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978-1-107-56556-2 - Lectures on the Theory of Water Waves

Edited by Thomas J. Bridges, Mark D. Groves and David P. Nicholls

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