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Preface

These are the proceedings of a week-long workshop entitled ‘The Bloch–Kato conjectures for the Riemann zeta function at the odd positive integers’, which was held at the Indian Institute of Science Education and Research (IISER), Pune, India, in July 2012. This workshop was immediately followed by the 2012 Pan Asian Number Theory (PANT) conference, and there was a considerable overlap between the participants of the two events. The workshop was organized by J. Coates, S. Maity, A. Raghuram, A. Saikia and R. Sujatha, and had a total of 18 lectures given by D. Blasius, J. Coates, R. Greenberg, G. Kings, S. Lichtenbaum, A. Raghuram, A. Saikia and R. Sujatha. These proceedings contain not only expanded versions of all the lectures given at the workshop, but also several invited articles on related material.

The Riemann zeta function $\zeta(s)$ is the oldest L -function, and it remains one of the most important functions in modern number theory. The values of $\zeta(s)$ at even positive integers and odd negative integers have been known since the time of Euler. However, its values at odd positive integers remain mysterious to this day. The Bloch–Kato conjectures, also known as the Tamagawa number conjectures, provide a conjectural framework to understand the special values of all motivic L -functions. The principal aim of the workshop was to prove the Bloch–Kato conjecture in the simplest non-critical example of the values of $\zeta(s)$ at odd positive integers $s > 1$.

On behalf of all the organizers, it gives me great pleasure to thank Dr Soumen Maity and Ms Suvarna Gharat, who went far beyond the call

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of duty in handling all the local logistics for both the workshop and the conference. As a co-editor of this volume, I am grateful to Ms Ayesha Fatima and Mr Jeeten Patel, both of whom are IISER alumni, for providing critical help with LaTeX, and for putting together different tex files into one coherent volume. I also thank Professor K.N. Ganesh, Director of IISER Pune, and Dr V.S. Rao, the then Registrar of IISER Pune, for their generous and untiring support. Indeed, the Bloch–Kato workshop and the PANT conference went a long way towards establishing IISER Pune as an important centre for mathematics in India. Finally, on behalf of all the lecturers and participants, I express my gratitude to the International Center for Theoretical Sciences for its financial support for the workshop and the conference.

A. Raghuram

Coordinator for Mathematics

Indian Institute of Science Education and Research

Pune, February 6th, 2014.