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978-1-107-45946-5- Current Developments in Algebraic Geometry

Edited by Lucia Caporaso, James McKernan, Mircea Mustață and Mihnea Popa

Frontmatter

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Algebraic geometry is one of the most diverse fields of research in mathematics. It has had an incredible evolution over the past century, with new subfields constantly branching off and spectacular progress in certain directions and, at the same time, with many fundamental unsolved problems still to be tackled.

In the spring of 2009 the first main workshop of the MSRI algebraic geometry program served as an introductory panorama of current progress in the field, addressed to both beginners and experts. This volume reflects that spirit, offering expository overviews of the state of the art in many areas of algebraic geometry. Prerequisites are kept to a minimum, making the book accessible to a broad range of mathematicians. Many chapters present approaches to long-standing open problems by means of modern techniques currently under development and contain questions and conjectures to help spur future research.

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# Current Developments in Algebraic Geometry

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## Preface

In the spring of 2009, the entire MSRI semester was devoted to Algebraic Geometry. The first main workshop, held during the week of January 26–30 and entitled “Classical Algebraic Geometry Today”, served as an introduction to the whole program. It was designed to give a panorama of current progress in the field, of interest to experts, but presented in an introductory manner in order to make it accessible to those with only a general background. As a result, all talks contained a substantial expository and didactic component for the younger audience for whom the workshop was primarily intended.

One of the goals of the workshop was to reflect the diversity of Algebraic Geometry and its most fascinating recent developments. As such, it covered a wide range of topics such as birational geometry and classification, rational curves on algebraic varieties, moduli spaces of abelian varieties, derived categories and Fourier-Mukai transforms, curve counting invariants, holomorphic symplectic manifolds, and Hodge theory. Topics like modern enumerative geometry, moduli of varieties of general type, and combinatorial geometry, although represented at this event as well, were more strongly emphasized at other workshops organized throughout the semester.

The speakers at the workshop were, in alphabetical order: Mark de Cataldo, Olivier Debarre, David Eisenbud, Sam Grushevsky, Christopher Hacon, Joe Harris, Daniel Huybrechts, Jun-Muk Hwang, János Kollár, Robert Lazarsfeld, Alina Marian, Kieran O’Grady, Martin Olsson, Rita Pardini, Giuseppe Pareschi, Richard Thomas, Burt Totaro, and Yuri Tschinkel.

We hope that this volume closely reflects the spirit of the workshop. Intended to give an overview of the state of the art in the field, it is addressed to both beginners and specialists. As one of the fields in mathematics with the longest history, Algebraic Geometry has had an incredible evolution over the past century, with new subfields constantly branching off, with spectacular progress in certain directions, but at the same time, with many fundamental unsolved problems still to be tackled. The focus of the volume is on approaches to long-standing open problems (in other words, problems with a “classical” flavor) by means of modern techniques currently under development. Many of the surveys contain questions and conjectures which will hopefully lead to a flurry of activity and wonderful new results in the future.

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The volume contains fifteen contributions. Most of them are expanded versions of the speakers' lectures and, as mentioned above, cover a wide range of topics:

- birational geometry (T. de Fernex and C. Hacon, B. Totaro)
- rational curves and differential geometry (J.-M. Hwang)
- moduli spaces of abelian varieties in connection with the Schottky problem (S. Grushevsky) and log-geometry (M. Olsson)
- moduli spaces via period maps (O. Debarre)
- derived categories in the study of K3 surfaces (D. Huybrechts) and irregular varieties (G. Pareschi)
- anabelian geometry (F. Bogomolov and Y. Tschinkel)
- holomorphic symplectic manifolds (K. O'Grady)
- classical problems on interpolation (J. Harris) and projections (R. Beheshti and D. Eisenbud)
- classification of surfaces of general type (M. Mendes Lopes and R. Pardini)
- Hodge theory of character varieties (M. de Cataldo)
- quotients of algebraic varieties (J. Kollár, appendix by C. Raicu).

We would like to thank the authors for the effort they put into submitting truly valuable contributions. We are grateful to Silvio Levy, the MSRI editor, for his help in preparing this volume, and to the organizers of the MSRI semester, Brendan Hassett, Sándor Kovács and Ravi Vakil, for suggesting that we organize the original workshop and for logistical help. Special thanks are due to Brendan Hassett, who co-organized the workshop together with us, and who initiated and further supported the idea of these proceedings.

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