

Cambridge University Press

978-1-107-49224-0 - Euclid's Book on Divisions of Figures: With a Restoration Based on Woepecke's text and on the Practica Geometriae of Leonardo Pisano

Raymond Clare Archibald

Excerpt

[More information](#)

I.

Proclus, and Euclid's book On Divisions.

1. Last in a list of Euclid's works "full of admirable diligence and skilful consideration," Proclus mentions, without comment, *περὶ διαιρέσεων βιβλίον*¹. But a little later² in speaking of the conception or definition of *figure* and of the divisibility of a figure into others differing from it in kind, Proclus adds: "For the circle is divisible into parts unlike in definition or notion, and so is each of the rectilinear figures; this is in fact the business of the writer of the Elements in his Divisions, where he divides given figures, in one case into like figures, and in another into unlike³."

De Divisionibus by Muhammed Bagdedinus and the Dee MS.

2. This is all we have from Greek sources, but the discovery of an Arabian translation of the treatise supplies the deficiency. In histories of Euclid's works (for example

¹ *Procli Diadochi in primum Euclidis elementorum librum commentarii* ex rec. G. Friedlein, Leipzig, 1873, p. 69. Reference to this work will be made by "Proclus."

² Proclus¹, p. 144.

³ In this translation I have followed T. L. HEATH, *The Thirteen Books of Euclid's Elements*, I, Cambridge, 1908, p. 8. To Heath's account (pp. 8–10) of Euclid's book *On Divisions* I shall refer by "Heath."

"Like" and "unlike" in the above quotation mean, not "similar" and "dissimilar" in the technical sense, but "like" or "unlike *in definition or notion*": thus to divide a triangle into triangles would be to divide it into "like" figures, to divide a triangle into a triangle and a quadrilateral would be to divide it into "unlike" figures. (Heath.)

Cambridge University Press

978-1-107-49224-0 - Euclid's Book on Divisions of Figures: With a Restoration Based on

Woepcke's text and on the Practica Geometriae of Leonardo Pisano

Raymond Clare Archibald

Excerpt

[More information](#)

2 EUCLID'S BOOK ON DIVISIONS OF FIGURES I [2

those by Hankel⁴, Heiberg⁵, Favaro⁶, Loria⁷, Cantor⁸, Hultsch⁹, Heath³) prominence is given to a treatise *De Divisionibus*, by one “Muhammed Bagdedinus.” Of this in 1563¹⁰ a copy (in Latin) was given by John Dee to Commandinus who published it in Dee's name and his own in 1570¹¹. Recent writers whose publications appeared before 1905 have generally supposed that Dee had somewhere discovered an Arabian original of Muhammed's work and had given a Latin translation to Commandinus. Nothing contrary to this is indeed explicitly

⁴ H. HANKEL, *Zur Geschichte der Mathematik*, Leipzig, 1874, p. 234.

⁵ J. L. HEIBERG, *Litterargeschichtliche Studien über Euklid*, Leipzig, 1882, pp. 13–16, 36–38. Reference to this work will be made by “Heiberg.”

⁶ E. A. FAVARO “Preliminari ad una Restituzione del libro di Euclide sulla divisione delle figure piane,” *Atti del reale Istituto Veneto di Scienze, Lettere ed Arti*, 16, 1883, pp. 393–6. “Notizie storico-critiche sulla Divisione delle Aree” (Presentata li 28 gennaio, 1883), *Memorie del reale Istituto Veneto di Scienze, Lettere ed Arti*, XXII, 129–154. This is by far the most elaborate consideration of the subject up to the present. Reference to it will be made by “Favaro.”

⁷ G. LORIA, “Le Scienze esatte nell' antica Grecia. Libro II, Il periodo aureo della geometria Greca.” *Memorie della regia Accademia di Scienze, Lettere ed Arti in Modena*, XI₂, 1895, pp. 68–70, 220–221. *Le Scienze esatte nell' antica Grecia*, Seconda edizione. Milano, 1914, pp. 250–252, 426–427.

⁸ M. CANTOR, *Vorlesungen über Geschichte der Mathematik*, 1₃, 1907, pp. 287–8; 11₂, 1900, p. 555.

⁹ F. HULTSCH, Article “Eukleides” in Pauly-Wissowa's *Real-Encyclopädie der Class. Altertumswissenschaften*, VI, Stuttgart, 1909, especially Cols. 1040–41.

¹⁰ When Dee was in Italy visiting Commandinus at Urbino.

¹¹ *De superficierum divisionibus liber Machometo Bagdedino ascriptus nunc primum Joannis Dee Londinensis & Federici Commandini Urbinatis opera in lucem editus*. Federici Commandini de eadem re libellus. Pisauri, MDLXX. In the same year appeared an Italian translation: *Libro del modo di dividere le superficie attribuito a Machometo Bagdedino. Mandato in luce la prima volta da M. G. Dee...e da M. F...Commandino...Tradotti dal Latino in volgare da F. Viani de' Malatesti...* In Pesaro, del MDLXX... 4 unnumbered leaves and 44 numbered on one side.

An English translation from the Latin, with the following title-page, was published in the next century: *A Book of the Divisions of Superficies: ascribed to Machomet Bagdedine. Now put forth, by the pains of John Dee of London, and Frederic Commandine of Urbin. As also a little Book of Frederic Commandine, concerning the same matter. London Printed by R. & W. Leybourn, 1660*. Although this work has a separate title page and the above date, it occupies the last fifty pages (601–650) of a work dated a year later: *Euclid's Elements of Geometry in XV Books...to which is added a Treatise of Regular Solids by Campane and Flussas likewise Euclid's Data and Marinus Preface thereunto annexed. Also a Treatise of the Divisions of Superficies ascribed to Machomet Bagdadine, but published by Commandine, at the request of John Dee of London; whose Preface to the said Treatise declares it to be the Worke of Euclide, the Author of the Elements. Published by the care and Industry of John Leeke and George Serle, Students in the Mathematics. London...MDCLXI*.

A reprint of simply that portion of the Latin edition which is the text of Muhammed's work appeared in: ΕΥΚΛΕΙΔΟΥ ΤΑ ΣΩΖΟΜΕΝΑ. *Euclidis quae supersunt omnia. Ex rescensione Davidis Gregorii...Oxoniae...MDCCIII*. Pp. 665–684: “ΕΥΚΛΕΙΔΟΥ ΩΣ ΟΙΟΝΤΑΙ ΤΙΝΕΣ, ΠΕΡΙ ΔΙΑΙΡΕΣΕΩΝ ΒΙΒΛΙΟΣ. Euclidis, ut quidam arbitrantur, de divisionibus liber—vel ut alii volunt, Machometi Bagdedini liber de divisionibus superficierum.”

Cambridge University Press

978-1-107-49224-0 - Euclid's Book on Divisions of Figures: With a Restoration Based on Woepecke's text and on the Practica Geometriae of Leonardo Pisano

Raymond Clare Archibald

Excerpt

[More information](#)

2] MSS. OF MUHAMMED BAGDEDINUS AND DEE 3

stated by Steinschneider when he writes in 1905¹², "Machomet Bagdadinus (= aus Bagdad) heisst in einem alten MS. Cotton (jetzt im Brit. Mus.) der Verfasser von: de Superficierum divisione (22 Lehrsätze); Jo. Dee aus London entdeckte es und übergab es T. Commandino..." For this suggestion as to the place where Dee found the MS. Steinschneider gives no authority. He does, however, give a reference to Wenrich¹³, who in turn refers to a list of the printed books ("Impressi") of John Dee, in a life of Dee by Thomas Smith¹⁴ (1638–1710). We here find as the third in the list, "Epistola ad eximium Ducis Urbini Mathematicum, Fredericum Commandinum, praefixa libello Machometi Bagdedini de superficierum divisionibus...Pisauri, 1570. Exstat MS. in Bibliotheca Cottoniana sub Tiberio B ix."

Then come the following somewhat mysterious sentences which I give in translation¹⁵: "After the preface Lord Ussher [1581–1656], Archbishop of Armagh, has these lines: It is to be noted that the author uses Euclid's Elements translated into the Arabic tongue, which Campanus afterwards turned into Latin. Euclid therefore seems to have been the author of the Propositions [of *De Divisionibus*] though not of the demonstrations, which contain references to an Arabic edition of the Elements, and which are due to Machometus of Bagded or Babylon." This quotation from Smith is reproduced, with various changes in punctuation and typography, by Kästner¹⁶. Consideration of the latter part of it I shall postpone to a later article (5).

¹² M. STEINSCHNEIDER, "Die Europäischen Übersetzungen aus dem Arabischen bis Mitte des 17. Jahrhunderts." *Sitzungsberichte der Akademie der Wissenschaften in Wien* (Philog.-histor. Klasse) CLI, Jan. 1905, Wien, 1906. Concerning "171. Muhammed" cf. pp. 41–2. Reference to this paper will be made by "Steinschneider."

¹³ J. G. WENRICH, *De auctorum Graecorum versionibus*. Lipsiae, MDCCXLII, p. 184.

¹⁴ T. SMITH, *Vitae quorundam eruditissimorum et illustrium virorum...* Londini...MDCCVII, p. 56. It was only the first 55 pages of this "Vita Joannis Dee, Mathematici Angli," which were translated into English by W. A. Ayton, London, 1908.

¹⁵ "Post praefationem haec habet D. Usserius Archiepiscopus Armachanus. *Notandum est autem, Auctorem hunc Euclide usum in Arabicam linguam converso, quem postea Campanus Latinum fecit. Auctor igitur propositionum videtur fuisse Euclides: demonstrationum, in quibus Euclides in Arabico codice citatur, Machometus Bagded sive Babylonius.*"

It has been stated that Campanus (13. cent.) did not translate Euclid's Elements into Latin, but that the work published as his (Venice, 1482—the first printed edition of the *Elements*) was the translation made about 1120 by the English monk Athelhard of Bath. Cf. HEATH, *Thirteen Books of Euclid's Elements*, I, 78, 93–96.

¹⁶ A. G. KÄSTNER, *Geschichte der Mathematik...* Erster Band...Göttingen, 1796, pp. 272–3. See also "Zweyter" Band, 1797, pp. 46–47.

Cambridge University Press

978-1-107-49224-0 - Euclid's Book on Divisions of Figures: With a Restoration Based on Woeypke's text and on the Practica Geometriae of Leonardo Pisano

Raymond Clare Archibald

Excerpt

[More information](#)

4 EUCLID'S BOOK ON DIVISIONS OF FIGURES I [3-4

3. Following up the suggestion of Steinschneider, Suter pointed out¹⁷, without reference to Smith¹⁴ or Kästner¹⁶, that in Smith's catalogue of the Cottonian Library there was an entry¹⁸ under "Tiberius¹⁹ B ix, 6": "Liber Divisionum Mahumeti Bag-dadini." As this MS. was undoubtedly in Latin and as Cottonian MSS. are now in the British Museum, Suter inferred that Dee simply made a copy of the above mentioned MS. and that this MS. was now in the British Museum. With his wonted carefulness of statement, Heath does not commit himself to these views although he admits their probable accuracy.

4. As a final settlement of the question, I propose to show that Steinschneider and Suter, and hence also many earlier writers, have not considered all facts available. Some of their conclusions are therefore untenable. In particular:

(1) In or before 1563 Dee did *not* make a copy of any Cottonian MS.;

(2) The above mentioned MS. (Tiberius, B. ix, 6) was *never*, in its entirety, in the British Museum;

(3) The inference by Suter that this MS. was probably the Latin translation of the tract from the Arabic, made by Gherard of Cremona (1114-1187)—among the lists of whose numerous translations a "liber divisionum" occurs—*should be accepted with great reserve*;

(4) The MS. which Dee used can be stated with absolute certainty and this MS. did not, in all probability, afterwards become a Cottonian MS.

(1) Sir Robert Bruce Cotton, the founder of the Cottonian Library, was born in 1571. The Cottonian Library was not, therefore, in existence in 1563 and Dee could not then have copied a Cottonian MS.

(2) The Cottonian Library passed into the care of the nation shortly after 1700. In 1731 about 200 of the MSS.

¹⁷ H. SUTER, "Zu dem Buche 'De Superficierum divisionibus' des Muhammed Bagdedinus." *Bibliotheca Mathematica*, VI, 3, 321-2, 1905.

¹⁸ T. SMITH, *Catalogus Librorum Manuscriptorum Bibliothecae Cottonianae...* Oxonii, ...MDCXCVI, p. 24.

¹⁹ The original Cottonian library was contained in 14 presses, above each of which was a bust; 12 of these busts were of Roman Emperors. Hence the classification of the MSS. in the catalogue.

Cambridge University Press

978-1-107-49224-0 - Euclid's Book on Divisions of Figures: With a Restoration Based on Woepcke's text and on the Practica Geometriae of Leonardo Pisano

Raymond Clare Archibald

Excerpt

[More information](#)

4] MSS. OF MUHAMMED BAGDEDINUS AND DEE 5

were damaged or destroyed by fire. As a result of the parliamentary inquiry Casley reported²⁰ on the MSS. destroyed or injured. Concerning Tiberius IX, he wrote, "This volume burnt to a crust." He gives the title of each tract and the folios occupied by each in the volume. "Liber Divisionum Mahumeti Bag-dadini" occupied folios 254–258. When the British Museum was opened in 1753, what was left of the Cottonian Library was immediately placed there. Although portions of all of the leaves of our tract are now to be seen in the British Museum, practically none of the writing is decipherable.

(3) Planta's catalogue²¹ has the following note concerning Tiberius IX: "A volume on parchment, which once consisted of 272 leaves, written about the XIV. century [not the XII. century, when Gherard of Cremona flourished], containing eight tracts, the principal of which was a 'Register of William Cratfield, abbot of St Edmund'" [d. 1415]. Tracts 3, 4, 5 were on music.

(4) On "A° 1583, 6 Sept." Dee made a catalogue of the MSS. which he owned. This catalogue, which is in the Library of Trinity College, Cambridge²², has been published²³

²⁰ D. CASLEY, p. 15 ff. of *A Report from the Committee appointed to view the Cottonian Library...Published by order of the House of Commons*. London, MDCCXXXII (British Museum MSS. 24932). Cf. also the page opposite that numbered 120 in *A Catalogue of the Manuscripts in the Cottonian Library...with an Appendix containing an account of the damage sustained by the Fire in 1731; by S. Hooper... London :... MDCCCLXXVII.*

²¹ J. PLANTA, *A Catalogue of the Manuscripts in the Cottonian Library deposited in the British Museum. Printed by command of his Majesty King George III...* 1802.

In the British Museum there are three MS. catalogues of the Cottonian Library:

(1) *Harleian MS.* 6018, a catalogue made in 1621. At the end are memoranda of loaned books. On a sheet of paper bearing date Novem. 23, 1638, Tiberius B IX is listed (folio 187) with its art. 4: "liber divisione Machumeti Bagdedini." The paper is torn so that the name of the person to whom the work was loaned is missing. The volume is not mentioned in the main catalogue.

(2) *MS. No.* 36789, made after Sir Robert Cotton's death in 1631 and before 1638 (cf. *Catalogue of Additions to the MSS. in British Museum, 1900–1905...* London, 1907, pp. 226–227), contains, apparently, no reference to "Muhammed."

(3) *MS. No.* 36682 A, of uncertain date but earlier than 1654 (*Catalogue of Additions...l.c.* pp. 188–189). On folio 78 verso we find Tiberius B IX, Art. 4: "Liber divisione Machumeti Bagdedini."

A "Muhammed" MS. was therefore in the Cottonian Library in 1638.

The anonymously printed (1840?) "Index to articles printed from the Cotton MSS., & where they may be found" which may be seen in the British Museum, only gives references to the MSS. in "Julius."

²² A transcription of the Trinity College copy, by Ashmole, is in MS. Ashm. 1142. Another autograph copy is in the British Museum: Harleian MS. 1879.

²³ *Camden Society Publications*, XIX, London, M.DCCC.XLII.

Cambridge University Press

978-1-107-49224-0 - Euclid's Book on Divisions of Figures: With a Restoration Based on Woepecke's text and on the Practica Geometriae of Leonardo Pisano

Raymond Clare Archibald

Excerpt

[More information](#)

6 EUCLID'S BOOK ON DIVISIONS OF FIGURES I [4-5

under the editorship of J. O. Halliwell. The 95th item described is a folio parchment volume containing 24 tracts on mathematics and astronomy. The 17th tract is entitled "Machumeti Bagdedini liber divisionum." As the contents of this volume are entirely different from those of Tiberius IX described above, in (3), it seems probable that there were two copies of "Muhammed's" tract, while the MS. which Dee used for the 1570 publication was undoubtedly his own, as we shall presently see. If the two copies be granted, there is no evidence against the Dee copy having been that made by Gherard of Cremona.

5. There is the not remote possibility that the Dee MS. was destroyed soon after it was catalogued. For in the same month that the above catalogue was prepared, Dee left his home at Mortlake, Surrey, for a lengthy trip in Europe. Immediately after his departure "the mob, who execrated him as a magician, broke into his house and destroyed a great part of his furniture and books²⁴..." many of which "were the written bookes²⁵." Now the Dee catalogue of his MSS. (MS. O. iv. 20), in Trinity College Library, has numerous annotations²⁶ in Dee's handwriting. They indicate just what works were (1) destroyed or stolen ("Fr.")²⁷ and (2) left ("T.")²⁸ after the raid. Opposite the titles of the tracts in the volume including the tract "liber divisionum," "Fr." is written, and opposite the title "Machumeti Bagdedini liber divisionum" is the following note: "Curavi imprimi Urbini in Italia per Federicum Commandinum exemplari descripto ex vetusto isto monumento (?) per me ipsum." Hence, as stated above, it is now definitely known (1) that the MS. which Dee used was his own, and (2) that some 20 years after he made a copy, the MS. was stolen and probably destroyed²⁹.

On the other hand we have the apparently contradictory

²⁴ *Dictionary of National Biography*, Article, "Dee, John."

²⁵ "The compendious rehearsall of John Dee his dutifull declaration A. 1592" printed in *Chetham Miscellanies*, vol. I, Manchester, 1851, p. 27.

²⁶ Although Halliwell professed to publish the Trinity MS., he makes not the slightest reference to these annotations.

²⁷ "Fr." is no doubt an abbreviation for *Furatum*.

²⁸ "T.", according to Ainsworth (*Latin Dictionary*), was put after the name of a soldier to indicate that he had survived (*superstes*). Whence this abbreviation?

²⁹ The view concerning the theft or destruction of the MS. is borne out by the fact that in a catalogue of Dee's Library (British Museum MS. 35213) made early in the seventeenth century (*Catalogue of Additions and Manuscripts... 1901*, p. 211), Machumeti Bagdedini is not mentioned.

Cambridge University Press

978-1-107-49224-0 - Euclid's Book on Divisions of Figures: With a Restoration Based on Woepecke's text and on the Practica Geometriae of Leonardo Pisano

Raymond Clare Archibald

Excerpt

[More information](#)

5-6] MSS. OF MUHAMMED BAGDEDINUS AND DEE 7

evidence in the passage quoted above (Art. 2) from the life of Dee by Smith¹⁴ who was also the compiler of the Catalogue of the Cottonian Library. Smith was librarian when he wrote both of these works, so that any definite statement which he makes concerning the library long in his charge is not likely to be successfully challenged. Smith does not however say that Dee's "Muhammed" MS. was in the Cottonian Library, and if he knew that such was the case we should certainly expect some note to that effect in the catalogue¹⁸; for in three other places in his catalogue (Vespasian B x, A II₁₃, Galba E VIII), Dee's original ownership of MSS. which finally came to the Cottonian Library is carefully remarked. Smith does declare, however, that the Cottonian MS. bore, "after the preface," certain notes (which I have quoted above) by Archbishop Ussher (1581-1656). Now it is not a little curious that these notes by Ussher, who was not born till after the Dee book was printed, should be practically identical with notes in the printed work, just after Dee's letter to Commandinus (Art. 3). For the sake of comparison I quote the notes in question³⁰; "To the Reader.—I am here to advertise thee (kinde Reader) that this author which we present to thee, made use of Euclid tranflated into the Arabick Tongue, whom afterwards Campanus made to speake Latine. This I thought fit to tell thee, that fo in fearching or examining the Propofitions which are cited by him, thou mightest not fometime or other trouble thy selfe in vain, Farewell."

The Dee *MS.* as published did not have any preface. We can therefore only assume that Ussher wrote in a MS. which *did* have a preface the few lines which he may have seen in Dee's printed book.

6. Other suggestions which have been made concerning "Muhammed's" tract should be considered. Steinschneider asks, "Ob identisch de Curvis superficibus, von einem Muhammed, MS. Brit. Mus. Harl. 623⁶ (I, 191)³¹?" I have examined this MS. and found that it has nothing to do with the subject matter of the Dee tract.

But again, Favaro states³²: "Probabilmente il manoscritto

³⁰ This quotation from the Leeke-Serle Euclid¹¹ is an exact translation of the original.

³¹ This should be 625⁶ (I, 391).

³² Favaro, p. 140. Cf. Heiberg, p. 14. This suggestion doubtless originated with Offerdinger³⁸, p. [1].

Cambridge University Press

978-1-107-49224-0 - Euclid's Book on Divisions of Figures: With a Restoration Based on

Woepcke's text and on the Practica Geometriae of Leonardo Pisano

Raymond Clare Archibald

Excerpt

[More information](#)

8 EUCLID'S BOOK ON DIVISIONS OF FIGURES I [6

del quale si servi il Dee è lo stesso indicato dall' Heilbronner³³ comme esistente nella Biblioteca Bodleiana di Oxford." Under date "6. 3. 1912" Dr A. Cowley, assistant librarian in the Bodleian, wrote me as follows: "We do not possess a copy of Heilbronner's Hist. Math. Univ. In the old catalogue of MSS. which he would have used, the work you mention is included—but is really a printed book and is only included in the catalogue of MSS. because it contains some manuscript notes—

"Its shelf-mark is Savile T 20.

"It has 76 pages in excellent condition. The title page has: De Superficierum | divisionibus liber | Machometo Bagdadino | ascriptus | nunc primum Joannis Dee | ... | opera in lucem editus | ...Pisauri MDLXX.

"The MS. notes are by Savile, from whom we got the collection to which this volume belongs."

The notes were incorporated into the Gregory edition¹¹ of the Dee tract. Here and elsewhere³⁴ Savile objected to attributing the tract to Euclid as author³⁵. His arguments

³³ J. C. HEILBRONNER, *Historia matheseos Universae*...Lipsiae, MDCCXLII, p. 620: ("Manuscripta mathematica in Bibliotheca Bodlejiana") "³⁴ Mohammedis Bagdadeni liber de superficierum divisionibus, cum Notis H. S."

³⁴ H. SAVILE, *Praelectiones tresdecim in principium elementorum Euclidis, Oxonii habitae M.DC.XX*. Oxonii..., 1621, pp. 17–18.

³⁵ Dee's statement of the case in his letter to Commandinus (Leeke-Serle Euclid¹¹, cf. note 30) is as follows: "As for the authors name, I would have you understand, that to the very old Copy from whence I writ it, the name of MACHOMET BAGDEDINE was put in ziphers or Characters, (as they call them) who whether he were that *Albategnus* whom *Copernicus* often cites as a very considerable Author in Astronomie; or that Machomet who is said to have been *Alkindus's* scholar, and is reported to have written somewhat of the art of Demonstration, I am not yet certain of: or rather that this may be deemed a Book of our *Euclide*, all whose Books were long since turned out of the Greeke into the Syriack and Arabick Tongues. Whereupon, It being found some time or other to want its Title with the *Arabians* or *Syrians*, was easily attributed by the transcribers to that most famous Mathematician among them, Machomet: which I am able to prove by many testimonies, to be often done in many Moniments of the Ancients; ...yea further, we could not yet perceive so great acuteness of any *Machomet* in the Mathematicks, from their moniments which we enjoy, as everywhere appears in these Problems. Moreover, that *Euclide* also himself wrote one Book *περί διαίρέσεων*, that is to say, of *Divisions*, as may be evidenced from Proclus's Commentaries upon his first of *Elements*: and we know none other extant under this title, nor can we find any, which for excellencie of its treatment, may more rightfully or worthily be ascribed to *Euclid*. Finally, I remember that in a certain very ancient piece of Geometry, I have read a place cited out of this little Book in expresse words, even as from almost (*sic*) certain work of *Euclid*. Therefore we have thus briefly declared our opinions for the present, which we desire may carry with them so much weight, as they have truth in them.... But whatsoever that Book of *Euclid* was concerning Divisions, certainly this is such an one as may be both very profitable for the studies of many, and also bring much honour and renown to every most noble ancient Mathematician; for the most excellent acutenesse of the invention, and the most accurate discussing of all the Cases in each Probleme...."

Cambridge University Press

978-1-107-49224-0 - Euclid's Book on Divisions of Figures: With a Restoration Based on Woepcke's text and on the Practica Geometriae of Leonardo Pisano

Raymond Clare Archibald

Excerpt

[More information](#)

6-7]

THE WOEPCKE-EUCLID MS.

9

are summed up, for the most part, in the conclusions of Heiberg followed by Heath: "the Arabic original could not have been a direct translation from Euclid, and probably was not even a direct adaptation of it; it contains mistakes and unmathematical expressions, and moreover does not contain the propositions about the division of a circle alluded to by Proclus. Hence it can scarcely have contained more than a fragment of Euclid's work."

The Woepcke-Euclid MS.

7. On the other hand Woepcke found in a MS. (No. 952. 2 Arab. Suppl.) of the Bibliothèque nationale, Paris, a treatise in Arabic on the division of plane figures, which he translated, and published in 1851³⁶. "It is expressly attributed to Euclid in the MS. and corresponds to the description of it by Proclus. Generally speaking, the divisions are divisions into figures of the same kind as the original figures, e.g. of triangles into triangles; but there are also divisions into 'unlike' figures, e.g. that of a triangle by a straight line parallel to the base. The missing propositions about the division of a circle are also here: 'to divide into two equal parts a given figure bounded by an arc of a circle and two straight lines including a given angle' and 'to draw in a given circle two parallel straight lines cutting off a certain part of a circle.' Unfortunately the proofs are given of only four propositions (including the two last mentioned) out of 36, because the Arabian translator found them too easy and omitted them." That the omission is due to the translator and did not occur in the original is indicated in two ways, as Heiberg points out. Five auxiliary propositions (Woepcke 21, 22, 23, 24, 25) of which no use is made are introduced. Also Woepcke 5 is: "...and we divide the triangle by a construction analogous to the preceding construction"; but no such construction is given.

The four proofs that are given are elegant and depend

³⁶ F. WOEPCKE, "Notice sur des traductions Arabes de deux ouvrages perdus d'Euclide" *Journal Asiatique*, Septembre-Octobre, 1851, XVIII₄, 217-247. Euclid's work *On the division (of plane figures)*: pp. 233-244. Reference to this paper will be made by "Woepcke." In *Euclidis opera omnia*, vol. 8, now in the press, there are "Fragmenta collegit et disposuit J. L. Heiberg," through whose great courtesy I have been enabled to see the proof-sheets. First among the fragments, on pages 227-235, are (1) the Proclus references to *περὶ διαίρεσεων*, and (2) the Woepcke translation mentioned above. In the article on Euclid in the last edition of the *Encyclopaedia Britannica* no reference is made to this work or to the writings of Heiberg, Hultsch, Steinschneider and Suter.

Cambridge University Press

978-1-107-49224-0 - Euclid's Book on Divisions of Figures: With a Restoration Based on Woepcke's text and on the *Practica Geometriae* of Leonardo Pisano

Raymond Clare Archibald

Excerpt

[More information](#)

10 EUCLID'S BOOK ON DIVISIONS OF FIGURES I [7-10

only on the propositions (or easy deductions from them) of the *Elements*, while Woepcke 18 has the true Greek ring: "to apply to a straight line a rectangle equal to the rectangle contained by AB , AC and *deficient by a square*."

8. To no proposition in the Dee MS. is there word for word correspondence with the propositions of Woepcke but in content there are several cases of likeness. Thus, Heiberg continues,

- Dee 3 = Woepcke 30 (a special case is Woepcke 1);
- Dee 7 = Woepcke 34 (a special case is Woepcke 14);
- Dee 9 = Woepcke 36 (a special case is Woepcke 16);
- Dee 12 = Woepcke 32 (a special case is Woepcke 4).

Woepcke 3 is only a special case of Dee 2; Woepcke 6, 7, 8, 9 are easily solved by Dee 8. And it can hardly be chance that the proofs of exactly these propositions in Dee should be without fault. That the treatise published by Woepcke is no fragment but the complete work which was before the translator is expressly stated³⁷, "fin du traité." It is moreover a well ordered and compact whole. Hence we may safely conclude that Woepcke's is not only Euclid's own work but the whole of it, except for proofs of some propositions.

9. For the reason just stated the so-called *Wiederherstellung* of Euclid's work by Ofterdinger³⁸, based mainly on Dee, is decidedly misnamed. A more accurate description of this pamphlet would be, "A translation of the Dee tract with indications in notes of a certain correspondence with 15 of Woepcke's propositions, the whole concluding with a translation of the enunciations of 16 of the remaining 21 propositions of Woepcke not previously mentioned." Woepcke 30, 31, 34, 35, 36 are not even noticed by Ofterdinger. Hence the claim I made above ("Introductory") that the first real restoration of Euclid's work is now presented. Having introduced Woepcke's text as one part of the basis of this restoration, the other part demands the consideration of the

Practica Geometriae of Leonardo Pisano (Fibonacci).

10. It was in the year 1220 that Leonardo Pisano, who occupies such an important place in the history of mathematics

³⁷ Woepcke, p. 244.

³⁸ L. F. OFTERDINGER, *Beiträge zur Wiederherstellung der Schrift des Euklides über der Theilung der Figuren*, Ulm, 1853.