Part 1

Contemporary Issues in Learning and Teaching Mathematics

CHAPTER 1

Introduction: the learning and teaching of mathematics

Learning outcomes

After studying this chapter, you should be able to:

- discuss the importance of mathematics in modern society, including fundamental aspects of mathematical literacy and numeracy
- describe the Proficiency Strands of the Australian Curriculum: Mathematics
- summarise the key features of some important theories about student learning
- identify some ways in which teachers can promote student engagement in mathematics lessons.

Introduction

The purpose of this chapter is to consider the nature of mathematics, its importance in the lives of twenty-first century students, and to discuss some key theories about how learning takes place. The chapter begins with a description of some of the key knowledge, skills and understandings that today's students need in order to participate fully in modern society. Next, we discuss some of the significant developments in theories about how students learn mathematics and some implications of these theoretical approaches for your work as a teacher. The chapter concludes with a look at how we might engage students more productively in mathematics lessons in order to cultivate positive attitudes to the subject among students and to improve their learning outcomes.

At various points in the chapter we shall discuss some relevant scholarship and research that may provide guidance for your work of teaching mathematics in secondary classrooms. We will also explore some of the HOTmaths resources (www.hotmaths.com.au) that you could use in your lessons and you will have an opportunity to reflect on their significance.

KEY TERMS

- **Behaviourism:** a theory of learning that is mainly concerned with observable behaviour and assumes that learners respond passively to external stimuli.
- **Constructivism:** a cognitivist theory of learning that views the learner as an active constructor of knowledge.
- **Engagement:** the level of attentiveness, interest and curiosity that students exhibit when they are learning.
- **Financial literacy:** the ability to manage money and financial risks effectively and responsibly in order to achieve one's financial goals.
- **Gestaltism:** a theory that suggests that the mind processes wholes first, before attending to constituent parts.
- **Numeracy:** the ability to understand and work flexibly and efficiently with numbers and other mathematical concepts to solve a variety of problems across a range of contexts.
- **Socioculturalism:** a view of learning that emphasises how learning does not take place solely within the mind of the individual, but also through one's interactions with others.
- **Statistical literacy:** the ability to understand, interpret and evaluate data from a variety of sources.

Mathematics for the twenty-first century

The Melbourne Declaration on Educational Goals for Young Australians (Ministerial Council on Education, Employment, Training and Youth Affairs, MCEETYA, 2008a) is the framework document that will effectively set the national school education agenda for the coming decade. One of the central themes in the Declaration is the need to improve the educational outcomes for all students since this is 'central to the nation's social and economic prosperity and will position young people to live fulfilling, productive and responsible lives' (MCEETYA , 2008a, p. 7).

We live in a world of rapid change and development, especially in terms of advances in technology. As information and communication technologies become more sophisticated, they exert an ever-increasing influence on how we conduct our daily lives. Many of these technological tools, such as databases and spreadsheets, are underpinned by mathematics (Siemon et al., 2011). Mathematical understanding is crucial in laying a strong foundation for study beyond secondary school in a range of disciplines, such as engineering, business and finance. In addition, developing mathematical understanding and becoming more numerate helps students to interpret a range of practical situations, allowing them to make more informed decisions in their everyday lives. For example, reading timetables, interpreting graphs, converting fractional amounts when cooking, and so on.

The Shape of the Australian Curriculum: Mathematics (National Curriculum Board, 2009) also notes the need for a mathematically literate workforce:

> Successful mathematics learning also provides a workforce that is appropriately educated in mathematics to contribute productively in an ever-changing global economy, with both rapid revolutions in technology and global and local social challenges. An economy competing globally requires substantial numbers of proficient workers able to learn, adapt, create, interpret and analyse mathematical information. (2009, p. 4)

Mathematics education plays a crucial role in driving national productivity and prosperity. School mathematics lays a foundation on which to build the mathematical and scientific literacy every citizen needs to thrive in an increasingly technology-dependent world. But what kinds of mathematical knowledge, skills and understandings do today's students require if they are to be full and active members of our modern, highly technological age? To answer this question we first need to consider the very nature of mathematics and what it means to do mathematics.

The Dutch mathematician Hans Freudenthal developed an approach to teaching known as realistic mathematics education, which he based on learning mathematics by solving well-chosen and carefully sequenced problems taken from daily life. Freudenthal believed that by attempting to solve such problems, students would gradually develop and enrich their mathematical understanding. Although he died in 1990, his work has continued to be very influential in mathematics education reform, not only in Holland, but throughout the world. He once described mathematics as:

> an activity of solving problems, of looking for problems, but it is also an activity of organizing a subject matter. This can be a matter from reality which has to be organized according to mathematical patterns if problems from reality have

> to be solved. It can also be a mathematical matter, new or old results, of your own or others, which have to be organized according to new ideas, to be better understood, in a broader context, or by an axiomatic approach. (Freudenthal, 1971, pp. 413–14)

As you read this quotation, and perhaps you need to do so more than once, note how Freudenthal begins with the many different kinds of activities people do in their everyday lives that involve mathematics. He also refers to more abstract mathematics, but in doing so he emphasises mathematics as a *process* of inquiry, particularly through his reference to finding and solving problems, taking a systematic approach in organising one's thinking about these problems, and making generalisations from their solutions to develop mathematical understanding. Another important feature of Freudenthal's description is the underlying assumption that mathematics is an essentially human endeavour because it is always contextualised as a social and cultural activity. From Freudenthal's point of view, mathematics education has its beginnings in the socio-cultural world of each student and the essential aim of school mathematics is to strive to raise students' awareness of the essentially mathematical nature of this everyday human activity (Ryan & Williams, 2007).

If we take Freudenthal's description as our starting point, we can begin to identify some of the particular mathematical knowledge and skills that are important for today's students. These are discussed in the following sections.

Statistical literacy

Data are all around us and twenty-first century students need considerable skill in order to comprehend the large quantity of information they are presented with each day through the media and other sources. Indeed, to be an active participant in our democratic society requires the ability to deal with ever increasing amounts of data. Many occupations, such as actuaries, business analysts and meteorologists, also require an ever greater appreciation of data in far more sophisticated ways than has previously been the case. These jobs require a sound grasp of statistics – a powerful tool that enables us to make sense of our data-driven world. We can use statistics to identify trends over time and make meaningful comparisons among and between data sets. We can also extrapolate from data in order to hypothesise and predict what might occur in the future for the purposes of planning and decision-making.

Every day, people use numbers and comparisons of numbers to justify decisions they make about their lives. Statistics is a way of making sense of a number of factors or costs when comparing these activities and providing

evidence to support our choices. In **statistical literacy**, students learn how to add rigour to numerical claims and comparisons and how to examine other people's claims using statistical evidence and reasoning. Statistical literacy is the ability to make sense of data; it is one's ability to understand, interpret and evaluate data from a variety of sources. Statistical literacy includes the ability to know where to look and how to find data appropriate to your specific needs and purposes. Being statically literate means understanding and making sense of data when it is presented to you. It involves familiarity with statistical terminology in order to communicate your understanding clearly to others. And statistical literacy is also about adopting a critical stance to deal with data that may be displayed in an ambiguous or misleading fashion (Gal, 2002). As with many other areas of mathematics, we would also want our students to develop positive attitudes and confidence in their statistical abilities so that they can be informed and engaged participants in society.

Some of the activities that could be used to promote statistical literacy include the development of surveys and questionnaires and a discussion about how the wording and ordering of the items can influence responses. Students could consider if they need to survey the entire population (e.g. all of the students in a school) or if a sample would be sufficient. This could lead to discussions about the sample size and the need for the sample to be representative of the population. Raw data often needs to be 'cleaned up' by, for example, disregarding anomalous results. In doing so, students could consider the impact of outliers on measures such as the mean of a set of scores. When the data are collected, decisions need to be made about how it could be represented (in tabular form or graphically). This could lead to discussion about the kinds of graphs that are best suited to representing data that are categorical, numerically discrete or continuous. Then the data need to be analysed so that trends or other patterns can be identified. When dealing with data from other sources, students need to develop a healthy scepticism for claims made by considering how the data were collected and by whom, and for what purpose. This could lead to a discussion of biased or misleading data.

Financial literacy

Another important aspect of mathematics in the twenty-first century is the area of **financial literacy**. Financial literacy is significant because the ability to manage personal finances 'is a core skill in today's world. It affects quality of life, the opportunities individuals and families can pursue, their sense

of security and the overall economic health of Australian society' (National Consumer and Financial Literacy Framework, 2011, p. 5). A recent report by the Australian Securities & Investments Commission (ASIC) defines financial literacy as 'being able to understand and negotiate the financial landscape, manage money and financial risks effectively and responsibly, and pursue and attain financial and lifestyle goals' (ASIC, 2014, p. 6). Financial literacy is therefore seen as a core life skill for twenty-first century students so that they can fully participate in modern society. The increasingly complex nature of the world today and the greater number of alternatives for dealing with discretionary income mean that students need to develop the knowledge and skills to take charge of their own financial future. Helping students understand financial issues is particularly important nowadays since they can be expected to deal with more and more sophisticated financial products and services. Levels of financial risk are also rising, especially in areas such as managing savings and investments, purchasing a home, planning for retirement and ensuring the ability to pay for future healthcare needs.

However, surveys and assessment data show that young adults typically have very low levels of financial literacy. For example, in 2012 the Programme for International Student Assessment (PISA) conducted the first financial literacy assessment of approximately 29 000 secondary school students aged 15 years. The survey was conducted in 18 countries and covered financial issues such as understanding a bank statement, calculating the long-term cost of a loan and knowing how insurance works. The results, which were released in July 2014, showed no significant difference between the performance of boys and girls. But they did indicate that students from relatively high socio-economic backgrounds tended to do better in financial literacy than less advantaged students. Importantly, the results also revealed that skills in mathematics and reading were very closely related to financial literacy.

The National Consumer and Financial Literacy Framework (2011, p. 5) sets out some of the key aspects of financial literacy that students should have the opportunities to learn. These include:

- how to manage their finances and plan for needs and wants, now and into the future
- the language of money, how to navigate the ever-changing consumer and financial landscape and where to go to for assistance
- the rights and responsibilities of consumers in modern society and the wider impact of everyday consumer and financial decisions
- the skills needed to develop a range of enterprising behaviours.

The range of skills that students need to develop in order to engage in the kinds of enterprising behaviours envisaged in the Framework document include adaptability, initiative, communication, problem solving, planning and organising, analysing issues and managing identified risks. Many of these skills are closely aligned with some of the central themes of the Australian Curriculum: Mathematics (Australian Curriculum, Assessment and Reporting Authority, 2011). This is especially the case with the working mathematically processes, which we discuss later in the chapter.

Numeracy

Today's students must achieve basic **numeracy** skills such as the ability to count, measure, compare and sequence. Students need to develop competence in many everyday tasks, such as budgeting, shopping, travel and leisure activities, and so on. In some respects, these skills can be learned and practised through simple exercises such as those typically found in school textbooks. However, most mathematics educators argue that to be fully numerate, students must do more than acquire and master basic mathematical routines and algorithms. These skills are necessary and essential, but not sufficient for numeracy (Australian Association of Mathematics Teachers, 1997).

The numeracy demands placed on twenty-first century learners cover much more than just number skills and the ability to recall basic facts such as the multiplication table. Numeracy involves a deep understanding of mathematical concepts and skills from across the discipline, including numerical, spatial, graphical, statistical and algebraic topics. Students need to develop their mathematical thinking strategies as well as more general thinking skills through problem-solving activities which are grounded in realistic contexts. Perso (2011) identifies the following as fundamental characteristics of students who are numerate:

- knowing how to read situations and determine if or what mathematics is needed
- knowing how to choose the methods and tools they will use and explain and justify their reasons based on context
- knowing how to apply the methods, models and tools they have chosen
- knowing how to critique their own (and others') methods and determine whether they make sense in context
- knowing how to communicate their process and results appropriately depending on purpose and audience.

The report of the National Numeracy Review (Commonwealth of Australia, 2008) reinforces the need to view numeracy more broadly than just the acquisition of basic numerical skills:

> Students need to learn mathematics in ways that enable them to recognise when mathematics might help to interpret information or solve practical problems, apply their knowledge appropriately in contexts where they will have to use mathematical reasoning processes, choose mathematics that makes sense in the circumstances, make assumptions, resolve ambiguity and judge what is reasonable. (2008, p. xi)

In other words, students today must not only develop disciplinary knowledge of basic mathematical processes and skills, they also require a deep conceptual understanding as well. That will enable them to engage in problem solving and critical analysis, apply their knowledge creatively in unfamiliar contexts, and communicate their ideas using appropriate mathematical language and symbols (Office of the Chief Scientist, 2014).

ACTIVITY 1.1

An example of a HOTmaths activity that could be used to develop students' numeracy skills is the 'Exploring discounts' HOTsheet. You can access this HOTsheet by logging in to HOTmaths and selecting the 'HOTmaths Global' Course list and the 'Early Secondary' Course. From the Topic list choose 'Using percentages', then the 'Discounts' Lesson. The HOTsheet can be found in the Resources tab.

The activity is designed to help students learn about mental computation methods for calculating percentage discounts and is linked to the Australian Curriculum: Mathematics learning outcome ACMNA187 (Solve problems involving the use of percentages, including percentage increases and decreases, with and without digital technologies).

The advantages of the HOTmaths activity are its clear links to real-life contexts that are familiar to students (calculating discounts on items they wish to purchase) and the emphasis on mental computation (e.g. calculating the amount of the discount using multiples of 10%). Students are also encouraged to make an estimate before calculating and must develop their own strategies to solve a range of problems. At the conclusion of the activity, the teacher can lead a discussion in which the students share their estimation techniques and compare the relative merits of each approach in order to ascertain the most efficient mental computation methods.

HOTsheet

EXPLORING DISCOUNTS

TASK 1 **Use 10%**

10% is the same as one tenth. Use 10% to complete these calculations.

1 Find 30% discount on $90. 30% = 3 × 10% Discount = 3 × 10% of $90 = 3 × $9 =	**2** Find 40% discount on $500. 40% = 50% – 10% 50% of $500 = 10% of $500 = Discount = =
3 Find 15% discount on $120. 5% is half of 10%. 10% of $120 = 5% of $120 = Discount =	**4** Find 60% discount on $8.

Figure 1.1 Exploring discounts HOTsheet

REFLECTIVE QUESTIONS

Read over the 'Exploring discounts' HOTsheet (see Activity 1.1) and then consider the following:

- Why is it important to contextualise the activity using relevant examples and exercises?
- Why is it important to emphasise mental computation and estimation?
- How does the activity promote numeracy?

Mathematical proficiency and the Australian Curriculum

Another way to think of the kinds of mathematical thinking strategies that twenty-first century learners need is found in the work of Kilpatrick, Swafford and Findell (2001). The authors describe five components or *strands* of