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978-1-107-59345-9 - Theoretical Mechanics: An Introductory Treatise on the Principles of Dynamics

A. E. H. Love

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AN INTRODUCTORY TREATISE

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PRINCIPLES OF DYNAMICS

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THEORETICAL MECHANICS

AN INTRODUCTORY TREATISE

ON THE

PRINCIPLES OF DYNAMICS

WITH APPLICATIONS AND NUMEROUS EXAMPLES

BY

A. E. H. LOVE, M.A., D.Sc., F.R.S.

SEDLEIAN PROFESSOR OF NATURAL PHILOSOPHY IN THE UNIVERSITY OF OXFORD.

HONORARY FELLOW OF QUEEN'S COLLEGE, OXFORD. FORMERLY FELLOW

AND LECTURER OF ST JOHN'S COLLEGE, CAMBRIDGE

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PREFACE

THE book, of which this is the third edition, is intended as a text-book of Dynamics, for the use of students who have some acquaintance with the methods of the Differential and Integral Calculus. Its scope includes the subjects usually described as Elementary Dynamics, Dynamics of a Particle, and Dynamics of a Rigid Body moving in two dimensions. It also includes an attempt to trace the logical development of the theory.

Within the chosen range of subject-matter there are many topics, such as the oscillation of a pendulum through wide angles, which it would be inappropriate to omit although they are difficult for a beginner. Articles dealing with such topics are marked with an asterisk. A student reading the subject for the first time, and without the guidance of a teacher, is advised to confine his attention to the unmarked Articles and the unmarked collections of Examples inserted in the text. Many of these Examples are well-known theorems, and some of them are referred to in subsequent demonstrations. Collections of miscellaneous Examples are appended to most of the Chapters. It is hoped that these will be useful to teachers and to students engaged in revising their work. A few of them, which I have not found in Examination papers, are taken from the well-known collections of Besant, Routh, and Wolstenholme.

The works which were most useful to me in regard to matters of theory, when I was writing the first edition of this book, were Kirchhoff's *Vorlesungen über mathematische Physik (Mechanik)*, Pearson's *Grammar of Science*, and Mach's *Science of Mechanics*. The last ought to be in the hands of all students who wish to follow the history of dynamical ideas. In regard to methods for the treatment of particular questions, I am conscious of a deep obligation to the teaching of Mr R. R. Webb.

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PREFACE

For the second edition the book was largely re-written and entirely re-arranged. In the present edition few changes have been made in the text. New Articles, which have been added, are marked with the letter A, thus “102 A.” The number of miscellaneous Examples has been reduced considerably, but it is hoped that the most interesting have been retained, and that these are still sufficiently numerous.

A. E. H. LOVE.

OXFORD.

April, 1921.

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