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Frontmatter
More information

LINEAR ALGEBRAS

by

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PREFACE

THE theory of linear associative algebras (or closed systems of hypercomplex numbers) is essentially the theory of pairs of reciprocal linear groups (§ 52) or the theory of certain sets of matrices or bilinear forms (§ 53). Beginning with Hamilton's discovery of quaternions seventy years ago, there has been a rapidly increasing number of papers on these various theories. The French Encyclopedia of Mathematics devotes more than a hundred pages to references and statements of results on this subject (with an additional part on ordinary complex numbers). However, the subject is rich not merely in extent, but also in depth, reaching to the very heart of modern algebra.

The purpose of this tract is to afford an elementary introduction to the general theory of linear algebras, including also non-associative algebras. It retains the character of a set of lectures delivered at the University of Chicago in the Spring Quarter of 1913. The subject is presented from the standpoint of linear algebras and makes no use either of the terminology or of theorems peculiar to the theory of bilinear forms, matrices or groups (aside of course from §§ 52–54, which treat in ample detail of the relations of linear algebras to those topics).

Part I relates to definitions, concrete illustrations, and important theorems capable of brief and elementary proof. A very elementary proof is given of Frobenius's theorem which shows the unique place of quaternions among algebras. The remarkable properties of Cayley's algebra of eight units are here obtained for the first time in a simple manner, without computations. Other new results and new points of view will be found in this introductory part.

In presenting in Parts II and IV the main theorems of the general theory, it was necessary to choose between the expositions by Molien, Cartan and Wedderburn (that by Frobenius being based upon bilinear forms and hence outside our plan of treatment). We have not presented the theory of Molien partly because his later proofs depend upon the theory of groups and partly because certain of his earlier proofs have not yet been made correctly by his methods. The more general paper by Wedderburn is based upon a rather abstract calculus of complexes, comparable with the theory of abstract groups (§ 61). In compensation, he obtains in relatively brief space the main theorems not only for the usual cases of complex and real algebras, but also for algebras the coordinates of whose numbers range over any field (stated in § 56).

In order that our treatment of the general theory shall be elementary and concrete and shall use but a very few concepts easily kept in mind, we have confined our exposition (in Part II) to the classical case of algebras whose numbers have ordinary complex coordinates and given a careful revision of the theory as presented in Cartan's fundamental paper. Running parallel with the general theory is an illustrative example treated independently but in the spirit of the theory. While we thereby lose the generalization to a general field, we gain the important normalized sets of units, first given by Scheffers under certain restrictions, and so obtain the analogues of important theorems on the canonical forms of groups of linear transformations or of sets of matrices or bilinear forms.

I am much indebted to Professor Wedderburn of Princeton and Miss Hazlett of Chicago for suggestions made after careful readings of the proofs. My thanks are due to the editors for the opportunity to participate in this useful series of tracts. Finally, I am under obligations to the officials of the University Press for complying with all of my requests as regards the form of this tract, and for expeditious publication in spite of the distance travelled by the proofs. The quality of the printing speaks for itself.

L. E. D.

Chicago,
May, 1914.

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