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978-1-107-60101-7 - Encyclopedia of Mathematics and its Applications: Convex Bodies:
The Brunn–Minkowski Theory: Second Expanded Edition

Rolf Schneider

Table of Contents

[More information](#)

Contents

	<i>Preface to the second edition</i>	<i>page</i> ix
	<i>Preface to the first edition</i>	xi
	<i>General hints to the literature</i>	xv
	<i>Conventions and notation</i>	xix
1	Basic convexity	1
	1.1 Convex sets and combinations	1
	1.2 The metric projection	9
	1.3 Support and separation	11
	1.4 Extremal representations	15
	1.5 Convex functions	19
	1.6 Duality	32
	1.7 Functions representing convex sets	44
	1.8 The Hausdorff metric	60
2	Boundary structure	74
	2.1 Facial structure	74
	2.2 Singularities	81
	2.3 Segments in the boundary	90
	2.4 Polytopes	104
	2.5 Higher regularity and curvature	112
	2.6 Generalized curvatures	126
	2.7 Generic boundary structure	132
3	Minkowski addition	139
	3.1 Minkowski addition and subtraction	139
	3.2 Summands and decomposition	156
	3.3 Additive maps	172
	3.4 Approximation and addition	183
	3.5 Minkowski classes and additive generation	189
4	Support measures and intrinsic volumes	208
	4.1 Local parallel sets	208

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Rolf Schneider

Table of Contents

[More information](#)

vi	<i>Contents</i>	
	4.2 Steiner formula and support measures	211
	4.3 Extensions of support measures	228
	4.4 Integral-geometric formulae	236
	4.5 Local behaviour of curvature and area measures	265
5	Mixed volumes and related concepts	275
	5.1 Mixed volumes and mixed area measures	275
	5.2 Extensions of mixed volumes	290
	5.3 Special formulae for mixed volumes	295
	5.4 Moment vectors, curvature centroids, Minkowski tensors	312
	5.5 Mixed discriminants	322
6	Valuations on convex bodies	329
	6.1 Basic facts and examples	329
	6.2 Extensions	332
	6.3 Polynomiality	340
	6.4 Translation invariant, continuous valuations	346
	6.5 The modern theory of valuations	365
7	Inequalities for mixed volumes	369
	7.1 The Brunn–Minkowski theorem	369
	7.2 The Minkowski and isoperimetric inequalities	381
	7.3 The Aleksandrov–Fenchel inequality	393
	7.4 Consequences and improvements	399
	7.5 Wulff shapes	410
	7.6 Equality cases and stability	418
	7.7 Linear inequalities	440
8	Determination by area measures and curvatures	447
	8.1 Uniqueness results	447
	8.2 Convex bodies with given surface area measures	455
	8.3 The area measure of order one	464
	8.4 The intermediate area measures	473
	8.5 Stability and further uniqueness results	478
9	Extensions and analogues of the Brunn–Minkowski theory	489
	9.1 The L_p Brunn–Minkowski theory	489
	9.2 The L_p Minkowski problem and generalizations	498
	9.3 The dual Brunn–Minkowski theory	507
	9.4 Further combinations and functionals	512
	9.5 Log-concave functions and generalizations	516
	9.6 A glimpse of other ramifications	525
10	Affine constructions and inequalities	528
	10.1 Covariogram and difference body	528
	10.2 Qualitative characterizations of ellipsoids	535
	10.3 Steiner symmetrization	536

Cambridge University Press

978-1-107-60101-7 - Encyclopedia of Mathematics and its Applications: Convex Bodies:
The Brunn–Minkowski Theory: Second Expanded Edition

Rolf Schneider

Table of Contents

[More information](#)

<i>Contents</i>		vii
10.4	Shadow systems	541
10.5	Curvature images and affine surface areas	543
10.6	Floating bodies and similar constructions	560
10.7	The volume product	563
10.8	Moment bodies and centroid bodies	566
10.9	Projection bodies	569
10.10	Intersection bodies	580
10.11	Volume comparison	583
10.12	Associated ellipsoids	587
10.13	Isotropic measures, special positions, reverse inequalities	595
10.14	L_p zonoids	606
10.15	From geometric to analytic inequalities	610
10.16	Characterization theorems	614
Appendix	Spherical harmonics	623
	<i>References</i>	629
	<i>Notation index</i>	715
	<i>Author index</i>	719
	<i>Subject index</i>	729