Constructing Dynamic Triangles Together

Rational thinking as exemplified in mathematical cognition is immensely important in the modern world. This book documents how a group of three eighth-grade girls developed specific group practices typical of such thinking in an online educational experience. A longitudinal case study tracks the team during eight hour-long sessions, following the students' meaning-making processes through their mutual chat responses preserved in computer logs coordinated with their geometric actions. The examination of data focuses on key areas of the team's development: its effective team *collaboration*, its productive mathematical *discourse*, its enacted use of dynamic-geometry *tools*, and its ability to identify and construct dynamic-geometry *dependencies*. This detailed study of group cognition serves as a paradigmatic example of computer-supported collaborative learning, incorporating a unique model of human-computer interaction analysis applied to the use of innovative educational technology. A valuable resource for researchers, instructors, and students alike, it offers concrete suggestions for improving educational practice.

Gerry Stahl is Professor Emeritus at Drexel University's College of Computing and Informatics. He is the founding editor of the *International Journal of Computer-Supported Collaborative Learning*, and his publications include *Group Cognition: Computer Support for Building Collaborative Knowledge*, *Translating Euclid: Designing a Human-Centered Mathematics*, and *Studying Virtual Math Teams*.

LEARNING IN DOING: SOCIAL, COGNITIVE, AND COMPUTATIONAL PERSPECTIVES

BOOKS IN THE SERIES

The Construction Zone: Working for Cognitive Change in School
Denis Newman, Peg Griffin, and Michael Cole

Situated Learning: Legitimate Peripheral Participation
Jean Lave and Etienne Wenger

Street Mathematics and School Mathematics
Terezinha Nunes, David William Carraher, and Analucia Dias Schliemann

Understanding Practice: Perspectives on Activity and Context
Seth Chaiklin and Jean Lave, Editors

Distributed Cognitions: Psychological and Educational Considerations
Gavriel Salomon, Editor

The Computer as Medium
Peter Bøgh Anderson, Berit Holmqvist, and Jens F. Jensen, Editors

Sociocultural Studies of Mind
James V. Wertsch, Pablo del Rio, and Amelia Alvarez, Editors

Sociocultural Psychology: Theory and Practice of Doing and Knowing
Laura Martin, Katherine Nelson, and Ethel Tobach, Editors

Mind and Social Practice: Selected Writings of Sylvia Scribner
Ethel Tobach et al., Editors

(*continued after index*)

Constructing Dynamic Triangles Together

The Development of Mathematical Group Cognition

GERRY STAHL
Drexel University

CAMBRIDGE UNIVERSITY PRESS

Shaftesbury Road, Cambridge CB2 8EA, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India

103 Penang Road, #05–06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of Cambridge University Press & Assessment, a department of the University of Cambridge.

We share the University's mission to contribute to society through the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781107566873

First published 2016
First paperback edition 2025

A catalogue record for this publication is available from the British Library

Library of Congress Cataloging-in-Publication data
Stahl, Gerry.
Constructing dynamic triangles together : the development of mathematical group cognition / Gerry Stahl, Drexel University.
pages cm. – (Learning in doing)
Includes bibliographical references and index.
ISBN 978-1-107-12791-3 (hardback)
1. Geometry – Study and teaching (Middle school) 2. Middle school education. 3. Middle school students. 4. Mathematical ability. I. Title.
QA461.S73 2016
516.0071'2–dc23 2015024940

ISBN 978-1-107-12791-3 Hardback
ISBN 978-1-107-56687-3 Paperback

Contents

Figures

Logs

Series Foreword

This series for Cambridge University Press is widely known as an international forum for studies of situated learning and cognition. Innovative contributions are being made by anthropology; by cognitive, developmental, and cultural psychology; by computer science; by education; and by social theory. These contributions are providing the basis for new ways of understanding the social, historical, and contextual nature of learning, thinking, and practice that emerge from human activity. The empirical settings of these research inquiries range from the classroom to the workplace, to the high-technology office, and to learning in the streets and in other communities of practice. The situated nature of learning and remembering through activity is a central fact. It may appear obvious that human minds develop in social situations and extend their sphere of activity and communicative competencies. But cognitive theories of knowledge representation and learning alone have not provided sufficient insight into these relationships. This series was born of the conviction that new exciting interdisciplinary syntheses are under way as scholars and practitioners from diverse fields seek to develop theory and empirical investigations adequate for characterizing the complex relations of social and mental life, and for understanding successful learning wherever it occurs. The series invites contributions that advance our understanding of these seminal issues.

Roy Pea
Christian Heath
Lucy Suchman

Preface

Rational thinking as exemplified in mathematical cognition is of undeniable importance in the modern world. This book documents how a group of three eighth-grade girls developed specific practices typical of such thinking through involvement in an online educational experience. The presentation begins by discussing the methodological approach adopted in analyzing the development of mathematical group cognition. An extended case study then tracks the team of students step by step through its eight-hour-long progression. Concluding sections draw the consequences for the theory of group cognition and for educational practice.

The book investigates the display of mathematical reasoning by the students discussing dependencies within a sequence of dynamic-geometry figures. By examining the network of their mutual chat responses preserved in computer logs coordinated with their geometric actions exhibited in a replayer, it is possible to follow in detail the meaning-making processes of the students and to observe how the team develops its mathematical group cognition by adopting a variety of group practices. The longitudinal data set provides a rich opportunity to observe cognitive development through the interplay of processes and practices identifiable at the individual, small-group, and community units of analysis.

The examination of data focuses on these areas of the team's development:

1. Its effective team *collaboration*,
2. Its productive mathematical *discourse*,
3. Its enacted use of dynamic-geometry *tools*, and
4. Its ability to identify and construct dynamic-geometry *dependencies* by:
   a. Dynamic *dragging* of geometric objects,
   b. Dynamic *construction* of geometric figures, and
   c. Dynamic *design* of dependencies in geometric relationships.

The analysis reveals how the three students contribute differently, but also appropriate each other's contributions. This successively advances the group's ability to collaborate effectively with group agency, to articulate mathematical ideas productively by applying increasingly meaningful mathematical terminology, and to engage in dynamic-geometry challenges using mastered software functionality. The shared digital workspace supports group exploration and testing of geometric conjectures, while sequenced curricular topics guide student discoveries. These affordances help the students advance to new levels of individual and group mathematical cognition through the situated adoption of many specific group practices for productive collaboration, mathematical discourse, and dynamic-geometry problem solving.

The result is a detailed case study of the Virtual Math Teams Project as a paradigmatic example of computer-supported collaborative learning, incorporating a unique model of human-computer interaction analysis applied to the use of innovative educational technology.

Acknowledgments

The data analyzed here is from WinterFest 2013, an effort of the Virtual Math Teams (VMT) Project, a long-term collaboration among researchers at Drexel University, the Math Forum, and Rutgers University at Newark. WinterFest 2013 involved teachers in New Jersey who were participating in a professional development course at Rutgers, and their students.

The analysis of the data was discussed at many weekly research meetings of the VMT project team. Regular participants in the VMT discussions were: Stephen Weimar, Annie Fetter, Tony Mantoan, Michael Khoo, Sean Goggins, Diler Öner, Murat Çakir, Arthur Powell, Muteb Alqahtani, and Loretta Dicker. They each contributed in multiple ways to the generation and analysis of the data. In particular, the analysis of Session 3 herein borrows from the parallel analysis in Öner & Stahl (2015b) and the analysis of Session 8 incorporates content and figures from Çakir & Stahl (2015). Excerpts of this data were also discussed at an all-day workshop at ICLS 2014 (Stahl, 2014d), which is documented at www.gerrystahl.net/vmt/icls2014, including the full dataset for the Cereal Team.

The VMT Project was generously funded by a sequence of grants from the U.S. National Science Foundation covering from 2003 to 2016.