MODERN HIGHER ALGEBRA

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CAMBRIDGE
UNIVERSITY PRESS
1938

CAMBRIDGE
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781107544628

Published December 1937
Second impression August 1938
First paperback edition 2015

A catalogue record for this publication is available from the British Library

ISBN 978-1-107-54462-8 Paperback

PREFACE

During the present century modern abstract algebra has become more and more important as a tool for research not only in other branches of mathematics but even in other sciences. Many discoveries in abstract algebra itself have been made during the past ten years and the spirit of algebraic research has definitely tended toward more abstraction and rigor so as to obtain a theory of greatest possible generality. In particular the concepts of group, ring, integral domain, and field have been emphasized.

The notion of an abstract group is fundamental in all science, and it is certainly proper to begin our subject with this concept. Commutative additive groups are made into rings by assuming closure with respect to a second operation having some of the properties of ordinary multiplication. Integral domains and fields are rings restricted in special ways and may be thought of as respective generalizations of ordinary integers and rational numbers.

These fundamental concepts and their more elementary properties are the basis for modern algebra. They are certainly abstract notions but their ultimate absorption by the reader of modern algebra is absolutely necessary and the best place for them is at the beginning. This mode of presentation has not been used in the present textbooks on algebra in the English language but is the customary presentation in all of the more recent texts in foreign languages. We treat the concepts in our first two chapters and present what is basic not only for what follows in the text but for all modern algebra and algebraic number theory.

Our exposition continues in Chapters III, IV, and V with the theory of matrices with elements in a completely general field. Recent trends in algebraic investigation have made it important to know the extent of the validity of the classical theorems on matrices. It is no more difficult to carry out the proofs, where they are valid, for general fields instead of the classical case of subfields of the field of all complex numbers. But it is true that the proofs and results of the classical theory are not always valid. This is brought out clearly in Chapter V, where it is necessary to restrict the types of fields considered.

Essential clarifications in the arguments used in proving matrix theorems are obtained here by extensive use of elementary transformations. These transformations are familiar, except in name, to any reader who has had college training in the theory of determinants and make our proofs of a non-computational character and easy to understand. The author has also attempted to give as much as possible of the algebraic manipulative technique which he has used in his recent investigations in algebras and on Riemann matrices.

The final chapter on matrices presents a rather novel complete generalization of the theory of symmetric matrices which arises naturally in the algebraic geometric study of Riemann matrices. It is here that even the elementary theorems on symmetric matrices are not valid unless the fundamental assumption is made that the so-called characteristic of the field is not two.

The Galois theory is of great importance in algebra and the theory of algebraic numbers. The older treatments defined the Galois group of an equation with distinct roots as a certain group of permutations on these roots. This treatment is not very simple and the final theorems not in a very good form for certain algebraic applications. One may say that the essential trouble is that the Galois group is defined as a subgroup of the group of all permutations. The more modern treatment is that of the theory of the Galois group of a normal field. This is the set of all automorphisms of the given field and the fact that we take *all* automorphisms makes our proofs quite simple. We present this treatment in Chapters VI–IX. The first of these chapters gives all of the results needed from the theory of finite groups. Chapter VII gives the essentials of the theory of algebraic extensions of a given field, and in Chapter VIII the Galois theory of fields is given and applied to obtain as a consequence the Galois theory of equations. The final chapter of this set is an application of the theory to obtain structure theorems for the simplest type of algebraic extension, the cyclic field.

The theory of linear associative algebras is a fundamental, if quite advanced, branch of modern algebra. It is natural, however, to introduce this subject from the matrix point of view and we do so in Chapter X. Many quite abstract notions are made concrete by such a treatment, and a quite adequate introduction to the theory is made in this way without going at all deeply into the abstract structure theorems on algebras.

Our exposition closes with an introduction to the theory of p-adic numbers. This subject is best studied by considering the general theory of fields with a valuation, and we do so here. The author has collected his material from a large number of sources and hopes that the present exposition is an adequate foundation of the theory. It is certainly true that no progress can be made in reading modern papers on algebraic numbers and their applications to the theory of algebras without a knowledge of p-adic number theory. It is equally true that it has heretofore been necessary to read a forbidding number of articles in order to get even a meager acquaintance with the theory.

The author has written the present text as a foundation for future exposition of the modern theory of algebraic numbers and class fields, and of the theory of linear associative algebras. He wishes to acknowledge as sources for much of the material L. E. Dickson's *Modern Algebraic Theories* and *Algebren und ihre Zahlentheorie*, A. Speiser's *Theorie der Gruppen von endlicher Ordnung*, B. L. van der Waerden's *Moderne Algebra*, J. H. M. Wedderburn's *Lectures on Matrices*, as well as a number of papers by C. Chevalley, H. Hasse, J. Kürschák, and A. Ostrowski. These latter were used as sources, particularly in the last two chapters. Final thanks are due to Mr. Sam Perlis, a graduate student at the University of Chicago, who has materially assisted in the preparation of the manuscript; to Drs. Daniel Dribin and Nathan Jacobson, who have read it critically; and to his colleagues of the Department of Mathematics of the University of Chicago, who have given very valuable advice on its preparation.

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UNIVERSITY OF CHICAGO
October 1, 1936

Cambridge University Press
978-1-107-54462-8 - Modern Higher Algebra
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Frontmatter
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