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978-1-107-46249-6 - O-Minimality and Diophantine Geometry: London Mathematical Society Lecture Note Series: 421

Edited by G. O. Jones and A. J. Wilkie

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