CAMBRIDGE

Cambridge University Press
978-1-107-49575-3 — Physics for the IB Diploma Exam Preparation Guide
K. A. Tsokos
Excerpt
More Information

1

MEASUREMENTS AND UNCERTAINTIES

This chapter covers the following topics:

- ☐ Fundamental and derived units
- ☐ Significant figures and scientific notation
- ☐ Order-of-magnitude estimates
- ☐ Random and systematic errors
- ☐ Uncertainties, gradients and intercepts
- ☐ Linearisation of graphs
- ☐ Vectors and scalars

1.1 Units

It is a fascinating fact that all physical quantities have units that can be expressed in terms of those for just seven **fundamental** quantities.

DEFINITIONS

FUNDAMENTAL UNITS The seven fundamental quantities in the S.I. system (the IB syllabus uses only the first six) and their units are:

- Time — second (s)
- Length — meter (m)
- Mass — kilogram (kg)
- Temperature — kelvin (K)
- Quantity of matter — mole (mol)
- Electric current — ampere (A)
- Luminous intensity — candela (cd)

DERIVED UNITS All other quantities have **derived** units, that is, combinations of the fundamental units. For example, the derived unit for force (the newton, N) is obtained using $F = ma$ to be $\text{kg}\,\text{m}\,\text{s}^{-2}$ and that for electric potential difference (the volt, V) is obtained using $W = qV$ to be $\frac{\text{J}}{\text{C}} = \frac{\text{N m}}{\text{A s}} = \frac{\text{kg m s}^{-2}\,\text{m}}{\text{A s}} = \text{kg m}^2\,\text{s}^{-3}\,\text{A}^{-1}$.

SIGNIFICANT FIGURES There is a difference between stating that the measured mass of a body is 283.64 g and saying it is 283.6 g. The implication is that the uncertainty in the first measurement is ±0.01 g and that in the second is ±0.1 g. That is, the first measurement is more precise – it has more **significant figures** (s.f.). When we do operations with numbers (multiplication, division, powers and roots) we must express the result to the same number of s.f. as in the least precisely known number in the operation (Table 1.1).

Table 1.1

Number		Number of s.f.	Scientific notation
34		2	3.4×10^1
3.4		2	3.4×10^0 or just 3.4
0.0340	Zeros in front do not count but zeros at the end *in a decimal* do count.	3	3.4×10^{-2}
340	Zeros at the end *in an integer* do not count.	2	3.4×10^2

CAMBRIDGE

Cambridge University Press
978-1-107-49575-3 — Physics for the IB Diploma Exam Preparation Guide
K. A. Tsokos
Excerpt
More Information

Thus the kinetic energy of a mass of 2.4 kg (2 s.f.) moving at $14.6\,\mathrm{m\,s^{-1}}$ (3 s.f.) is given as $E_K = \frac{1}{2} \times 2.4 \times 14.6^2 = 255.792\ \mathrm{J} \approx 260 = 2.6 \times 10^2\ \mathrm{J}$ (2 s.f.). Similarly, the acceleration of a body of mass 1200 kg (2 s.f.) acted upon by a net force of 5250 N (3 s.f.) is given as $\frac{5250}{1200} = 4.375 \approx 4.4\,\mathrm{m\,s^{-2}}$ (2 s.f.).

TEST YOURSELF 1.1

The force of resistance from a fluid on a sphere of radius r is given by $F = 6\pi\eta r v$, where v is the speed of the sphere and η is a constant. What are the units of η?

TEST YOURSELF 1.2

The radius R of the fireball t seconds after the explosion of a nuclear weapon depends only on the energy E released in the explosion, the density ρ of air and the time t. Show that the quantity $\frac{Et^2}{\rho}$ has units of $\mathrm{m^5}$ and hence that $R \approx \left(\frac{Et^2}{\rho}\right)^{\frac{1}{5}}$. Calculate the energy released if the radius of the fireball is 140 m after 0.025 s. (Take $\rho = 1.0\,\mathrm{kg\,m^{-3}}$.)

1.2 Uncertainties

DEFINITIONS

RANDOM UNCERTAINTIES Uncertainties due to the inexperience of the experimenter and the difficulty of reading instruments. Taking an average of many measurements leads to a more accurate result. The average of n measurements $x_1, x_2, \ldots, x_n$ is $\bar{x} = \frac{x_1 + x_2 + \ldots + x_n}{n}$.

SYSTEMATIC UNCERTAINTIES Uncertainties due mainly to incorrectly calibrated instruments. They cannot be reduced by repeated measurements.

ACCURATE MEASUREMENTS Measurements that have a small systematic error.

PRECISE MEASUREMENTS Measurements that have a small random error.

Nature of Science. A key part of the scientific method is recognising the errors that are present in the experimental technique being used, and working to reduce these as much as possible. In this section you have learned how to calculate errors in quantities that are combined in different ways and how to estimate errors from graphs. You have also learned how to recognise systematic and random errors.

No matter how much care is taken, scientists know that their results are uncertain. However, they need to distinguish between inaccuracy and uncertainty, and to know how confident they can be about the validity of their results. The search to gain more accurate results pushes scientists to try new ideas and refine their techniques. There is always the possibility that a new result may confirm a hypothesis for the present, or it may overturn current theory and open a new area of research. Being aware of doubt and uncertainty are key to driving science forward.

Normally we express uncertainties to just one significant figure. However, if a more sophisticated statistical analysis of the data has taken place there is some justification for keeping two significant figures.

In general, for a quantity Q we have

$$Q = \underbrace{Q_0}_{\text{measured value}} \pm \underbrace{\Delta Q}_{\text{absolute uncertainty}}, \quad \frac{\Delta Q}{Q_0} = \text{fractional uncertainty}, \quad \frac{\Delta Q}{Q_0} \times 100\% = \text{percentage uncertainty}$$

As an example, consider a measurement of the length of the side of a cube, given as 25 ± 1 mm. The 25 mm represents the **measured value** of the length and the ±1 mm represents the **absolute uncertainty** in the measured value. The ratio $\frac{1}{25} = 0.04$ is the **fractional uncertainty** in the length, and $\frac{1}{25} \times 100\% = 4\%$ is the **percentage uncertainty** in the length.

Suppose quantities *a*, *b* and *c* have been measured with absolute uncertainties, respectively, of Δa, Δb and Δc. If we use *a*, *b* and *c* to calculate another quantity Q, these uncertainties will result in an uncertainty in Q. The (approximate) rules for calculating the uncertainty ΔQ in Q are:

- If $Q = a \pm b \pm c$, then $\Delta Q = \Delta a + \Delta b + \Delta c$. That is, for addition and subtraction, **add the absolute uncertainties**.
- If $Q = \frac{ab}{c}$, then $\frac{\Delta Q}{Q} = \frac{\Delta a}{a} + \frac{\Delta b}{b} + \frac{\Delta c}{c}$. That is, for multiplication and/or division **add the fractional uncertainties**.
- If $Q = \frac{a^n}{b^m}$, then $\frac{\Delta Q}{Q} = n\frac{\Delta a}{a} + m\frac{\Delta b}{b}$. In particular, if $Q = \sqrt{ab}$ or $Q = \sqrt{\frac{a}{b}}$, then $\frac{\Delta Q}{Q} = \frac{1}{2}\frac{\Delta a}{a} + \frac{1}{2}\frac{\Delta b}{b}$.

Model Answer 1.1

The volume of a cylinder of base radius *R* and height *H* is given by $V = \pi R^2 H$. The volume of a cylinder is measured to 10% and height to 4%. Estimate the percentage uncertainty in the radius.

First solve for the variable whose uncertainty we want to estimate: $R = \sqrt{\frac{V}{\pi H}}$.

Hence $\frac{\Delta R}{R} = \frac{1}{2}\left(\frac{\Delta V}{V} + \frac{\Delta H}{H}\right) = \frac{1}{2} \times (10 + 4) = 7\%$.

TEST YOURSELF 1.3

The resistance of a lamp is given by $R = \frac{V}{I}$. The uncertainty in the voltage is 4% and the uncertainty in the current is 6%. What is the absolute uncertainty in a calculated resistance value of 24 Ω?

TEST YOURSELF 1.4

Each side of a cube is measured with a fractional uncertainty of 0.02. Estimate the percentage uncertainty in the volume of the cube.

TEST YOURSELF 1.5

The period of oscillation of a mass *m* at the end of a spring of spring constant *k* is given by $T = 2\pi\sqrt{\frac{m}{k}}$.

What is the percentage uncertainty in the period if *m* is measured with a percentage uncertainty of 4% and the *k* with a percentage uncertainty of 6%?

hint

Avoid the common mistake of saying that the uncertainty is $\sqrt{(4\% + 6\%)} \approx 3\%$.

Cambridge University Press
978-1-107-49575-3 — Physics for the IB Diploma Exam Preparation Guide
K. A. Tsokos
Excerpt
More Information

1 Measurements and Uncertainties

Error bars

Suppose that we want to plot the point $(3.0 \pm 0.1, 5.0 \pm 0.2)$ on a set of x and y axes. First we plot the point with coordinates (3.0, 5.0) and then show the uncertainties as error bars (Figure 1.1). The horizontal error bar will have length $2 \times 0.1 = 0.2$ and the vertical will have length $2 \times 0.2 = 0.4$.

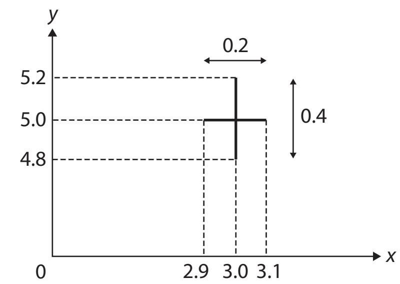

Figure 1.1

DEFINITIONS

BEST-FIT LINE The curve or straight line that goes through all the error bars; an example is shown in Figure 1.2. Note that a 'line' may be straight or curved.

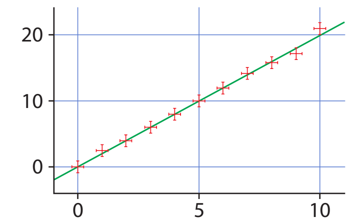

Figure 1.2

Finding slopes

To find the slope (or gradient) of a curve at a particular point (here at $x = 1.0 \times 10^{-2}$ m), draw the tangent to the curve at that point (Figure 1.3).

Choose two points *on the tangent* that are as far apart as possible. Note in this case that the units on the horizontal axis are 10^{-2} m, and that the slope has a negative value.

$$\text{Slope} = \frac{6.0 - 1.2}{0.0 - 2.0 \times 10^{-2}}\ \frac{\text{volt}}{\text{m}}$$
$$= -2.4 \times 10^{2}\ \text{V m}^{-1}$$

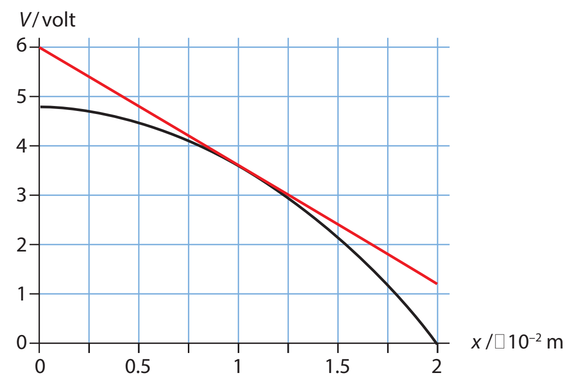

Figure 1.3

Estimating areas under curves

To estimate the area under the black curve in Figure 1.4, draw a straight line (red) from the point (0, 6) to the point (4, 1.5).

It is easy to calculate the area of the trapezium under the straight line, as

$$\frac{(6 + 1.5)}{2} \times 4 = 15.0.$$

Now estimate the number of small squares in the space between the straight line and the curve, and subtract this from the total, to give the area under the curve. There are about 53 squares.

Each one has area of $0.25 \times 0.25 = 0.0625$ square units, so the area between the curve and the straight line is about 53 0.0625 = 3.3.

So the area under the curve is about $15.0 - 3.3 = 11.7$ square units.

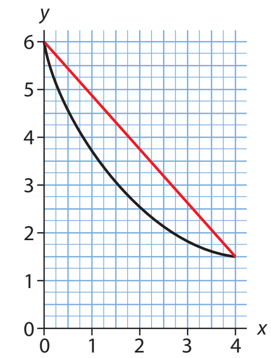

Figure 1.4

Cambridge University Press
978-1-107-49575-3 — Physics for the IB Diploma Exam Preparation Guide
K. A. Tsokos
Excerpt
More Information

Getting straight-line graphs

If we know the relationship between two variables we can usually arrange to plot the data in such a way that we get a straight line. Bear in mind that the standard equation of a straight line is

$$y = \underbrace{m}_{\text{gradient}} x + \underbrace{c}_{\text{vertical intercept}}$$

where we plot the variable y on the vertical axis and the variable x on the horizontal axis.

If the straight line goes through the origin ($c = 0$), we say that y is **proportional** to x.

If the best-fit line is not straight or if it does not go through the origin, then *either* of these reasons is sufficient to claim that y is *not* proportional to x.

As an example, consider the relationship $T = 2\pi\sqrt{\frac{m}{k}}$ for the period T of a mass m undergoing oscillations at the end of a spring of spring constant k. Compare this equation and the general straight-line equation:

$$\begin{array}{ccccc} T & = & \dfrac{2\pi}{\sqrt{k}} & \times & \sqrt{m} \\ \downarrow & & & & \downarrow \\ y & = & \underbrace{\dfrac{2\pi}{\sqrt{k}}}_{\text{constants}} & \times & x \end{array}$$

By identifying $T \leftrightarrow y$ and $\sqrt{m} \leftrightarrow x$ we get the equation of a straight line, $y = \frac{2\pi}{\sqrt{k}}x$. So if we plot T on the vertical axis and $\sqrt{m}$ on the horizontal axis we should get a straight line whose gradient is $\frac{2\pi}{\sqrt{k}}$. Alternatively, we may write:

$$\begin{array}{ccccc} T^2 & = & \dfrac{4\pi^2}{k} & \times & m \\ \downarrow & & & & \downarrow \\ y & = & \underbrace{\dfrac{4\pi^2}{k}}_{\text{constants}} & \times & x \end{array}$$

By identifying $T^2 \leftrightarrow y$ and $m \leftrightarrow x$ we get the equation of a straight line, $y = \frac{4\pi^2}{k}x$. So if we plot T^2 on the vertical axis and m on the horizontal axis we should get a straight line whose gradient is $\frac{4\pi^2}{k}$.

A different procedure must be followed if the variables are related through a power relation such as $F = kr^n$, where the constants k and n are unknown. Taking natural logs (or logs to any other base), we have:

$$\begin{array}{ccc} \ln F & = \ln k + n \times & \ln r \\ \downarrow & & \downarrow \\ y & = \ln k + n \times & x \end{array}$$

and so plotting $\ln F$ versus $\ln r$ should give a straight line with gradient n and vertical intercept $\ln k$.

A variation of this is used for an exponential equation such as $A = A_0 e^{-\lambda t}$, where A_0 and λ are constants. Here we can take the logs of both sides to get $\ln A = \ln A_0 - \lambda t$, and so:

$$\begin{array}{ccc} \ln A & = \ln A_0 - \lambda \times & t \\ \downarrow & & \downarrow \\ y & = \ln A_0 - \lambda \times & x \end{array}$$

CAMBRIDGE

Cambridge University Press
978-1-107-49575-3 — Physics for the IB Diploma Exam Preparation Guide
K. A. Tsokos
Excerpt
More Information

Plotting $\ln A$ on the vertical axis and t on the horizontal then gives a straight line with gradient $-\lambda$ and vertical intercept $\ln A_0$.

TEST YOURSELF 1.6

Copy Table 1.2 and fill in the blank entries.

Table 1.2

Equation	Constants	Variables to be plotted to give straight line	Gradient	Vertical intercept
$P = kT$	k			
$v = u + at$	u, a			
$v^2 = 2as$	a			
$F = \frac{kq_1q_2}{r^2}$	k, q_1, q_2			
$a = -\omega^2 x$	ω^2			
$V = \frac{kq}{r}$	k, q			
$T^2 = \frac{4\pi^2}{GM}R^3$	G, M			
$I = I_0 e^{-aT}$	I_0, a			
$\lambda = \frac{h}{\sqrt{2mqV}}$	h, m, q			
$F = av + bv^2$	a, b			
$E = \frac{1}{2}m\omega^2\sqrt{A^2 - x^2}$	m, ω^2, A			
$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$	f			

TEST YOURSELF 1.7

State what variables must be plotted so that we get a straight line for the relation $d = ch^{0.8}$, where c is a constant.

Estimating uncertainties in measured quantities

Useful simple rules are for estimating the uncertainty in a measured quantity is:

For *analogue meters*, use half of the smallest scale division. For example, for an ordinary meter rule the smallest scale division is 1 mm and so the uncertainty is ±0.5 mm. If this is used, for example, to measure the length of a rod, this uncertainty applies to the position of each end of the rod, for a total uncertainty of ±1 mm in the rod's length.

For *digital meters*, use the smallest division. For example, with a digital voltmeter that can read to the nearest hundredth of a volt, take the uncertainty as ±0.01V. For an ammeter that can read to the nearest tenth of an ampere, take the uncertainty as ±0.1 A.

TEST YOURSELF 1.8

Estimate the reading and the uncertainty in each of the instruments in Figure 1.5.

Figure 1.5

Annotated Exemplar Answer 1.1

The period of a pendulum is measured to be $T = (2.20 \pm 0.05)$ s. Calculate the value of T^2, including its uncertainty. **[3]**

$T^2 = 2.20^2 = 4.84\ s^2$ — The value of T^2 is correct, and with the correct units.

$\Delta T^2 = 0.05^2 = 0.0025\ s^2$ — The value of ΔT is 0.05 s, but you cannot square the uncertainty in T to find ΔT^2, the uncertainty in T^2. Make sure you know how to find fractional uncertainties when there are powers, and remember the protocol for the number of significant figures in the uncertainty.

Here the fractional uncertainty in T is $\frac{\Delta T}{T} = \frac{0.05}{2.20} = 0.022 \approx 0.02$ (rounded to 1 s.f. as the uncertainty is greater than 2%). The power in T^2 is 2, so multiply the fractional uncertainty in T by 2 to find the fractional uncertainty in T^2, that is, $0.02 \times 2 = 0.04$. So $\Delta T^2 = 0.04 \times 4.84 \approx 0.2\ s^2$.

So $T^2 = (4.84 \pm 0.0025)\ s^2$.

The final answer gains no marks because the wrong method was used to find ΔT. One way to spot errors like this is to ask if the final answer is sensible. Here the uncertainty is given to 2 s.f. in the third and fourth decimal places, when the value has two decimal places, so the answer must be wrong. The correct answer is $T^2 = (4.8 \pm 0.2)\ s^2$.

1/3

Uncertainty in the measured value of a gradient (slope)

To find the uncertainty in the gradient of the (straight) best-fit line, draw lines of maximum and minimum gradient. You must judge these by eye, taking into account *all* error bars, not just those of the first and last data points. Calculate these two gradients, m_{max} and m_{min}. A simple estimate of the uncertainty in the gradient is then $\frac{m_{max} - m_{min}}{2}$.

TEST YOURSELF 1.9

Electrons that have been accelerated through a potential difference V enter a region of magnetic field B, where they are bent into a circular path of radius r. Theory suggests that $r^2 = \frac{2m}{qB^2}V$, where q is the electron's charge and m is its mass. Table 1.3 shows values of the potential difference V and radius r obtained in an experiment.

Cambridge University Press
978-1-107-49575-3 — Physics for the IB Diploma Exam Preparation Guide
K. A. Tsokos
Excerpt
More Information

Table 1.3

Radius r / cm ±0.1 cm	Potential difference V / V	r^2 / cm^2
4.5	500	±
4.9	600	±
5.3	700	±
5.7	800	±
6.0	900	±

a Explain why a graph of r^2 against V will result in a straight line.

b State the slope of the straight line in **a** in terms of the symbols m, q, B.

c Copy Table 1.3 and in the right-hand column insert values of the radius squared, including its uncertainty.

Figure 1.6 shows the data points plotted on a set of axes.

d Draw error bars for the all the data points.

e Draw a best-fit line for these data points.

f Calculate the gradient of the best-fit line, including its uncertainty.

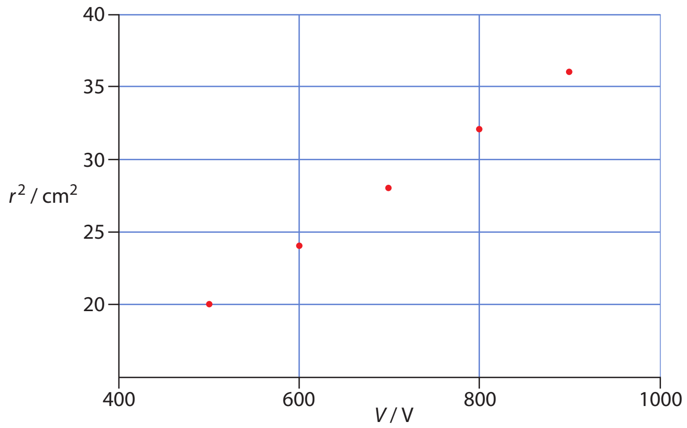

Figure 1.6

The magnetic field used in this experiment was $B = 1.80 \times 10^{-3}$ T.

g Calculate the value that this experiment gives for the charge-to-mass ratio $\left(\frac{q}{m}\right)$ of the electron. Include the uncertainty in the calculated value.

1.3 Vectors and scalar quantities

DEFINITIONS

VECTOR A physical quantity that has both magnitude and direction. It is represented by arrows. The length of the arrow gives the magnitude of the vector. The direction of the arrow is the direction of the vector. Examples of vectors are displacement, velocity, acceleration, force, momentum and electric/gravitational/magnetic fields.

SCALARS A physical quantity with magnitude but not direction. A scalar can be positive or negative. Examples are distance, speed, mass, time, work/energy, electric/gravitational potential and temperature.

Cambridge University Press
978-1-107-49575-3 — Physics for the IB Diploma Exam Preparation Guide
K. A. Tsokos
Excerpt
More Information

Adding vectors: have ***a*** and ***b*** start at the same point, O (Figure 1.7a). Draw the parallelogram whose two sides are ***a*** and ***b***. Draw the diagonal starting at O.

Subtracting vectors: have ***a*** and ***b*** start at the same point, O (Figure 1.7b). To find ***b*** – ***a*** draw the vector from the tip of ***a*** to the tip of ***b***.

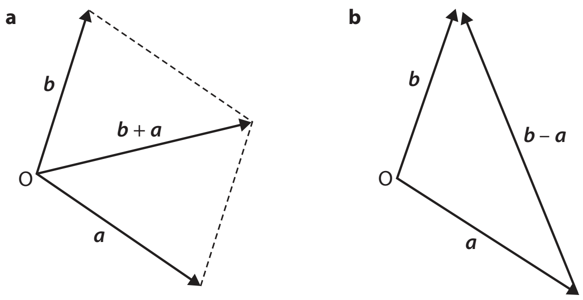

Figure 1.7 **a** Addition; **b** subtraction.

Components of vectors

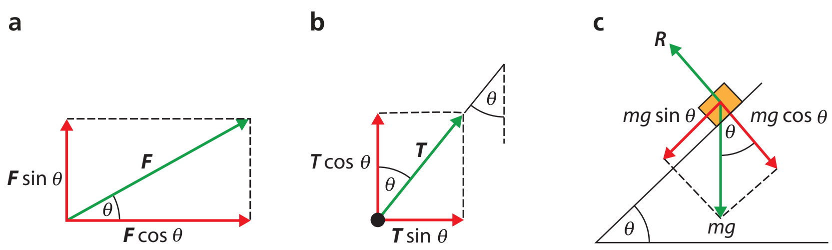

Figure 1.8 Components of vectors.

The component adjacent to the angle θ involves $\cos\theta$ and that opposite to θ involves $\sin\theta$.

- Draw the forces.
- Put axes.
- Get components.
- Choose as one of your axis the direction in which the body moves or would move if it could.

TEST YOURSELF 1.10

A river is 16 m wide. A boat can travel at 4.0 m s^{-1} with respect to the water and the current has a speed of 3.0 m s^{-1} with respect to the shore, directed to the right (Figure 1.9). The boat is rowed in such a way as to arrive at the opposite shore directly across from where it started. Calculate the time taken for the trip.

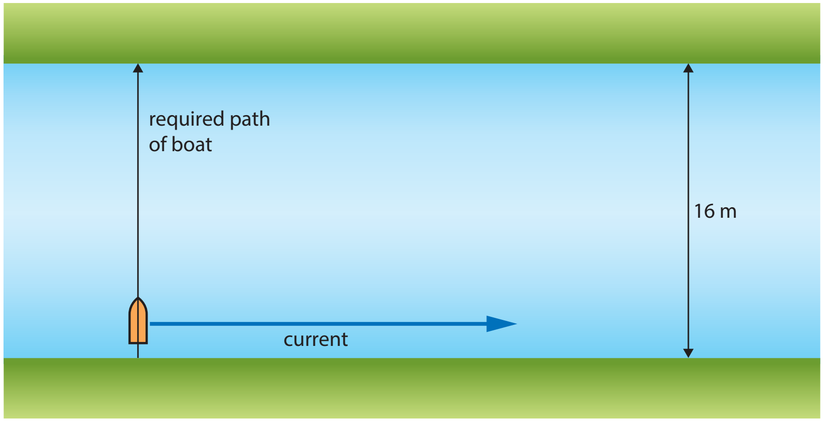

Figure 1.9

Cambridge University Press
978-1-107-49575-3 — Physics for the IB Diploma Exam Preparation Guide
K. A. Tsokos
Excerpt
More Information

1.4 Order-of-magnitude estimates

Tables 1.4, 1.5 and 1.6 give typical values for various distances, masses and times. You are not expected to know these by heart but you must have a *general* idea of such sizes, masses and durations.

Table 1.4

	Length / m
Radius of observable universe	10^{26}
Distance to the Andromeda galaxy	10^{22}
Diameter of the Milky Way galaxy	10^{21}
Distance to Proxima Centauri (star)	10^{16}
Diameter of solar system	10^{13}
Distance to the Sun	10^{11}
Radius of the Earth	10^{7}
Size of a cell	10^{-5}
Size of a hydrogen atom	10^{-10}
Size of an average nucleus	10^{-15}
Planck length	10^{-35}

Table 1.5

	Mass / kg
The universe	10^{53}
The Milky Way galaxy	10^{41}
The Sun	10^{30}
The Earth	10^{24}
Boeing 747 (empty)	10^{5}
An apple	0.2
A raindrop	10^{-6}
A bacterium	10^{-15}
Mass of smallest virus	10^{-21}
A hydrogen atom	10^{-27}
An electron	10^{-30}

Table 1.6

	Time / s
Age of the universe	10^{17}
Time of travel for light from nearby star (Proxima Centauri)	10^{8}
One year	10^{7}
One day	10^{5}
Period of a heartbeat	1
Period of red light	10^{-15}
Time of passage of light across an average nucleus	10^{-23}
Planck time	10^{-43}

TEST YOURSELF 1.11

Estimate the weight of an apple.

TEST YOURSELF 1.12

Estimate the number of seconds in a year.

TEST YOURSELF 1.13

Estimate the time taken by light to travel across a nucleus.