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978-1-107-60100-0 - Fusion Systems in Algebra and Topology

Michael Aschbacher, Radha Kessar and Bob Oliver

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