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978-1-107-55497-9 - Recent Progress in the Theory of the Euler and Navier–Stokes Equations

Edited by James C. Robinson, José L. Rodrigo, Witold Sadowski and Alejandro Vidal-López

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Edited by

JAMES C. ROBINSON

University of Warwick

JOSÉ L. RODRIGO

University of Warwick

WITOLD SADOWSKI

Uniwersytet Warszawski, Poland

ALEJANDRO VIDAL-LÓPEZ

Xi'an Jiaotong-Liverpool University, P. R. China



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Preface

This volume is the result of a workshop, “The Navier-Stokes Equations in Venice”, which took place in the Palazzo Pesaro-Papafava in Venice (part of the *Warwick in Venice* program), April 8th–12th, 2013.

Several of the speakers agreed to write review papers related to their contributions to the workshop, while others have written more traditional research papers. We believe that this volume therefore provides an accessible summary of a wide range of active research topics, along with some exciting new results, and we hope that it will prove a useful resource for both graduate students new to the area and to more established researchers.

We would like to express their gratitude to the following sponsors of the workshop and the writing of this volume of proceedings: JCR was supported by an EPSRC Leadership Fellowship (grant EP/G007470/1). JLR is currently supported by the European Research Council (ERC grant agreement n. 616797).

Finally it is a pleasure to thank Chiara Croff, the Venice administrator of *Warwick in Venice*, for her assistance during the organization of the workshop.

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Contributors

Those contributors who presented their work at the Warwick in Venice meeting are indicated by a star in the following list. We have also included the addresses of the editors.

Hugo Beirão da Veiga*

*Dipartimento di Matematica,
Università di Pisa,
Via F. Buonarroti 1/c, Pisa. Italy.
bveiga@dma.unipi.it*

Zachary Bradshaw

*Department of Mathematics,
University of British Columbia,
1984 Mathematics Road,
Vancouver, B.C.
Canada V6T 1Z2
zbradshaw@math.ubc.ca*

Weiwei Hu

*Department of Mathematics,
University of Southern California,
Los Angeles, CA 90089. USA.
weiwei@usc.edu*

Giovanni P. Galdi*

*Department of Mechanical Engineering and Materials Science,
University of Pittsburgh,
Pittsburgh, PA 15261. USA.
galdi@pitt.edu*

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Contributors

John D. Gibbon*

*Department of Mathematics,
Imperial College London,
London, SW7 2AZ. UK.*
j.d.gibbon@ic.ac.uk

Zoran Grujić

*Department of Mathematics,
Kerchof Hall,
University of Virginia,
Charlottesville, VA 22904. USA.*
zg7c@virginia.edu

Igor Kukavica*

*Department of Mathematics,
University of Southern California,
Los Angeles, CA 90089. USA.*
kukavica@usc.edu

Adam Larios

*Department of Mathematics,
University of Nebraska-Lincoln,
Lincoln, NE 68588-0130. USA.*
alarios@unl.edu

Pierre Gilles Lemarié–Rieusset*

*Laboratoire de Mathématiques et Modélisation d'Évry,
UMR CNRS 8071,
Université d'Évry,
France.*
plemarie@univ-evry.fr

Giusy Mazzone

*Department of Mechanical Engineering and Materials Science,
University of Pittsburgh,
Pittsburgh, PA 15261. USA.*
gim20@pitt.edu

Benjamin Pooley

*Mathematics Institute, University of Warwick,
Coventry, CV4 7AL. UK.*
B.C.Pooley@warwick.ac.uk

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James C. Robinson*Mathematics Institute, University of Warwick,
Coventry, CV4 7AL. UK.*

j.c.robinson@warwick.ac.uk

Marco Romito**Dipartimento di Matematica,
Università di Pisa,
Largo Bruno Pontecorvo 5,
56127 Pisa, Italia.*

romito@dm.unipi.it

Edriss S. Titi**Department of Mathematics,
Texas A&M University,
3368 TAMU, College Station, TX 77843–3368. USA.*

titi@math.tamu.edu

✉

*Department of Computer Science and Applied Mathematics,
Weizmann Institute of Science,
Rehovot 76100. Israel.*

edriss.titi@weizmann.ac.il

Alejandro Vidal-López**Department of Mathematical Sciences,
Xi'an Jiaotong-Liverpool University,
Suzhou 215123, China P. R.*

Alejandro.Vidal@xjtlu.edu.cn

Fei Wang*Department of Mathematics,
University of Southern California,
Los Angeles, CA 90089. USA.*

wang828@usc.edu

Mohammed Ziane**Department of Mathematics,
University of Southern California,
Los Angeles, CA 90089. USA.*

ziane@math.usc.edu