

Cambridge University Press

978-1-107-49224-0 - Euclid's Book on Divisions of Figures: With a Restoration Based on  
Woepcke's text and on the Practica Geometriae of Leonardo Pisano

Raymond Clare Archibald

Frontmatter

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# EUCLID'S BOOK ON DIVISIONS OF FIGURES

(περὶ διαιρέσεων βιβλίον)

WITH A RESTORATION BASED ON  
WOEPCKE'S TEXT

AND ON THE

*PRACTICA GEOMETRIAE*  
OF LEONARDO PISANO

BY

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TO  
MY OLD TEACHER AND FRIEND  
ALFRED DEANE SMITH  
PROFESSOR OF GREEK AND LATIN  
AT MOUNT ALLISON UNIVERSITY  
FOR FORTY-FOUR YEARS  
SCHOLAR OF GREAT ATTAINMENTS  
THE WONDER OF ALL WHO KNOW HIM  
THESE PAGES ARE AFFECTIONATELY DEDICATED

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## INTRODUCTORY

EUCLID, famed founder of the Alexandrian School of Mathematics, was the author of not less than nine works. Approximately complete texts, all carefully edited, of four of these, (1) the *Elements*, (2) the *Data*, (3) the *Optics*, (4) the *Phenomena*, are now our possession. In the case of (5) the *Pseudaria*, (6) the *Surface-Loci*, (7) the *Conics*, our fragmentary knowledge, derived wholly from Greek sources, makes conjecture as to their content of the vaguest nature. On (8) the *Porisms*, Pappus gives extended comment. As to (9), the book *On Divisions (of figures)*, Proclus alone among Greeks makes explanatory reference. But in an Arabian MS., translated by Woepcke into French over sixty years ago, we have not only the enunciations of all of the propositions but also the proofs of four of them.

Whilst elaborate restorations of the *Porisms* by Simson and Chasles have been published, no previous attempt has been made (the pamphlet of Ofterdinger is not forgotten) to restore the proofs of the book *On Divisions (of figures)*. And, except for a short sketch in Heath's monumental edition of Euclid's *Elements*, nothing but passing mention of Euclid's book *On Divisions* has appeared in English.

In this little volume I have attempted :

(1) to give, with necessary commentary, a restoration of Euclid's work based on the Woepcke text and on a thirteenth century geometry of Leonardo Pisano.

(2) to take due account of the various questions which arise in connection with (a) certain MSS. of "Muhammed Bagdedinus," (b) the Dee-Commandinus book on divisions of figures.

(3) to indicate the writers prior to 1500 who have dealt with propositions of Euclid's work.

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(4) to make a selection from the very extensive bibliography of the subject during the past 400 years.

In the historical survey the MSS. of "Muhammed Bagdedinus" play an important rôle, and many recent historians, for example Heiberg, Cantor, Hankel, Loria, Suter, and Steinschneider, have contributed to the discussion. As it is necessary for me to correct errors, major and minor, of all of these writers, considerable detail has to be given in the first part of the volume; the brief second part treats of writers on divisions before 1500; the third part contains the restoration proper, with its thirty-six propositions. The Appendix deals with literature since 1500.

A score of the propositions are more or less familiar as isolated problems of modern English texts, and are also to be found in many recent English, German and French books and periodicals. But any approximately accurate restoration of the work as a whole, in Euclidean manner, can hardly fail of appeal to anyone interested in elementary geometry or in Greek mathematics of twenty-two centuries ago.

In the spelling of Arabian names, I have followed Suter.

It is a pleasure to have to acknowledge indebtedness to the two foremost living authorities on Greek Mathematics. I refer to Professor J. L. Heiberg of the University of Copenhagen and to Sir Thomas L. Heath of London. Professor Heiberg most kindly sent me the proof pages of the forthcoming concluding volume of Euclid's *Opera Omnia*, which contained the references to Euclid's book *On Divisions of Figures*. To Sir Thomas my debt is great. On nearly every page that follows there is evidence of the influence of his publications; moreover, he has read this little book in proof and set me right at several points, more especially in connection with discussions in Note 113 and Paragraph 50.

R. C. A.

BROWN UNIVERSITY,

June, 1915.

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