A LOCUS WITH 25920 LINEAR SELF-TRANSFORMATIONS

(1) **The fundamental notation.** In a space of four dimensions we may use for homogeneous coordinates the six

$$x_1, x_2, x_3, x_4, x_5, x_6,$$

connected by the equation $x_1+x_2+\ldots+x_6=0$. Throughout we shall use ϵ for the cube-root of unity, exp $(2\pi i/3)$. If l, m, n, p, q, r denote the numbers 1, 2, 3, 4, 5, 6, in any order, the symbol $(lp.mq.nr)$ shall denote the point for which $x_l=x_p=1$, $x_m=x_q=\epsilon$, $x_n=x_r=\epsilon^2$. The same point is then represented by the symbol $(mq.nr.lp)$, or by the symbol $(nr.lp.mq)$. Occasionally, a couple such as l, p may be spoken of as a *duad*, and the symbol $(lp.mq.nr)$, whose three duads contain all the numbers 1, 2, ..., 6; may be spoken of as a *syntheme* (after Sylvester). Similarly the symbol $[lp.mq.nr]$ shall denote the linear function of the coordinates $x_l+x_p+\epsilon(x_m+x_q)+\epsilon^2(x_n+x_r)$; the prime represented by $[lp.mq.nr]=0$ is the same as either of those denoted by $[mq.nr.lp]=0$, or $[nr.lp.mq]$. Further (lp) shall denote the point whose coordinates are $x_l=1$, $x_p=-1$, with $x_m=x_q=x_n=x_r=0$, and $[lp]$ shall denote the linear function x_l-x_p.

We consider the square scheme of synthemes which, for facility of reference, is printed as frontispiece of the volume, of which the columns are denoted respectively by $\{A\}$, $\{B\}$, $\{C\}$, $\{D\}$, $\{E\}$, $\{F\}$ and the rows by $\{A_0\}$, $\{B_0\}$, $\{C_0\}$, $\{D_0\}$, $\{E_0\}$, $\{F_0\}$. In each column, and in each row, all the fifteen duads of two numbers from 1, 2, 3, 4, 5, 6 occur, each once; the thirty synthemes which occur are all different, but to a syntheme occurring in any row and column there corresponds a syntheme, occurring in the same column and row, differing from the former syntheme by the interchange of the order of the second and third duads. Such a scheme is referred to by Sylvester (*Coll. Papers*, I (1844), p. 92, and *Coll. Papers*, II (1861), p. 265), and may be used in connexion with the theory of the Pascal lines of six points of a conic (Baker, *Principles of Geometry* II (1930), p. 221). But in both these cases the order of the duads in any syntheme is indifferent, while here this order is of the essence of the notation. In this scheme, any duad occurs once in any row, and once in any column; and any two duads that occur once together occur also together in another syntheme, but in reverse order. The scheme thus represents thirty points, each of which can also be characterized by the row and column in which it appears, and denoted by a symbol (PQ_0), or $(Q_0 P)$, where P is one of $A, B, \ldots, F$, and Q_0 is one of $A_0, B_0, \ldots, F_0$. For instance $(14.36.25)$ may be denoted by (AB_0). It *will be convenient then to denote* $[14.36.25]$ by $[AB_0]$; *and so in general.*

It may then be verified that in any column, as in any row, the five points represented by the synthemes form a *simplex* in the space of four dimensions, any four of these defining a prime (or space of three dimensions). In fact, the points in the first column, other than the first of these, $(14.36.25)$, or (AB_0), define the prime $[14.25.36] = 0$, or $[A_0 B] = 0$; and the points in the first row, other than the first of these, $(14.25.36)$, or $(A_0 B)$, define the prime $[14.36.25] = 0$, or $[AB_0] = 0$. Or, generally, if P, Q denote two of the letters $A, B, \ldots, F$, the points of the column $\{P\}$ other than (PQ_0) determine the prime $[P_0 Q] = 0$; and the points of the row $\{Q_0\}$ other than (PQ_0) equally lie in this prime $[P_0 Q] = 0$. (This prime, $[P_0 Q] = 0$, thus contains eight points, and we shall see that it also contains the point $(P_0 Q)$, as well as the three points (lp), (mq), (nr), if $(PQ_0) = (lp.mq.nr)$.) As we have said, in all cases the synthemes $(P_0 Q)$, or $[P_0 Q]$, are obtained respectively from (PQ_0), or $[PQ_0]$, by interchange of the second and third duads of the syntheme.

We shall speak of the simplex, whose angular points are those of the column $\{P\}$, as a *pentahedron* $\{P\}$, and equally of the simplex,