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Lectures on the Theory of Water Waves

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Preface

Waves on the surface of the ocean are a dramatic and beautiful phenomena that impact every aspect of life on the planet. At small length scales, ripples driven by surface tension on the surface of these "water waves" affect remote sensing of surface and underwater obstacles. At intermediate scales, waves on the surface and the interface between internal layers of water of differing densities affect shipping, coastal morphology, and near-shore navigation. At larger lengths, tsunamis and hurricane-generated waves can cause devastation on a global scale. Additionally, water waves play a crucial role at all length scales in the exchange of momentum and thermal energy between the ocean and atmosphere that, in turn, affect the global weather system and climate.

From a mathematical viewpoint, the water wave equations pose severe challenges for rigorous analysis, modelling, and numerical simulation. The governing equations are widely accepted and there has been substantial research into their validity. However, a rigorous theory of their solutions is extremely complex due not only to the fact that the water wave problem is a classical free boundary problem, where the domain shape is unknown, but also because the boundary conditions are strongly nonlinear. The level of difficulty is such that the theory has merely begun to answer the fundamental questions that must be addressed before our understanding can be considered "adequate." For instance, it is well known that the very existence of solutions to the equations that describe fluid motion, even in the absence of free boundaries, is one of the most difficult unanswered questions in mathematics (indeed it is one of the Clay Mathematics Institute Millennium Prize Problems).

In July and August of 2014 a four-week programme on the theory of water waves was convened at the Isaac Newton Institute for Mathematical Sciences in Cambridge,[a] with over 50 leading researchers in the theory of water waves from fifteen countries across the globe. The aim was to share recent research ideas and identify strategies for future directions. The key

[a] https://www.newton.ac.uk/event/tww/

themes of the conference were (a) the initial value problem (well-posedness, singularities, numerical methods, simulations), (b) existence of classes of waves (travelling waves, standing waves, multi-periodic waves, solitary waves, patterns, waves with vorticity and viscosity), (c) stability of waves (analytical and numerical methods, wave interactions, rogue waves, energetic stability), and (d) dynamical systems approaches (variational principles, modulation, centre-manifolds, spatial dynamics, bottom topography).

Interspersed among the various discussions, seminars, workshops, and other events, were lectures of an introductory and tutorial nature. The purpose of this volume is to compile a summary of those introductory lectures.

In week one the key theme was numerics. There were introductory talks by Benjamin Akers and David Nicholls on the theory and numerics of High-Order Perturbation of Surfaces methods, and a series of introductory talks on the Unified Transform Method and its application to water waves by Athanassios S. Fokas. All three of these speakers have provided chapters for this volume summarizing their lectures. In week two a London Mathematical Society Spitalfields event was held with four talks on rigorous analysis of nonlinear waves by Mark Groves, Guido Schneider, Steve Shkoller and Eugen Varvaruca, with the first two presenters providing chapters for this volume including an introduction to their talks. In week four a summer school on the theory of water waves was held, where early career researchers from the UK and overseas were invited to attend, with funding provided for the attending participants by the Newton Institute and the Office of Naval Research Global. The summer school had a cross section of talks including rigorous analysis of the initial value problem, numerical analysis of water waves, variational principles, shallow water hydrodynamics, the role of deterministic and random bottom topography, and modulation of nonlinear waves. The lecturers were David Ambrose, Onno Bokhove, Thomas Bridges, André Nachbin, and Sijue Wu, and they have all written up their lectures which are included as chapters herein.

The editors are grateful to the Newton Institute and the Director, John Toland, for supporting and facilitating the water waves programme. Special thanks are due to Christine West for excellent support leading up to and during the programme, and to Almarie Williams for her support during the planning and running of the Summer School.

Overall, the Newton programme covered a very wide range of material and provided a catalyst for future research. We hope this volume will contribute to the momentum that emerged from the programme and stimulate further interest in the fascinating combination of mathematics and water waves.