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Arithmetic and Geometry

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Preface

The trimester on "Arithmetic and Geometry" at the Hausdorff Research Institute for Mathematics (University of Bonn) took place in January – April 2013. In the next few pages the reader will find a list of the participants of the trimester, the descriptions of the session on Serre's conjecture, conducted by L.V. Dieulefait and J.-P. Wintenberger, and of the session on counting rational points on algebraic varieties, conducted by D.R. Heath-Brown, the programmes of the workshop on Serre's conjecture and of the final research conference, and a list of the talks given at the HIM during the trimester. The participants were invited to submit their papers for publication in this volume. The papers appearing in the volume have been carefully refereed.

Acknowledgements. We wish to record our gratitude to the Hausdorff Research Institute, and in particular to its director, Professor Dr W. Lück, for the hospitality and financial support we received.

L.V. Dieulefait, G. Faltings, D.R. Heath-Brown, Yu.I. Manin,
B.Z. Moroz, and J.-P. Wintenberger (editors).

Introduction

The main theme of the trimester was the interplay of different methods used in modern number theory. We wish to emphasize the new results and conjectures in arithmetic geometry, having direct bearing on the classical number theoretic problems. Two sessions, on the recently proved Serre's conjecture from 15 January to 14 February (organizers: L. Dieulefait and J.-P. Wintenberger) and on counting rational points on algebraic varieties from 15 March to 14 April (organizer: D.R. Heath-Brown), as well as a couple of shorter workshops, several seminars, and mini-courses were organized. The trimester culminated in a research conference from 15 to 19 April.

The aim of the session "Serre's conjecture" was to report on recent works linked to that conjecture, in particular about Galois representations and automorphic representations. During the weeks starting on 14 January and 21 January, Henri Carayol lectured on his work on the algebraic properties of Griffiths-Schmid varieties. The Griffiths-Schmid varieties are analytic varieties classifying Hodge structures. Studying their algebraic properties might be a step towards constructing Galois representations associated to automorphic representations appearing in the cohomology of these varieties. Our second theme related to the recent work of Michael Harris, Kai-Wen Lan, Richard Taylor and Jack Thorne, who have constructed Galois representations associated to not necessarily self-dual automorphic representations. The proof heavily relies on p-adic properties of automorphic representations.

The aim of the session "counting rational points on algebraic varieties" was to report on recent works on the existence, frequency and distribution of rational points on algebraic varieties. Thus the main themes were local to global principles, Manin's conjecture, developments of the Hardy-Littlewood method and the determinant method.

List of participants

Victor Abrashkin (University of Durham)
Rajender Adibhatla (Universität Regensburg)
Shabnam Akhtari (University of Oregon)
Paloma Bengoechea (College de France)
Denis Benois (Université Bordeaux 1)
Tobias Berger (University of Sheffield)
Yuri Bilu (IMB Université Bordeaux I)
Marco Boggi (University of Los Andes)
Pierre Le Boudec (Institute of Advanced Study)
Régis de la Bretéche (Institut de Mathématiques de Jussieu – Paris Rive Gauche, UMR 7586 Université Paris-Diderot)
Christophe Breuil (Université Paris-Sud)
Tim Browning (University of Bristol)
Jörg Brüdern (Universität Göttingen)
Roman Budylin (Steklov Mathematical Institute)
Alexandru Buium (University of New Mexico)
Alberto Cámara (University of Nottingham)
Henri Carayol (L'Université de Strasbourg et du CNRS)
Magnus Carlson (University of Stockholm)
Tommaso Centeleghe (Universität Heidelberg)
Huan Chen (Ecole normale superieure ENS)
Narasimha Kumar Cheraku (Ruprecht Karls Universität Heidelberg)
Przemyslaw Chojecki (Institut Mathématique de Jussieu)
Laurent Clozel (Université de Paris Sud 11)
Jean-Louis Colliot-Thélène (Université Paris-Sud)
David Mendes da Costa (University of Bristol)
Tuan Ngo Dac (Université de Paris 13)

Ulrich Derenthal (Universität München)
Fred Diamond (King's College London)
Rainer Dietmann (Royal Holloway, University of London)
Luis Dieulefait (Universitat de Barcelona)
Gerd Faltings (Max-Planck-Institut für Mathematik)
Ivan Fesenko (University of Nottingham)
Nuno Freitas (Universitat de Barcelona)
Wojciech Jerzy Gajda (The Adam Mickiewicz University)
David Geraghty (Institute for Advanced Study)
Sergey Gorchinskiy (Steklov Mathematical Institute)
Frank Gounelas (Humboldt University)
Xavier Guitart (Universität Duisburg-Essen)
Shuvra Gupta (University of Iowa)
Shai Haran (Technion – Israel Institute of Technology)
Michael Harris (Institut de Mathématiques de Jussieu)
Roger Heath-Brown (University of Oxford)
Florian Herzig (University of Toronto)
Alexander Ivanov (Universität Heidelberg)
Mikhail Ivanov (Saint-Petersburg State University)
Andrew Kresch (Universität Zürich)
Lars Kühne (SNS Pisa)
Boris Kunyavskii (Bar-Ilan University)
Leonid Kuzmin (National Research Center Kurchatov Institute)
Kai-Wen Lan (University of Minnesota)
Dmitry Logachev (Universidad Simon Bolivar)
Oliver Lorscheid (IMPA)
Daniel Loughran (Leibniz Universität Hannover)
Yuri Manin (Max-Planck-Institut für Mathematik)
Oscar Marmon (Georg-August-Universität Göttingen)
Lilian Matthiesen (University of Bristol)
David McKinnon (University of Waterloo)
Boris Moroz (Universität Bonn)
Jeanine Van Order (The Hebrew University of Jerusalem)
Denis Osipov (Steklov Mathematical Institute)
Ambrus Pál (Imperial College London)
Aftab Pande (Cidade Universitária Ilha do Fundao)
Alexej Parshin (Steklov Mathematical Institute)
Florian Pop (University of Pennsylvania)
Dinakar Ramakrishnan (Caltech)
Giovanni Rosso (Université Paris 13)

Mohamed Saidi (Exeter University)
Per Salberger (Chalmers University of Technology)
Damaris Schindler (University of Bristol)
Mehmet Haluk Sengun (University of Warwick)
Evgeny Shinder (Max-Planck-Institut für Mathematik)
Ceclia Salgado Guimaraes da Silva (UFRJ)
Alexei Skorobogatov (Imperial College London)
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Aleksander Smirnov (Steklov Institute of Mathematics in St. Petersburg)
Efthymios Sofos (University of Bristol)
Cesar Alejandro Soto Posada (Universität Tübingen)
Mike Swarbrick Jones (University of Bristol)
Jack Thorne (Harvard University)
Jacques Tilouine (Université Paris 13)
Yuri Tschinkel (New York University)
Pankaj Hemant Vishe (University of York)
Sergei Vostokov (University of St. Petersburg)
Yosuhiro Wakabayashi (Kyoto University)
Gabor Wiese (Université du Luxembourg)
Nicholas Williams (University of Exeter)
Jean-Pierre Wintenberger (Université Strasbourg)
Trevor Wooley (University of Bristol)
Yanhong Yang (Universität Mainz)
Adrin Zenteno (Universidad Nacional Autónoma de México)

Trimester Seminar

January 8, Jeanine Van Order, Iwasawa main conjectures for $GL(2)$ via Howard's criterion (abstract). In this talk, I will present the Iwasawa main conjectures for Hilbert modular eigenforms of parallel weight two in dihedral or anticyclotomic extensions of CM fields. The first part will include an overview of known results, as well as some discussion of open problems and applications (e.g. to bounding Mordell-Weil ranks), and should be accessible to the non-specialist. The second part will describe the p-adic L-functions in more detail, as well as the non-vanishing criterion of Howard (and its implications for the main conjectures).

January 15, Oliver Lorscheid, A blueprinted view on F_1-geometry (abstract). A blueprint is an algebraic structure that "interpolates" between multiplicative monoids and semirings. The associated scheme theory applies to several problems in F_1-geometry: Tits's idea of Chevalley groups and buildings over F_1, Euler characteristics as the number of F_1-rational points, total positivity, K-theory, Arakelov compactifications of arithmetic curves; and it has multiple connections to other branches of algebraic geometry: Lambda-schemes (after Borger), log schemes (after Kato), relative schemes (after Toen and Vaquie), congruence schemes (after Berkovich and Deitmar), idempotent analysis, analytic spaces and tropical geometry. After a brief overview and an introduction to the basic definitions of this theory, we focus on the combinatorial aspects of blue schemes. In particular, we explain how to realize Jacques Tits's idea of Weyl groups as Chevalley groups over F_1 and Coxeter complexes as buildings over F_1. The central concepts are the rank space of a blue scheme and the Tits category, which make the idea of "F_1-rational points" rigorous.

January 16, Jean-Pierre Wintenberger, Introduction to Serre's modularity conjecture (abstract). This lecture is intended for non-specialists. We state Serre's modularity conjecture and give some consequences and hints on its proof.

January 17, Henri Carayol, Realization of some automorphic forms and rationality questions (Part I) (abstract). In this first (and mostly introductory) talk I shall recall some (well-known) facts on the realization of automorphic forms in the cohomology groups of some geometric objects, and the relation with the arithmetic properties of such forms. I shall introduce locally symmetric varieties, Shimura varieties and the more exotic Griffiths-Schmid varieties. I shall discuss the case of automorphic forms whose archimedean component is a limit of discrete series. In the case of degenerate limits, the only known realization uses the coherent cohomology of Griffiths-Schmid varieties.

January 21, Günter Harder, Modular construction of mixed motives and congruences (Part I) (abstract). Starting from a Shimura variety S and its compactification S^v we construct certain objects, which can be thought of as being mixed motives. These mixed motives give rise to certain elements of Ext_1 groups. We can use the theory of Eisenstein cohomology to compute the Hodge-de-Rham extension classes of these extensions. We also have some conjectural formulas for these extensions as Galois modules. Assuming the correctness of these formulas for the Galois extension class we can derive congruences between eigenvalues of Hecke operators acting on the cohomology of different arithmetic groups, these congruences are congruences modulo primes l dividing certain special values of L-functions. These congruences have been verified experimentally in many cases. They imply the reduciblity of certain Galois-representations mod l.

January 22, Yuri Manin, Non-commutative generalized Dedekind symbols (abstract). Classical Dedekind symbol was introduced and studied in connection with functional equation of Dedekind eta-function. Later it was generalized and had multiple applications, in particular to topological invariants. I will define and study generalized Dedekind symbols with values in non-necessarily commutative groups, extending constructions of Sh. Fukuhara done in the commutative context. Basic examples of such symbols are obtained by replacing period integrals of modular forms by iterated period integrals. I will also explain the interpretation of such symbols in terms of non-commutative 1-cocycles.

January 23, Henri Carayol, Realization of some automorphic forms and rationality questions (Part II) (abstract). This talk is a continuation of part I.

January 24, Michael Harris, Eisenstein cohomology and construction of Galois representations (Part I) (abstract). I will report on some aspects of the joint work with Lan, Taylor, and Thorne, which attaches compatible families of l-adic Galois representations to a cuspidal cohomological automorphic

representation of $GL(n)$ of a CM field. Earlier work by many authors had treated the case where the automorphic representation is dual to its image under complex conjugation; under this hypothesis, the Galois representations in question, or closely related representations, can be obtained directly in the cohomology with twisted coefficients of Shimura varieties attached to unitary groups. Without the duality hypothesis, this is no longer possible; instead, the representations are constructed by p-adic approximation of Eisenstein cohomology classes by cuspidal classes in an appropriate (infinite-dimensional) space of p-adic modular forms. The lectures will concentrate on the construction of Eisenstein classes, the relation to p-adic modular forms and the definition of Galois representations by p-adic approximation.

January 25, Michael Harris, Eisenstein cohomology and construction of Galois representations (Part II) (abstract). This talk is a continuation of part I.

January 28, Günter Harder, Modular construction of mixed motives and congruences (Part II) (abstract). This talk is a continuation of part I.

January 28, Fred Diamond, The weight part of Serre's conjecture for $GL(2)$ over totally real fields (abstract). I will review the statement of the weight part of Serre's conjecture for $GL(2)$ over totally real fields. I will describe what has been proved by Gee and his coauthors, and give a brief overview of the methods.

January 30, Luis Dieulefait, Non-solvable base change for $GL(2)$ (abstract). We will show that any classical cuspidal modular form can be lifted to any totally real number field. The proof uses a recent Modularity Lifting Theorem proved by Barnet-Lamb, Gee, Geraghty and Taylor (plus a variant of it proved by Gee and the speaker) and another one by Kisin that is used in the "killing ramification" step. The core of the proof is the construction of a "safe" chain of congruences linking to each other any given pair of cuspforms. The safe chain that we will construct is also a key input in the proof of other cases of Langlands functoriality, but this will be explained in another talk (see the abstracts for the conference week).

January 31, Kai-Wen Lan, Galois representations for regular algebraic cuspidal automorphic representations over CM fields (part I) (abstract). I will report on my joint work with Michael Harris, Richard Taylor and Jack Thorne on the construction of p-adic Galois representations for regular algebraic cuspidal automorphic representations of $GL(n)$ over CM (or totally real) fields, without hypotheses on self-duality or ramification. (This should be considered as part III of a series of four talks, the first two being given by Michael Harris in the previous week.)

February 1, Kai-Wen Lan, Galois representations for regular algebraic cuspidal automorphic representations over CM fields (part II) (abstract). This is a continuation of part I.

February 12, David Geraghty, The Breuil-Mezard conjecture for quaternion algebras (abstract). The Breuil-Mezard conjecture relates the complexity of certain deformation rings for mod p representations of the Galois group of Q_p with the representation theory of $GL_2(F_p)$. Most cases of the conjecture were proved by Kisin who established a link between the conjecture and modularity lifting theorems. In this talk I will discuss a generalization of the conjecture to quaternion algebras (over an arbitrary finite extension of Q_p) and show how it follows from the original conjecture for $GL(2)$. This is a joint work with Toby Gee.

February 18, Boris Kunyavskii, Geometry and arithmetic of word maps in simple matrix groups (abstract). We wil discuss various geometric and arithmetic properties of matrix equations of the form

$$P(X_1, \dots, X_d) = A,$$

where the left-hand side is an associative non-commutative monomial in X_i's and their inverses, and the right-hand side is a fixed matrix. Solutions are sought in some group $G \subset GL(n, R)$. We will focus on the case where the group G is simple, or close to such. We will give a survey of classical and recent results and open problems concerning this equation, concentrating around the following questions (posed for geometrically and/or arithmetically interesting rings and fields R): is it solvable for any A?, is it solvable for a "typical" A?, does it have "many" solutions?, does the set of solutions possess "good" local–global properties?, to what extent does the set of solutions depend on A?. The last question will be discussed in some detail for the case $G = SL(2, q)$ and $d = 2$, where criteria for equidistribution were obtained in our recent joint work with T. Bandman.

February 19, Shai Haran, Non-additive geometry (abstract). We give a language for algebraic geometry based on non-additive generalized rings. In this language, number fields look more like curves over a finite field. The initial object of generalized rings is the "field with one element". This language "sees" the real and complex primes of a number field, and there is a compactificaton $\widetilde{\mathrm{Spec}}\ \mathfrak{O}_K$ of $\mathrm{Spec}\ \mathfrak{O}_K$, $\mathfrak{O}_K$ being the ring of integers of a number field K. The arithmetic surface $\widetilde{\mathrm{Spec}}\ \mathfrak{O}_K \ \Pi\ \widetilde{\mathrm{Spec}}\ \mathfrak{O}_K$ exists and is not reduced to its diagonal. And yet most of the Grothendieck algebraic geometry works with generalized rings replacing commutative rings.

February 20, Aleksander Smirnov, The internal and external problems of algebraic geometry over F_1 (abstract). An introduction to Durov's approach will be given. The theory will be illustrated with several explicit examples. Besides, we plan to discuss some problems caused by both the development of the theory and the demands of its applications.

February 21, Marina Viazovska, CM values of higher Green's functions and regularized Petersson products (abstract). Higher Green functions are real-valued functions of two variables on the upper half-plane, which are bi-invariant under the action of a congruence subgroup, have a logarithmic singularity along the diagonal, and satisfy the equation $\Delta f = k(1-k)f$; here Δ is a hyperbolic Laplace operator and k is a positive integer. The significant arithmetic properties of these functions were disclosed in the paper of B. Gross and D. Zagier "Heegner points and derivatives of L-series" (1986). In the particular case when $k = 2$ and one of the CM points is equal to $\sqrt{-1}$, the conjecture has been proved by A. Mellit in his Ph.D. thesis. In this lecture we prove that conjecture for arbitrary k, assuming that all the pairs of CM points lie in the same quadratic field. The two main parts of the proof are as follows. We first show that the regularized Petersson scalar product of a binary theta-series and a weight one weakly holomorphic cusp form is equal to the logarithm of the absolute value of an algebraic integer and then prove that the special values of weight k Green's function, occurring in the conjecture of Gross and Zagier, can be written as the Petersson product of that type, where the form of weight one is the $k-1$st Rankin-Cohen bracket of an explicitly given holomorphic modular form of weight $2-2k$ and a binary theta-series. Algebraicity of regularized Petersson products was also proved at about the same time by W. Duke and Y. Li by a different method; however, our result is stronger since we also give a formula for the factorization of the algebraic number in question.

February 26, David Mendes da Costa, Integral points on elliptic curves and the Bombieri-Pila bounds (abstract). In 1989, Bombieri and Pila found upper bounds for the number of integer points of (naive exponential) height at most B lying on a degree d affine plane curve C. In particular, these bounds are both uniform with respect to the curve C and the best possible with this constraint. It is conjectured though that if we restrict to curves with positive genus then the bounds can be broken. In this talk we shall discuss progress towards this conjecture in the case of elliptic curves and an application to counting rational points on degree 1 del Pezzo surfaces.

February 27, Lars Kühne, Effective and uniform results of André-Oort type (abstract). The André-Oort Conjecture (AOC) states that the irreducible components of the Zariski closure of a set of special points in a Shimura variety are special subvarieties. Here, a special variety means an irreducible

component of the image of a sub-Shimura variety by Hecke correspondence. The AOC is an analogue of the classical Manin-Mumford conjecture on the distribution of torsion points in abelian varieties. In fact, both conjectures are considered as special instances of the far-reaching Zilber-Pink conjecture(s). I will present a rarely known approach to the AOC that goes back to Yves André himself: Before the model-theoretic proofs of the AOC in certain cases by the Pila-Wilkie-Zannier approach, André presented in 1998 the first proof of the AOC in a non-trivial case, namely, a product of two modular curves. In my talk, I discuss several results in the style of André's method, allowing to compute all special points in a non-special curve of a product of two modular curves. These results are effective – as opposed to the results that could be obtained by the Pila-Wilkie-Zannier approach – and have also the further advantage of being uniform in the degrees of the curve and its field of definition. For example, this allows to show that, in fact, there are no two singular moduli x and y satisfying $x + y = 1$.

February 28, Nuno Freitas, Fermat-type equations of signature (r, r, p) (abstract). In this talk I plan to discuss how a modular approach via the Hilbert cusp-forms can be used to attack equations of the form $x^r + y^r = Cz^p$, where r is a fixed prime and p varies. We first relate a possible solution of that equation to solutions of several related Diophantine equations over certain totally real fields F. Then we attach Frey curves E over F to the solutions of the latter equations. After proving modularity of E and irreducibility of certain Galois representations attached to E we can use the modular approach. We apply the method to solve equations in the particular case of signature $(13, 13, p)$.

March 4, Leonid Kuzmin, l-adic regulator of an algebraic number field and Iwasawa theory (abstract). We give a new definition of the l-adic regulator, which makes sense for any (not necessarily totally real) algebraic number field, present a few results and conjectures, relating to that notion, and discuss the behaviour of the l-adic regulator in a $\mathbb{Z}_l$-cyclotomic extension of the field.

March 4, Alexander Ivanov, Arithmetic and anabelian geometry of stable sets of primes in number fields (abstract). We define a new class of sets – stable sets – of primes in number fields. For example, Chebotarev sets are very often stable. Those sets have positive (but arbitrary small) Dirichlet density and generalize the notion of a set of density 1, in the sense that arithmetic theorems like certain Hasse principles, the Grünwald-Wang theorem, the Riemann existence theorem, etc. hold for them. Geometrically, this allows to give examples of infinite sets with arbitrarily small positive density such that the corresponding arithmetic curves are algebraic $K(\pi, 1)$ and, using some further ideas, to generalize (a part of) the Neukirch-Uchida birational anabelian theorem to stable sets.

March 5, Tuan Ngo Dac, On the problem of counting shtukas (abstract). I will introduce the stacks of shtukas, explain its role in the Langlands program, and then report on my work on the problem of counting shtukas.

March 7, Rajender Adibhatla, Modularity of certain two-dimensional mod p^n representations of $G_{\mathbb{Q}}$ (abstract). For an odd rational prime p and integer $n > 1$, we consider certain continuous representations

$$\rho_n : G_{\mathbb{Q}} \to GL_2(\mathbb{Z}/p^n\mathbb{Z})$$

with fixed determinant, whose local restrictions "look like arising" from modular Galois representations, and whose mod p reductions are odd and irreducible. Under suitable hypotheses on the size of their images, we use deformation theory to lift ρ_n to ρ in characteristic 0. We then invoke a modularity lifting theorem of Skinner-Wiles to show that ρ is modular.

March 11, Frank Gounelas, Rationally connected varieties and free curves (abstract). The first part of this talk will be a general introduction to rationally connected varieties. I will then discuss various ways in which a variety can be "connected by curves of a fixed genus, mimicking the notion of rational connectedness". At least in characteristic zero, in the specific case of the existence of a single curve with a large deformation space of morphisms to a variety implies that the variety is in fact rationally connected. Time permitting I will discuss attempts to show this result in positive characteristic.

March 12, Tommaso Centeleghe, On the decomposition of primes in torsion fields of an elliptic curve (abstract). Let E be an elliptic curve over a number field K and N be a positive integer. In this talk we consider the problem of describing how primes P of K of good reduction for E and away from N decompose in the extension $K(E[N])|K$. As it turns out, the class Frob P in $\mathrm{Gal}(K(E[N]|K))$ can be completely described, apart of finitely many primes P, in terms of the error term $a_P(E)$ and the j-invariant of E. The Hilbert class polynomials, associated to imaginary quadratic orders, play a role in the description. The main result relies on a theorem on elliptic curves over finite fields.

March 13, Shuvra Gupta, Noether's problem and rationality of invariant spaces (abstract). In the early 1900s Emmy Noether asked the following question: If a group G acts faithfully on a vector space V (over a field k), is the field of invariants $k(V)^G$ rational, i.e. purely transcendental over k? The answer (for $k = \mathbb{Q}$ or a number field) in general is no, and we will discuss some consequences and variants of Noether's problem. We will also discuss the problem when the field k is algebraically closed, and techniques of testing rationality of the fields of invariants using unramified cohomology groups.

March 14, Mohamed Saidi, Some problems/results related to the Grothendieck anabelian section conjecture (abstract). I will discuss some (major) problems related to the Grothendieck anabelian section conjecture. I will discuss two new results related to these problems and the section conjecture. First result: there exists a local–global principle for torsors under the geometric prosolvable fundamental group of a proper hyperbolic curve over a number field. Second result: the passage in the section conjecture from number fields to finitely generated fields is possible under the assumption of finiteness of suitable Shafarevich-Tate groups.

March 18, Sergey Gorchinskiy, Parameterized differential Galois theory (abstract). Classical Galois theory studies symmetry groups of solutions of algebraic equations. Differential Galois theory studies symmetry groups of solutions of linear differential equations. We discuss the so-called parameterized differential Galois theory which studies symmetry groups of solutions of linear differential equations with parameters. The groups that arise are linear differential groups given by differential equations (not necessarily linear) on functions in parameters. We also discuss, in this connection, derivations on Abelian categories and differential Tannakian categories.

March 19, Alexandru Buium, The concept of linearity for an arithmetic differential equation (abstract). The concept of an ordinary differential equation has an arithmetic analogue in which the derivation operator is replaced by a Fermat quotient operator. We would like to understand which arithmetic differential equations should be considered as being "linear".

Classical linear differential equations arise from differential cocycles of linear algebraic groups into their Lie algebras and their differential Galois groups are algebraic groups with coefficients in the field of constants. On the other hand one can prove that there are no such cocycles in the arithmetic context. This leads one to introduce, in the arithmetic context, a new concept of "Lie algebra", "cocycles", "linear" equations, and "differential Galois groups"; the latter can be viewed as subgroups of the general linear group with coefficients in the algebraic closure of the "field with one element".

March 20, Roman Budylin, Adelic Bloch formula (abstract). Chern class $c_2(X)$ is involved in the functional equation for two-dimensional schemes. To get functional equation by the Tate method we need a local decomposition of the Chern class, satisfying some properties. Bloch proves that the second Chern class of a vector bundle with trivial determinant can be obtained by the boundary homomorphism for the universal central extension of the sheaf $\mathrm{SL}(\mathcal{O}_X)$. In the talk, this construction will be used to get an adelic formula for the second Chern class in terms of trivializations in scheme points. We will also discuss a generalization of this formula for c_n of vector bundles with $c_i = 0$ for $i < n$.

March 21, Alexej Parshin, A generalization of the Langlands correspondence and zeta-functions of a two-dimensional scheme (abstract). We introduce Abelian Langlands correspondence for algebraic surfaces defined over a finite field. When the surface is a semi-stable fibration over an algebraic curve, we define two operations, automorphic induction and base change, which connect this correspondence with the classical Langlands correspondence on the curve. Some conjectural properties of these operations imply the standard theorems for zeta- and L-functions on the surface (analytic continuation and functional equation). In this approach we do not need to use the étale cohomology theory.

March 22, Denis Osipov, Unramified two-dimensional Langlands correspondence (abstract). We will describe the local unramified Langlands correspondence for two-dimensional local fields (following an approach of M. Kapranov). For this goal, we will construct a categorical analogue of principal series representations of general linear groups of even degrees over two-dimensional local fields and describe their properties. The main ingredient of this construction is a central extension of a general linear group defined over a two-dimensional local field or over an adelic ring of a two-dimensional arithmetic scheme. We will prove reciprocity laws for such central extensions, i.e. splittings of the central extensions over some subgroups defined over rings constructed by means of points or by integral one-dimensional subschemes of a two-dimensional arithmetic scheme.

March 25, Rainer Dietmann, On quantitative versions of Hilbert's irreducibility theorem (abstract). If $f(X, Y)$ is an irreducible rational polynomial, then by Hilbert's irreducibility theorem for infinitely many rational specialisations of X the resulting polynomial in Y is still irreducible over the rationals. In this talk we want to discuss quantitative versions of this result, using recent advances from the determinant method on bounding the number of points on curves.

March 27, Roger Heath-Brown, Pairs of quadratic forms in 8 variables (abstract). We show that a smooth intersection of two quadrics in P^7, defined over a number field, satisfies the Hasse principle and weak approximation. The proof is based on the work of Colliot-Thélène, Sansuc and Swinnerton-Dyer on Chatelet surfaces, which enables one to reduce the problem to a purely local problem. The first part of the talk will discuss the background and the overall strategy of the proof, and the second part will look in a little more detail at some of the methods involved.

April 2, Pankaj Vishe, Cubic hypersurfaces and a version of the circle method over number fields (abstract). A version of the Hardy-Littlewood circle method is developed for number fields K and is used to show that any

non-singular projective cubic hypersurface over K of dimension ≥ 8 always has a K-rational point. This is a joint work with T. Browning.

April 3, Efthymios Sofos, Counting rational points on the Fermat surface (abstract). In this talk we shall discuss progress towards finding lower bound for the number of rational points of bounded height on the Fermat cubic surface. The argument is based on a uniform asymptotic estimate for the associated counting function on conics.

April 5, Jörg Brüdern and Trevor Wooley, Systems of cubic forms at the convexity barrier (abstract). We describe recent joint work concerning the validity of the Hasse principle for systems of diagonal cubic forms. The number of variables required meets the convexity barrier. Certain features of our methods are motivated by work of Gowers on Szemeredi's theorem.

April 8, Mike Swarbrick Jones, Weak approximation on cubic hypersurfaces of large dimension (abstract). A natural question in arithmetic geometry is to investigate weak approximation on varieties. If the dimension of the variety is large compared to the degree, our most successful tool is the circle method, however there are cases where using this is not feasible given our current state of knowledge. In this talk I will sketch a proof that weak approximation holds for generic cubic hypersurfaces of dimension at least 17, in particular discussing a fibration method argument that applies to the cases where the usual application of the circle method is not possible.

April 10, Arne Smeets, Local–global principles for fibrations in torsors under tori (abstract). This talk is a report on work in progress about local–global principles for varieties fibred over the projective line. In particular, we will study the Brauer-Manin obstruction to the Hasse principle and weak approximation for certain fibrations in torsors under tori, e.g. (multi-)norm form equations. Our results are conditional on Schinzel's hypothesis.

April 12, David Mendes da Costa, On uniform bounds for integral points on elliptic curves (abstract). In 1989, Bombieri and Pila proved that given a plane algebraic affine curve of degree d there are no more than $O(N^{1/d+\epsilon})$ integral points on the curve within a box of size $N \times N$. Moreover, the implied constant in their bound depended only on the degree of the curve and not on the equation. Such bounds are, in general, the best possible, however, it is believed that by restricting to curves which have positive genus one can do much better. In this talk we consider the problem of improving these uniform bounds for integral points on elliptic curves. An application of this work to degree one del Pezzo surfaces will be presented.

Workshop on Serre's conjecture

Monday, February 4

10:00 – 11:00 Fred Diamond (London King's college): Explicit Serre weights for two-dimensional Galois representations.

11:30 – 12:30 Denis Benois (Université Bordeaux 1): Trivial zeros of p-adic L-functions and Iwasawa theory.

14:30 – 15:30 Tommaso Centeleghe (Universität Heidelberg): Computing the number of certain mod p Galois representation.

16:00 – 17:00 Wojciech Gajda (UAM Poznań): Abelian varieties and l-adic representations.

Tuesday, February 5

10:00 – 11:00 Tobias Berger (University of Sheffield): Eisenstein congruences and modularity of Galois representations.

11:30 – 12:30 Christophe Breuil (Université de Paris-Sud): Ordinary representations of $\mathrm{GL}_n(\mathbb{Q}_p)$ and fundamental algebraic representations I.

14:30 – 15:30 Jeanine Van Order (EPFL Lausanne): Critical values of GL(2) Rankin-Selberg L-functions.

16:00 – 17:00 Luis Dieulefait (Universitat de Barcelona): Some new cases of Langlands functoriality solved.

Wednesday, February 6

10:00 – 11:00 Aftab Pande (Cidade Universitária Ilha do Fundao): Deformations of Galois representations and the theorems of Sato-Tate, Lang-Trotter and others.

11:30 – 12:30 Florian Herzig (University of Toronto): Ordinary representations of $\mathrm{GL}_n(\mathbb{Q}_p)$ and fundamental algebraic representations II.

14:30 – 15:30 Mehmet Sengun (University of Warwick): Mod p Cohomology of Bianchi groups and Mod p Galois representations.

16:00 – 17:00 Jack Thorne (Harvard University): Symmetric power functoriality for GL(2).

Thursday, February 7

10:00 – 11:00 Dinakar Ramakrishnan (California Institute of Technology): Picard modular surfaces, residual Albanese quotients, and rational points.

11:30 – 12:30 Jacques Tilouine (Université Paris 13): Image of Galois and congruence ideals, a program of a joint work with H. Hida.

Friday, February 8

10:00 – 11:00 David Geraghty (IAS Princeton): Modularity lifting beyond the "numerical coincidence" of the Taylor-Wiles method.

11:30 – 12:30 Florian Pop (University of Pennsylvania): Faithful representations of absolute Galois groups.

Abstracts

Denis Benois. Trivial zeros of p-adic L-functions and Iwasawa theory. We prove that the expected properties of Euler systems imply quite general Mazur-Tate-Teitelbaum type formulas for derivatives of p-adic L-functions. We also discuss the Iwasawa theory of p-adic representations in the trivial zero case.

Tobias Berger. Eisenstein congruences and modularity of Galois representations. I will report on joint work with Kris Klosin (CUNY) on congruences of Eisenstein series and cuspforms modulo prime powers and its application in proving the modularity of residually reducible Galois representations.

Tommaso Centeleghe. Computing the number of certain mod p Galois representations. We report on computations aimed to obtain, for a given prime p, the number $R(p)$ of two-dimensional odd mod p Galois representations of $G_{\mathbb{Q}}$ which are irreducible and unramified outside p. Thanks to Serre's conjecture, this amounts to a computation of the number of non-Eisenstein systems of Hecke eigenvalues arising from mod p modular forms of level one. Using well-known dimension formulas for modular forms (and the fact that any mod p eigensystem can be Tate-twisted to one arising from a weight $\leq p+1$),

an explicit upper bound $U(p)$ of $R(p)$ can be drawn. While discussing the reasons which might make the "error term" $U(p) - R(p)$ large, we stress how, in practice, one can control it from above using only one of the first Hecke operators. This gives a lower bound for $R(p)$, which coincides in many cases with $R(p)$ itself.

Fred Diamond. Explicit Serre weights for two-dimensional Galois representations. I will discuss joint work with Savitt making the set of Serre weights more explicit for indecomposable two-dimensional mod p representations of Galois groups over ramified extensions of $\mathbb{Q}_p$. In particular the results indicate a structure on the set of such weights.

Luis Dieulefait. Some new cases of Langlands functoriality solved. We combine the method of Propagation of Automorphy with recent Automorphy Lifting Theorems (A.L.T.) of Barnet-Lamb, Gee, Geraghty, and Taylor to prove some new cases of Langlands functoriality (tensor products and symmetric powers). In particular, we establish automorphy for lots of Galois representations of $G_{\mathbb{Q}}$ of arbitrarily large dimension (and their base changed counterparts). We also prove some variants of the available A.L.T., which are needed at some steps of our proof. Remark: Some technical improvements required to extend some A.L.T. to the case of "small primes" were accomplished with the kind cooperation of R. Guralnick and T. Gee.

Wojciech Gajda. Abelian varieties and l-adic representations. We will discuss monodromies for abelian varieties, and independence (in the sense of Serre) for families of some geometric l-adic representations over finitely generated fields.

David Geraghty. Modularity lifting beyond the "numerical coincidence" of the Taylor-Wiles method. Modularity lifting theorems have proven very useful since their invention by Taylor and Wiles. However, as explained in the introduction to Clozel-Harris-Taylor, they only apply in situations where a certain numerical coincidence holds. In this talk, I will describe a method to overcome this restriction. The method is conditional on the existence of Galois representations associated to integral cohomology classes (which can be established in certain cases). This is joint work with Frank Calegari.

Aftab Pande. Deformations of Galois Representations and the Theorems of Sato-Tate, Lang-Trotter and others. We construct infinitely ramified Galois representations ρ such that the sequences $a_l(\rho)$ have distributions in contrast to the statements of Sato-Tate, Lang-Trotter and others. Using similar methods, we deform a residual Galois representation for number fields and obtain an

infinitely ramified representation with very large image, generalizing a result of Ramakrishna.

Florian Pop. Faithful representations of absolute Galois groups. In his "Esquisse d'un Programme" Grothendieck suggested to study the absolute Galois group of the rationals via its representations on the algebraic fundamental group of natural categories of varieties, e.g. the Teichmueller modular tower. This leads to the intensive study of the so-called Grothendieck-Teichmueller group and its variants, and the I/OM (Ihara/Oda-Matsumoto conjecture). I plan to explain variants of I/OM, and discuss its state of the art.

Dinakar Ramakrishnan. Picard modular surfaces, residual Albanese quotients, and rational points. The Picard modular surfaces X are at the crossroads of rich interplay between geometry, Galois representations, and automorphic forms on $G = U(2,1)$ associated to an imaginary quadratic field K. The talk will introduce an ongoing project with M. Dimitrov on the quotients of the albanese variety $Alb(X)$ coming from residual automorphic forms on G, give examples with finite Mordell-Weil group, and investigate possible consequences, inspired by classical arguments of Mazur, for the K-rational points on X.

Mehmet Sengun. Mod p Cohomology of Bianchi Groups and Mod p Galois Representations. Given an imaginary quadratic field K with ring of integers R, consider the Bianchi group GL(2, R). It is suspected since the numerical investigations of Fritz Grunewald in the late 1970s that there is a connection between the Hecke eigenclasses in the mod p cohomology of (congruence subgroups of) Bianchi groups and the two-dimensional continuous mod p representations of the absolute Galois group of K. Most of the basic tools used for establishing this connection (and its surrounding problems) in the classical setting fail to work in the setting of Bianchi groups. The situation has an extra layer of complication by the fact that there are "genuinely mod p" Hecke eigenvalue systems, resulting from the existence of torsion in the integral cohomology. In this talk I will elaborate on the above, presenting numerical examples for illustration. Towards the end, I will also talk about how the "even" two-dimensional continuous mod p representations of the absolute Galois group of $\mathbb{Q}$ come into the picture.

Jack Thorne. Symmetric power functoriality for GL(2). We discuss some new automorphy lifting theorems, and their applications to the existence of new cases of Langlands' functoriality for GL(2). This is joint work with L. Clozel.

Jacques Tilouine. Image of Galois and congruence ideals, a programme of a joint work with H. Hida. In a recent preprint, H. Hida showed that the image of the Galois representation associated to a non-CM Hida family contains a congruence subgroup of GL(2) over Λ, whose level is given in terms of p-adic L-functions. We try to generalize this to Hida families for bigger groups, replacing p-adic L-functions by congruence ideals.

Jeanine Van Order. Critical values of GL(2) Rankin-Selberg L-functions. The aim of this talk is to explain the subtle but powerful link between the algebraicity of critical values of automorphic L-functions, the existence of associated p-adic L-functions, and the generic non-vanishing of these values, particularly in the setting of Rankin-Selberg L-functions of GL(2) over a totally real number field. More precisely, the aim is to explain how to extend the conjectures of Mazur to the non self-dual setting, thereby extending the works of Vatsal, Cornut and Cornut-Vatsal, via a combination of techniques from Iwasawa theory, analytic number theory and the theory of automorphic forms. If time permits, then some open problems will also be introduced.

The research conference

Monday, April 15

09:30 – 10:30 Trevor Wooley (University of Bristol): Applications of efficient congruencing to rational points.

11:00 – 12:00 Per Salberger (Chalmers University of Technology): Heath-Brown's determinant method and Mumford's geometric invariant theory.

13:30 – 14:30 Jeanine Van Order (EPFL): Stable Galois averages of Rankin-Selberg L-values and non-triviality of p-adic L-functions.

15:00 – 16:00 Przemyslaw Chojecki (Institut Mathématique de Jussieu): On mod p non-abelian Lubin-Tate theory for GL(2).

Tuesday, April 16

09:30 – 10:30 Alexei Skorobogatov (Imperial College London): Applications of additive combinatorics to rational points.

11:00 – 12:00 Yuri Bilu (IMB Université Bordeaux I): Integral points on modular curves.

13:30 – 14:30 Ulrich Derenthal (Universität München): Counting points over imaginary quadratic number fields.

15:00 – 16:00 Oscar Marmon (Universität Göttingen): The density of twins of k-free numbers.

Wednesday, April 17

09:30 – 10:30 Jörg Brüdern (Universität Göttingen): Random Diophantine equations.

11:00 – 12:00 Florian Pop (University of Pennsylvania): Local–global principles for rational points.

13:30 – 14:30 Mohamed Saidi (Exeter University): On the anabelian section conjecture over finitely generated fields.

15:00 – 16:00 Victor Abrashkin (University of Durham): p-extensions of local fields with Galois groups of nilpotence class less than p.

Thursday, April 18

09:30 – 10:30 Ambrus Pál (Imperial College London): New two-dimensional counter-examples to the local–global principle.

11:00 – 12:00 David McKinnon (University of Waterloo): Approximating points on varieties.

13:30 – 14:30 Tim Browning (University of Bristol): Norm forms as products of linear polynomials, I.

15:00 – 16:00 Lilian Matthiesen (University of Bristol): Norm forms as products of linear polynomials, II.

Friday, April 19

09:30 – 10:30 Gabor Wiese (Université du Luxembourg): Symplectic Galois representations and applications to the inverse Galois problem.

11:00 – 12:00 Damaris Schindler (University of Bristol): Manin's conjecture for certain smooth hypersurfaces in biprojective space.

13:30 – 14:30 Roger Heath-Brown (University of Oxford): Simultaneous representation of pairs of integers by quadratic forms.

15:00 – 16:00 Jean-Louis Colliot-Thélène (Université Paris-Sud): Strong approximation in a family.

Abstracts

Victor Abrashkin. p-extensions of local fields with Galois groups of nilpotence class less than p. Nilpotent analogue of the Artin-Schreier theory was developed by the author about twenty years ago. It has found already applications in an explicit description of the ramification filtration modulo p-th commutators and the proof of an analogue of the Grothendieck Conjecture for local fields. We remind basic constructions of this theory and indicate further progress in the study of local fields of mixed characteristic, especially, higher dimensional local fields.

Yuri Bilu. Integral points on modular curves. The problem of determination of rational points on modular curves reduces grosso mode to three types of curves of prime level, corresponding to three types of maximal subgroups of the linear group $\mathrm{GL}_2(\mathbb{F}_p)$: the curve $X_0(p)$, corresponding to the Borel subgroup; the curve $X_{sp}^+(p)$, corresponding to the normalizer of a split Cartan subgroup; the curve $X_{ns}^+(p)$, corresponding to the normalizer of a non-split Cartan subgroup. The rational points on the curves of the first two types are determined (almost) completely: Mazur (1978), B.-Parent-Rebolledo (2012). In particular, it is proved that for $p > 13$ the rational points are either cusps or the CM-points. Little is known, however, about the rational points on $X_{ns}^+(p)$. I will speak about recent progress in a simpler problem: classification of integral points on $X_{ns}^+(p)$ (i.e. rational points P such that $j(P) \in \mathbb{Z}$). My students Bajolet and Sha obtained a rather sharp upper bound for the size of integral points. Also, in a joint work with Bajolet we proved that for $7 < p < 71$ there are no integral points on $X_{ns}^+(p)$ other than the CM-points; this improves on a recent work of Schoof and Tzanakis, who proved that for $p = 11$.

Tim Browning, Lilian Matthiesen. Norm forms as products of linear polynomials. We report on recent progress using additive combinatorics to prove the Hasse principle and weak approximation for certain varieties defined by systems of equations involving norm forms. This is used to show that the Brauer-Manin obstruction controls weak approximation on normic bundles of the shape $N_K(x_1, \dots, x_n) = P(t)$, where $P(t)$ is a product of linear polynomials all defined over the rationals and K is an arbitrary degree n extension of the rationals.

Jörg Brüdern. Random Diophantine equations. We address the classical questions, concerning diagonal forms with integer coefficients. Does the Hasse principle hold? If there are solutions, how many? If there are solutions, what is the size of the smallest solutions? In a joint work with Dietmann, nearly optimal answers to such questions were obtained for almost all forms (in the sense typically attributed to "almost all" in the analytic theory of numbers) provided that the number of variables exceeds three times the degree of the forms under consideration.

Przemyslaw Chojecki. On mod p non-abelian Lubin-Tate theory for GL(2). Non-abelian Lubin-Tate theory for GL(2) describes the l-adic cohomology of the Lubin-Tate tower for $\mathrm{GL}(2, \mathbb{Q}_p)$ in terms of the Langlands program. Until recently, all results were stated under the assumption that p is different from l. We discuss the case when $l = p$ and give a partial description of the