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Teaching Secondary Mathematics

Technology plays a crucial role in contemporary mathematics education. *Teaching Secondary Mathematics* covers major contemporary issues in mathematics education, as well as how to teach key mathematics concepts from the Australian Curriculum: Mathematics. It integrates digital resources via Cambridge HOTmaths (www.hotmaths.com.au), a popular, award-winning online tool with engaging multimedia that help students and teachers learn and teach mathematical concepts. This book comes with a free 12-month subscription to Cambridge HOTmaths.

Each chapter is written by an expert in the field, and features learning objectives, definitions of key terms, and classroom activities – including HOTmaths activities and reflective questions.

Teaching Secondary Mathematics is a valuable resource for pre-service teachers who wish to integrate contemporary technology into teaching key mathematical concepts and engage students in the learning of mathematics.

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Teaching Secondary Mathematics

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Preface

Teaching Secondary Mathematics has been written to enrich the professional lives of secondary mathematics educators at any stage of their careers. Throughout the text the audience has access to key theoretical and philosophical perspectives that have been the focus of much empirical research and that underpin best instructional practices within the secondary mathematics classroom. Additionally, various contemporary issues influencing the planning, teaching and evaluation of mathematical learning experiences have been included for reader consideration.

Within each of the 12 chapters, the needs of adolescent learners have been outlined carefully, together with pedagogical approaches educators can use to respond effectively to these needs. At the conclusion of each chapter, readers are provided with various activities to consolidate and extend their learning. First, reflective questions that link closely to the topics presented are offered for individual or collaborative response. Second, a variety of engaging mathematics activities – which are strategically linked to Cambridge HOTmaths – provides teachers of middle and senior school students with resources to use immediately in the classroom. At the time of publication, *Teaching Secondary Mathematics* reflects the thinking and content available in the most current version of the Australian Curriculum.

The authors of the chapters are notable academics hailing from a broad range of Australian universities, and who in their current roles prepare the next generation of mathematics teachers in Australia. Their professional classroom experience and engagement with cutting-edge research combine to produce a clear 'mathematical voice' which resonates in the summaries, insights, teachings, questions and reflections presented. Although the text has been authored primarily for an Australian audience, it is hoped that this voice will inspire and challenge mathematics teachers everywhere for years to come.

> Gregory Hine January 2016

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How to use HOTmaths with this book

Once you have registered your HOTmaths access code (see the inside front cover of this book), for subsequent visits the below navigation instructions provide a general overview of the main HOTmaths features used within this textbook. Log in to your account via at www.hotmaths.com.au.

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This will take you to your Dashboard. The different HOTmaths streams can be accessed via the Course dropdown, located on the left-hand side of the toolbar. Here you can change the **Course list** (stream) and **Course** (year level), then select a **Topic** and **Lesson** from the respective lists.

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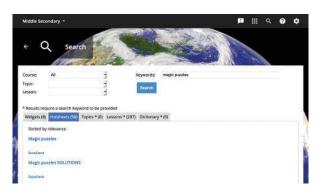
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Most lessons contain a number of interactive and printable activities, which can be accessed via the links on the right-hand side of the orange lesson toolbar. These include: **Resources**, **Walkthroughs** and **Questions**. The Resources tab contains the **demonstrations** (videos), **widgets** (animations) and **HOTsheets** (activities) associated with that particular lesson.



You can also access specific widgets and HOTsheets quickly using the search function, found on the right-hand side of the main toolbar. Upon searching the name of any widget or HOTsheet, the results page will display widgets based on the keywords searched – as denoted by the 'widget' tab being highlighted in blue. If you are looking for a HOTsheet, simply click onto the 'Hotsheets' tab and these results will appear. Using the 'Magic puzzles' HOTsheet as an example, searching this name and clicking on the 'HOTsheets' results tab will provide you a link to the Magic puzzles HOTsheet.



Please note that given its nature HOTmaths is constantly being updated. All pathways and references are up to date as of May 2016 and every effort has been made to provide you with an accurate picture of the functions within HOTmaths.

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