on projective space, 164
 on the 3-sphere, 163
 on the 3-torus, 122
 embedded submanifold, 27
 energy
 of deformation, 620–22
 density, 316
 hypersurface, 148; invariant form, 150
 internal, 179
 momentum vector, 195
 momentum tensor, 295
 of a path, 274
 rest, 195
 total, 148, 196
 entropy, 183
 empirical, 185

- equations of motion, 144
 - relativistic, 303
 - equilibrium equations, 622–4
 - euclidean metric in quantum fields, 551
 - Euler
 - characteristic, 423, 426
 - equations of fluid flow, 144
 - integrability condition, 166
 - principle of least action, 281
 - exact
 - form, 156
 - sequence, 598–600; homology, 604; homotopy, 600; short, 599
 - exp, 284
 - exponential map for a Lie group, 399, 403
 - extension theorem, 592
 - exterior
 - algebra, 68
 - covariant differential ∇ , 250, 430;
 - of a form section of a vector bundle, 488
 - covariant divergence ∇^* , 545
 - differential d , 73; coordinate expression, 76;
 - spatial \mathbf{d} , 141
 - form, 66; and vector analysis, 71
 - power operation, 588
 - product, 67; and determinants, 71; geometric meaning, 70
 - face, 335
 - Faraday's law, 121
 - Fermat's principle, 297
 - fermion, 650
 - fiber, 49, 415
 - bundle, 451, 594
 - coordinate, 416
 - over p , 416
 - space, 593
 - field strength, 64
 - Flamm paraboloid, 321
 - flow generated by a vector field, 32, 33
 - by invariant fields, 408
 - by Lie bracket, 129
 - straightened, 35
 - fluid flow, 30, 143–5
 - magnetohydrodynamic, 145
 - foliation, 173
 - force
 - classical, 195
 - Lorentz, 119
 - Minkowski, 195
 - form
 - bi-invariant, 561–3
 - Cartan, 562
 - Cauchy stress form, 617
 - closed, 156
 - exact, 156
 - exterior, 66
 - first fundamental, 202
 - harmonic, 370
 - heat 1-form, 179
 - integration of, 95–102; and pull-backs, 102
 - invariant, 395
 - Maurer–Cartan, 476
 - normal, 376
 - and pseudo-form, 122
 - p -form, 41
 - pseudo-, 86
 - pull-back, 77–82
 - second fundamental, 204, 309; and
 - expansion of the universe, 318, 319
 - stress: Cauchy, 617; Piola–Kirchhoff, 619–20
 - tangential, 376
 - of type Ad, 489, 490
 - vector bundle-valued, 429
 - vector-valued, $d\mathbf{r}$ and $d\mathbf{S}$, 203, 248
 - volume, 86, 88
 - with values in a Lie algebra, 475, 477
 - work 1-form, 179
- frame \mathbf{e} , 243
 - change of, 253
 - coordinate, 243
 - orthonormal, 255
 - of sections, 417
 - frame bundle, 453
 - Frobenius
 - chart, 167
 - theorem, 170
 - Frobenius–Schur relations, 656
 - functional derivative, 307
 - fundamental
 - group π_1 , 567–9, 578
 - theorem of algebra, 215
 - vector field, 455
 - \sqrt{g} , 88
 - Galloway's theorem, 578
 - gauge
 - bundle, 490
 - field, 255, 536
 - invariance, 441, 449, 533–6
 - particles: gluons, 540; mesons, 538; photons, 536
 - principle, 537
 - transformation, 255, 490; global, 535

- Gauss
 –Bonnet theorem, 215, 323, 462; as an index theorem, 465; generalized, 465–8
 curvature, 207
 equations, 229, 310, 311–14; relativistic meaning, 316–18
 formula for variation of area, 225
 law, 121
 lemma, 286
 linking or looping integral, 218
 normal map, 208, 215, 260
theorema egregium, 231, 317–18
- Gaussian coordinates, 284
- Gell-Mann
 Gell-Mann matrices, 540, 644
 Gell-Mann/Okuba mass formula, 651
- generalized
 momentum, 55
 velocity, 50
- general linear group $Gl(n)$, 254, 391
- general relativity, 291–322
- geodesic, 233, 271–4
 J. Bernoulli's theorem, 234
 in a bi-invariant metric, 563
 circle, 287
 closed, 281, 284
 completeness, 564
 curvature κ_g , 235, 239
 equation, 235
 null, 303
 polar coordinates, 287
 stability, 324, 326
 submanifold, 310; total, 311
- geodesy, 252
- gluons, 540
- gradient vector, 45
- Grassmann algebra (*see also* exterior algebra)
 manifold, 459
- Green's reciprocity, 639
- Green's theorem, 368
- group
 \mathbb{R} , \mathbb{Z} , \mathbb{Z}_2 , 336
 boundary, 344
 chain, 337
 cycle, 344
 de Rham, 356
 exact sequence, 598
 homology, 345
 homomorphism, 337, 398
- homotopy, 596
 quotient, 345
- \mathfrak{H} , 200
- Haar measure, 397, 541
- Hadamard's lemma, 126
- hairy sphere, 423
- Hamilton, on composing rotations, 499
- Hamilton's
 equations, 147
 principle, 154, 275
- Hamiltonian, 147
 flow, 148
 operator, 439
 relativistic, 196
 vector field, 148
- harmonic cochain, 633
- harmonic field, 376
- harmonic form, 370
 in a bi-invariant metric, 564
- Hawking singularity theorem, 579
- heat 1-form, 179
- helicity, 145
- Helmholtz decomposition, 372
- Hermitian
 adjoint \dagger , 392
 line bundle, 466
- Hessian matrix, 383
- Hilbert
 action principle, 308
 space inner product, 361
 variational approach, 305–8, 368
- Hodge
 * operator, 362
 codifferential d^* , 364
 decomposition, 372, 388
 theorem, 371
 theorem for normal forms, 381
 theorem for tangential forms, 377
- holomorphic, 158
- holonomic constraint, 175
- holonomy, 259
- homeomorphism, 13
- homogeneous space, 458
- homologous, 345
- homology group, 345–55
 relative, 379; sequence, 604
- homomorphism, 337, 398
 algebra, 78
 boundary, 338, 601
 induced, 337

- homotopically critical point, 382
- homotopy, 591
 - and homology, 603
 - covering homotopy, 592
 - free homotopy class, 282, 283
 - sequence for a bundle, 600–3
- homotopy groups π_k , 596–8
 - computation of, 605–8
 - and covering spaces, 605
 - of spheres, 597, 598
- Hopf
 - bundle, 473, 474
 - map and fibering, 606, 667
 - theorem, 427
- Hopf–Rinow theorem, 564
- horizontal distribution, 263–6, 481
- Hurewicz theorem, 603
- hypercharge, 646
- hyperelastic, 622
- hypersurface, 6
 - parallel, 286
 - 1- and 2-sided, 84
- immersion, 169, 173
- implicit function theorem, 5
- incidence matrix, 637
- inclusion map, 79
- index of a vector field (*see also* Kronecker index)
 - of a section, 466
- index theorem, 465
- indicator, 315
- infinitesimal generator, 399
- instanton, 550
 - winding number, 556, 560
- integrability condition, 166, 170, 174
- integrable
 - constraint, 175
 - distribution, 167
- integral
 - curve, 31
 - manifold, 166
- integrating factor, 183
- integration
 - of forms, 96–109
 - over manifolds, 104–9
 - of pseudoforms, 114–17
- interaction, 534
- interior product, 89
- intersection number, 219
- intrinsic, 234
 - derivative, 235
- invariant
 - form, 395
 - vector field, 395
 - volume form, 397
- inverse
 - function theorem, 29
 - image, 12
- involution, 167
- isometry, 230, 314
 - fixed set, 314
 - invariant, 231
- isotopic spin, 640, 646
- isotropic body, 653
- isotropy subgroup, 457
- J , 432
- Jacobi
 - determinant, 5
 - equation of geodesic variation, 273
 - field, 129, 273, 326–9
 - identity, 403
 - metric, 281
 - principle of least action, 281
 - rule for change of variables in an integral, 101
 - variational equation, 128
- Killing field, 528
 - equation, 529
- kinetic term, 535
- Kirchhoff’s current law (KCL), 636
- Kirchhoff’s voltage law (KVL), 636
- Klein bottle, 348
- Klein–Gordon equation, 502
- Kronecker
 - delta, generalized δ^j_i , 67
 - index of a vector field, 216
- Lagrange
 - bracket $\{ , \}$, 80, 100
 - deformation tensor, 82, 621
- Lagrange’s equations, 147
 - in a curved M^3 , 276
 - tensorial nature, 526
 - with electromagnetism, 439
- Lagrangian, 54
 - Dirac, 531
 - electromagnetic, 308
 - for particle in an electromagnetic field, 436–9
 - significance in special relativity, 437

- Lambert's formula, 290
- Lamé moduli, 655
- Laplace's formula for pressure in a bubble, 227
- Laplacian ∇^2 , 93, 305
and mean curvature, 305
- Laplace operator on a cochain, 633
- Laplace operator $\Delta = dd^* + d^*d$ on forms, 368–72
on a 1-form, 370
- leaf of a foliation, 173
maximal, 173
- Levi–Civita
connection, 242
equation, 297
parallel displacement, 237
- Lie algebra \mathfrak{g} , 402
Ad invariant scalar product, 543
- Lie bracket $[\cdot, \cdot]$, 126, 402
- Lie derivative \mathcal{L}_X
of a form, 132–8
of the metric tensor, 620
of the stress form, 626, 627
of a vector field, 125
- Lie group, 391–412
1-parameter subgroup, 398, 405–7, 564; on $SI(2, \mathbb{R})$, 407
compact, 541; averaging over, 541;
bi-invariant forms, 561–7
connection and curvature of, 580
- Lie subgroup and subalgebra, 410–12
- lifting paths, 277
in a bundle, 593
in a covering space, 574
- lifting spheres, 605
- light cone, 193
- lightlike, 193
- linear functional, 38
- linking number, 219
- Liouville's theorem, 148
- local
product, 49
trivialization, 417
- Lorentz
factor, 193
force, 119; covector, 120, 197
group, 504; and spinor representation of $SI(2, \mathbb{C})$, 509
metric, 192
transformation, 46, 198
- magnetic field \mathbf{B} , 119
1-form $\star\mathfrak{B}$, 121
2-form \mathfrak{B} , 120
and topology, 123, 387
- magnetohydrodynamics, 145
- manifold, 13, 19
closed, 120
complex, 21
integral, 166
mechanical, 180
orientable, 83
product, 15
pseudo-Riemannian, 45
Riemannian, 45
symplectic, 146
with boundary, 106
- map
canonical, 149
coordinate, 20
differentiable, 20
exponential, 284, 399
geographical, 230
inclusion, 79
of manifolds: critical points and values, 28;
regular points and values, 28
projection, 415
- matrix group, 394
- Maurer–Cartan
equations, 403, 477
form Ω , 476
- maximal
atlas, 15
torus, 393
- Maxwell's equations, 120–3, 198, 200, 536
on a curved space, 366–7
independence of, 200
on projective space, 164
on a 3-sphere, 163
on a torus, 122
- Mayer–Lie system, 174
- mean curvature, 207, 311, 529
and divergence, 224
- mesons, 538
Yukawa, 540
- metric
conformally related, 531
flat or locally euclidean, 263
Lorentz or Minkowski, 192
potentials, 293
pseudo-Riemannian, 45

- Riemannian, 45
 - spatial, 297
 - static, 292, 296
 - stationary, 291
 - tensor, 43
- minimal submanifold, 311, 528
 - surface, 227, 305
- minimization of arc length, 286
- Minkowski
 - electromagnetic field tensor, 197
 - force, 195
 - metric and space, 46, 192
- Möbius band, 18
- mode
 - normal, 65
 - zero, 465
- momentum
 - canonical, 439
 - classical, 194
 - density, 320, 322
 - 4-vector, 194
 - generalized, 55
 - kinematical, 436
 - operator, 439
- monopole bundle, 444, 473
- Morse
 - deformation, 47
 - equalities, 387, 428
 - index, 328, 384
 - inequalities, 385, 386
 - lacunary principle, 388
 - lemma, 384
 - polynomial, 385
 - theory, 382–8
 - type number, 385, 604
- multilinear, 58
- Myers's theorem, 576–8

- negative normal bundle, 665
- neighborhood, 12
- Noether's theorem, 527–9
- Nomizu's theorem, 530
- normal
 - bundle, 419, 616
 - coordinates, 287, 303
 - derivative, 364
 - map, 208
 - mode, 65
- nucleon
 - Heisenberg, 537
 - Yang–Mills, 538

- obstruction cocycle, 609–12
- one parameter group, 31
- open set, 11, 12
- orientability, 83
 - and curvature, 331
 - and homology, 349
 - and two-sidedness, 84
- orientable
 - bundle, 611, 665
 - manifold, 83
 - transverse, 115
- orientation, 82
 - of the boundary, 110
 - coherent, 341
 - transverse, 115
- orthogonal group, $O(n)$, 9, 392
 - $SO(n)$, 9, 392
- osculating plane, 191

- paper folding, 315
- parallel displacement, 237
 - independence of path, 260
- parallelizable, 252
- parameter, distinguished or affine, 272
- parameterized subset, 97
- partition of unity, 107
 - and Riemannian metrics, 109
- passes peaks and pits, 427
- path ordering, 555
- Pauli
 - algebra, 501
 - matrices, 493
- period of a form, 357
- periodic motion, 282
 - for double pendulum, 284
 - for rigid body, 331
- Pfaffian, 167
- phase, 448, 535
 - space, 55; extended, 151
- physical components, 48, 630
- Piola–Kirchhoff stress forms
 - first, 619
 - second, 619
- Poincaré
 - characteristic, 604
 - duality, 375
 - index theorem, 421–8
 - lemma and converse, 160
 - metric, 239, 258; geodesics, 274, 530
 - 1-form, 56; extended, 151

- Poincaré (*continued*)
 polynomial, 385
 2-form, 80; extended, 151, 437
- Poisson
 bracket (,), 154
 equation, 293, 371
- potential
 of a closed form, 158, 160–4
 global vector, 443, 448
 monopole, 444
 singularities, *see* Dirac string
- Poynting vector, 322
- principal
 bundle, 454, 458, 481
 directions, 207, 310
 normal, 191
 normal curvatures, 207, 310
- principle of least action, 281
- probability amplitude, 447
- projection, 49, 415
 homomorphism, 605
- projective space, 16, 85
 homogeneous coordinates, 17
 $\mathbb{R}P^n$, 16
 $\mathbb{C}P^n$, 22
- proper time, 193, 292
- pseudo-form, 86
 integration of, 114–17
- pseudo-Riemannian, 45
- pull-back
 of covariant tensors, 53, 77,
 79
 in elasticity, 81, 619
 and integration, 102
- pure gauge, 553
- quantization of a gauge field, 536
 topological, 261
- quark, 540
 up, down, and strange flavored, 641
- quasi-static, 179
- quaternion, 502
- quotient group, 345
- radius of curvature, 192, 221
- rate of deformation tensor,
 624–6
- regular points and values, 28
- relative
 boundaries, cycles, and homology groups,
 379–81
 homology sequence, 604
- relativistic equations of motion, 303
 mass, 194
- reparameterization, 101
- representation, 481
 adjoint, Ad, 486
 dual, 482
 irreducible, 654
 of a group, 481, 482
 reducible, 643
 tensor product, 482
- residue of a form, 159
- rest mass, 194
- retraction, 217
- Ricci
 curvature, 315, 374, 577
 identities 302
 tensor R_{ij} , 295
- Riemann
 –Christoffel curvature tensor, 229
 sectional curvature $K(X \wedge Y)$, 313–14
 sphere, 21
 theorem, 266
- Riemannian
 manifold and metric, 45; bi-invariant, 563; on
 a surface of revolution, 258
 connection, 242
- rigid body, 9, 331
- rotation group $SO(n)$, 392, 492
- Sard’s theorem, 29
- scalar curvature R , 296
- scalar product, 42
 global, 361
 of Hermitian matrices, 494
 nondegenerate, 42
- Schrödinger’s equation, 439
 in curved space, 442
 with an electromagnetic field, 440,
 443
- Schur’s lemma and corollary, 654, 655
- Schwarz’s formula, 228
- Schwarzschild solution, 320–2
 spatial metric, 298
- section, 50, 416, 466
 holomorphic, 467
 p -form section of a vector bundle,
 488
- sectional curvature, 313
- self adjoint, 205, 317
- self (anti) dual field, 549
- Serret–Frenet formulas, 196, 431
- Simon connection, 472

- simplex, 333
 - boundary, 335
 - face, 335
 - ordered, 335
 - orientation, 336
 - singular, 334
 - standard, 333
- simplicial complex, 343
- simply connected, 283, 329, 595
- singularity of a vector field, 422
- skeleton, 610
- smooth, 7
- soap bubbles and films, 226–8
- spacelike, 193
- space–time notation, 141
- spatial slice, 316
- special, 392
 - linear group, $Sl(n)$, 11, 392
 - orthogonal group $SO(n)$, 392
 - unitary group $SU(n)$, 392
- sphere lifting theorem, 605
- spin structure, 515–18
- spinor
 - adjoint, 532
 - bundle $\mathbb{S}M$, 517
 - connection, 518–21
 - cospinor, 513
 - Dirac or 4-component, 513
 - group $Spin(3)$, 497
 - “representation” of $SO(3)$, 497
 - “representation” of the Lorentz group, 509
 - 2-component, 497; left- and right-handed, 513
- stability, 324; subgroup, 457
- Stiefel
 - manifold, 459, 616
 - vector field, 426
- Stokes’s theorem, 111–14
 - generalized, 155
 - for pseudoforms, 117
- stored energy of deformation, 621
- strain energy, 652
- stress–energy–momentum tensor T_{ij} , 295
- stress forms
 - Cauchy, 617
 - first Piola–Kirchhoff, 619
 - second Piola–Kirchhoff, 619
- stress tensor, 295, 618
- structure constants, 402
 - in a bi-invariant metric, 566
- structure group of a bundle, 433, 452
 - reduction, 433
 - $SU(2) * U(1)$, 649
- $SU(n)$, 392, 493–7
- subalgebra, 411
- subgroup, 411
 - isotropy = little = stability, 457
- submanifold, 26
 - embedded, 27
 - framed, 115
 - immersed, 169
 - of M^n , 29
 - of \mathbb{R}^n , 4, 8
 - 1- and 2-sided, 84
 - with transverse orientation, 115
- submersion, 181
- summation convention, 59
- support, 107
- symmetries, 527–31
- symplectic
 - form, 146
 - manifold, 146
- Synge’s
 - formula, 325
 - theorem, 329
- tangent
 - bundle, 48; unit, 51
 - space, 7, 25
 - vector, 23
- Tellegen’s theorem, 638
- tensor
 - analysis, 298–303
 - Cauchy–Green, 82
 - contravariant, 59
 - covariant, 58
 - deformation, 82, 621
 - metric, 58
 - mixed, 60; linear transformation, 61
 - product, 59, 66; representation, 482
 - rate of deformation, 624
 - transformation law, 62
- theorema egregium*, 231
- thermodynamics
 - first law, 180
 - second law according to Lord Kelvin, 181; according to Caratheodory, 181
- Thom’s theorem, 349
- timelike, 193
- topological
 - invariants, 346
 - quantization, 468
- topological space, 12
 - compact, 13
- topology, 12
 - induced or subspace, 12

- torsion
 of a connection, 245; 2-form, 249
 of a space curve, 196
- torus, 16
 maximal, 393
- transformation group, 456
- transition matrix c_{UV} , 24, 254, 414
 for the cotangent bundle, 417
 for dual bundles, 417
 for tangent bundle, 417
 for tensor product bundle, 417
- transitive, 456
- translation (left and right), 393
- transversal to a submanifold, 34
- transverse orientation, 115
- triangulation, 346
- tunneling, 558
- twisted product, 415
- unitary group $U(n)$, 392
- universe
 static, 292
 stationary, 291
- vacuum state, 557, 558
 tunneling, 558
- variation
 of action, 154
 external, 523
 first, of arc length, 232; of area, 221, 322
 internal, 523
 of a map, 153
 of Ricci tensor, 306
 second, of arc length, 324–32
- variational
 derivative δ , 307, 526
 equation, 128
 principles of mechanics, 275–81
 vector, 128, 153, 272
- vector
 analysis, 92, 136–8
 bundle, 413–19; ν -valued form, 488
 contravariant or tangent, 23
 coordinate, 25
 covariant = covector = 1-form, 41
 as differential operator, 25
 field, 25; flow (1-parameter group) generated
 by, 32, 33; integral curve of, 31; along a
 submanifold, 269
 gradient, 45
- integral, 144, 308
- invariant, 395
- Killing, 528
- product, 92, 94, 103
- transformation law, 34
- ν -valued form, 248
- variational, 128, 153, 272
- velocity 4-vector, 193
- velocity field, 31
- virtual displacement, 276
- voltage as a cochain, 636
- volume
 bundle, 488
 form, 86, 88
 invariant: in mechanics, 148; on the energy
 hypersurface, 150; on the unit hyperboloid,
 200; on a Lie group, 397, 541; on $Sl(2, \mathbb{R})$,
 398
- vorticity, 145
- wedge product, *see* exterior product
- weight diagram, 647
- Weingarten equations, 204
- Weizenböck formulas, 370
- Weyl's
 equation for neutrinos, 515
 method of orthogonal projection, 639
 principle of gauge invariance, 441
 theorem on the fundamental group of a Lie
 group, 565, 581
- Whitney embedding theorem, 23
- winding number
 of a curve, 212
 of a Yang–Mills instanton, 560; in terms of
 field strength, 585–7
 of a Yang–Mills vacuum, 560
- work 1-form in thermodynamics, 179
- world line, 193
- wormhole, 446
- Yang–Mills
 action, 544
 analogy with electromagnetism, 547, 548, 550
 equations, 545
 field strength, 539
 instanton, 550; winding number, 560, 585
- Yukawa–Kemmer, 650
- \mathbb{Z}_2 , 336
- zero modes, 465