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Geometry, Topology, and Dynamics in Negative Curvature

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Preface

The geodesic flow on the unit tangent bundle of a closed surface of constant negative curvature is one of the earliest examples of an ergodic dynamical system. This was first proven by G. A. Hedlund in 1936. Soon afterwards, it was reproved by E. Hopf, who also generalized to the case of closed surfaces of variable negative curvature. Hopf's proof already indicated the relevance of negative curvature to the ergodicity of the geodesic flow. About 20 years later, Hopf's theorem was generalized by Anosov to geodesic flows on unit tangent bundles of higher dimensional closed negatively curved manifolds.

Around the same time, combining certain basic results in geometry and topology, it was observed that the fundamental group of a closed manifold M of negative curvature determines M up to homotopy equivalence. Mostow's celebrated strong rigidity theorem (1968) showed that, within the class of closed locally symmetric spaces M of non-compact type of dimension ≥ 3, the fundamental group $\pi_1(M)$ determines M up to isometry (possibly after scaling the metric by a positive constant). Further study of this rigidity phenomenon led to two important generalizations. On the one hand, Margulis established his super-rigidity theorem in higher rank. Using the harmonic map techniques of Eells-Sampson, versions of the super-rigidity theorem were established for certain rank 1 cases by Corlette, Jost-Yau and Mok-Siu-Yeung. On the other hand, it led Ballmann-Brin-Eberlein to introduce, in the mid 1980's, the notion of geometric rank. Generalizing a notion that existed for locally symmetric spaces, this culminated in the rank rigidity theorem due, independently, to Ballmann and Burns-Spazier. Around this time, Gromov realized that many results of this nature could be formulated and proved, in a synthetic way, in the more general setting of metric spaces of non-positive curvature. The study of this broader class of spaces resulted in certain new rigidity phenomena (such as quasi-isometric rigidity). It also allowed these techniques to be applied to the

study of certain infinite groups. Research in this direction has since exploded and created the whole new field of geometric group theory.

Concurrently, investigations continued on how the fundamental group $\pi_1(M)$ of a closed negatively curved manifold M constrained the topology and geometry of M. Farrell-Jones proved, in the late 1980s, the remarkable topological rigidity theorem, which showed that in dimensions $n \neq 3, 4$, the fundamental group $\pi_1(M)$ determines M up to homeomorphism. They also constructed examples of closed negatively curved manifolds which are homeomorphic but not diffeomorphic, thereby showing that smooth rigidity, as conjectured by Lawson-Yau, fails in general.

Besson-Courtois-Gallot reintroduced dynamical ideas to study, on a given closed locally symmetric manifold M, the special role played by the locally symmetric metric within the broader class of negatively curved metrics. They defined the notion of a natural map, and in the early 1990s used the natural map to show that, among all negatively curved metrics, the locally symmetric ones minimize the topological entropy.

Farrell-Jones-Ontaneda-Raghunathan constructed exotic PL-structures on certain closed real hyperbolic manifolds. These constructions have exposed certain limitations to the otherwise successful analytic methods (such as the harmonic map, the natural map and the Ricci flow) in taking the study of closed negatively curved manifolds further.

With many important questions still waiting to be resolved, one of the aims of the GTDNC conference was to bring together mathematicians working in different aspects of negative curvature, and to discuss some of the recent highlights of the field.

India has had its own share of participation in these exciting developments. Some of the leading players visited India (especially TIFR Mumbai) over the years, inspiring significant research activity on some of these topics, and forging great collaborations. A testimony to the fact that research in these areas flourished in India is that some of these mathematicians made multiple visits to India. Several of these visitors made extended visits and gave lecture courses at TIFR. Notes from some of these courses were written up by young students at TIFR, and appeared in the TIFR Lecture notes series. These visits also inspired articles and books that appeared elsewhere too, notable among these is the foundational article of Gromov that heralded the beginnings of geometric group theory. Atle Selberg made the first of his three visits to India in 1956, Armand Borel was a frequent visitor, Marcel Berger visited a couple of times, Dan Mostow visited in 1968, Gromov visited in 1984-85, Margulis visited in

1989, Tom Farrell first visited in 1993 and returned to India several times after that. In recent times Besson has been a regular visitor. Hamilton, Yau and Gabai have also delivered special lecture series.

The ICM 2010 provided a great occasion to get together again in India and discuss various facets of the fascinating area of negative curvature. The ICM 2010 satellite conference *Geometry, Topology and Dynamics in Negative Curvature* was held in the beautiful ambiance of the Raman Research Institute in Bangalore during 2-7 August, 2010. The conference was a big event, with 35 one hour lectures, and two open problem sessions. We had 20 participants from abroad, 23 participants from Bangalore, and 24 participants from other cities throughout India. A large number of photos and videos of talks from the conference are posted on the conference webpage:

<http://www.icts.res.in/archive/program/details/91/>

We hope that the articles in these proceedings provide some snippets of the conference.

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