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Overview

From the observational data of Supernovae Type Ia (SN Ia) accumulated by the year 1998, Riess *et al.* [1] in the High-redshift Supernova Search Team and Perlmutter *et al.* [2] in the Supernova Cosmology Project Team independently reported that the present Universe is accelerating. The source for this late-time cosmic acceleration was dubbed "dark energy." Despite many years of research (see e.g., the reviews [3, 4, 5, 6, 7]) its origin has not been identified yet. Dark energy is distinguished from ordinary matter species such as baryons and radiation, in the sense that it has a negative pressure. This negative pressure leads to the accelerated expansion of the Universe by counteracting the gravitational force. The SN Ia observations have shown that about 70% of the present energy of the Universe consists of dark energy.

The expression "dark energy" may be somewhat confusing in the sense that a similar expression, "dark matter," has been used to describe a pressureless matter (a non-relativistic matter) that interacts very weakly with standard matter particles. The existence of dark matter was already pointed out by Zwicky in the 1930s by comparing the dispersion velocities of galaxies in the Coma cluster with the observable star mass. Since dark matter does not mediate the electromagnetic force, its presence is mainly inferred from gravitational effects on visible matter. Dark matter can cluster by gravitational instability (unlike standard dark energy) so that local structures have been formed in the Universe. In fact it is observationally known that dark matter has played a crucial role for the growth of large-scale structure such as galaxies and clusters of galaxies. The energy fraction of dark matter in the present universe is about 25%, whereas that of baryons is about 4%. The black body radiation, which dominated over the other matter components in the past, shares only about 0.005% of the present total energy density.

In modern cosmology it is believed that another cosmic acceleration called "inflation" occurred in the very early Universe prior to the radiation-dominated epoch. The idea of inflation was originally proposed in the early 1980s by a number of people [8, 9, 10, 11] to solve several cosmological problems such as the flatness and horizon problems. Inflation also provides a causal mechanism for the origin of large-scale structure in the Universe. The temperature anisotropies of the Cosmic Microwave Background (CMB) observed by the Cosmic Background Explorer (COBE) in 1992 [12] showed that the fluctuation spectrum is nearly scale-invariant.[1] This is consistent with theoretical predictions of the power spectrum of density perturbations originated from quantum fluctuations of a scalar field generated during inflation. After 2003, the Wilkinson Microwave Anisotropy Probe (WMAP) group has provided high-precision observational data of CMB anisotropies [13, 14, 15]. This has given strong support for the existence of an inflationary period as well as dark energy.

After the end of inflation the Universe entered the radiation-dominated epoch during which light elements such as helium and deuterium were formed. Since the energy density of radiation decreases faster than that of non-relativistic matter such as dark matter and baryons, the radiation-dominated era is eventually followed by the matter-dominated epoch around the redshift $z = 3000$. The temperature anisotropies observed by COBE and WMAP occur on the last scattering surface at which electrons were trapped by hydrogen to form atoms. After this *decoupling epoch* photons can freely move to us without experiencing Thomson scattering. The decoupling corresponds to the redshift $z \simeq 1090$. According to the WMAP 5-year data [15], the energy components at the decoupling epoch are dark matter (63%), radiations (25%) [photons (15%) and neutrinos (10%)], and baryons (12%) with at most a tiny amount of dark energy. We will often make reference to the cosmological parameters measured by WMAP in the course of this book.

The formation of structure (galaxies, clusters) started in the matter-dominated epoch, i.e. when the pressureless dark matter began to dominate the total energy density of the Universe. Baryons also contribute to the formation of large-scale structure to some extent. During the matter era the energy density of dark energy needs to be suppressed compared to that of dark matter in order to allow sufficient growth of large-scale structure. If dark energy couples to dark matter with some interaction (as in the coupled quintessence scenario [16, 17]), then dark energy also affects the past expansion history of the Universe as well as the structure formation. It is possible to place bounds on the strength of such couplings from the observations of CMB and of galaxy clustering. In addition to the experiments of direct and indirect dark matter search (see e.g., [18, 19, 20, 21]), like those at the Large Hadron Collider (LHC) at CERN and in underground, ground, and space

[1] J. Mather and G. Smoot won the Nobel Prize in 2006 for the measurement of the black body spectrum and the discovery of the temperature anisotropy of CMB.

facilities, cosmological observations will shed light on the relation between dark matter and dark energy.

While the energy density of dark matter evolves as $\rho_m \propto a^{-3}$ (a is the scale factor of an expanding Universe), the dark energy density is nearly constant in time ($\rho_{\rm DE} \propto a^{-n}$ with n probably close to 0). Hence the latter energy density eventually catches up with the former. The onset of the cosmic acceleration occurs around the redshift $z \sim 1$, although there is still uncertainty for its precise value because of the model-dependence. We live then in a special epoch of the cosmic acceleration in the long expansion history of the Universe. The problem why the accelerated expansion of the Universe started around today is often called the "coincidence problem." The standard radiation- and matter-dominated eras are sandwiched by two periods of cosmic acceleration – inflation and dark energy.

The simplest candidate for dark energy is the so-called cosmological constant Λ, whose energy density remains constant [22]. Originally the cosmological constant was introduced by Einstein in 1917 to realize a static Universe in the framework of General Relativity [23]. In fact the Einstein equations allow the freedom to add the constant Λ term. The cosmological constant works as a negative pressure against gravity so that the two effects can balance each other. However, after the discovery of the expansion of the Universe by Hubble from the measurement of recession speeds of distant galaxies, Einstein abandoned the idea of adding the Λ term to the Einstein equations. At the late stage of his career, he regretted having introduced Λ as his "*biggest blunder*" (or so is told by George Gamow). In fact, there was nothing to regret: after 1998 the cosmological constant revived again as a form of dark energy responsible for the late-time acceleration of the Universe.

From the viewpoint of particle physics, the cosmological constant appears as vacuum energy density. If we sum up zero-point energies of all normal modes of some field and take the cut-off scale of the momentum at the Planck scale, the vacuum energy density is estimated to be $\rho_{\rm vac} \simeq 10^{74}\,{\rm GeV}^4$. This is much larger than the observed value of dark energy: $\rho_\Lambda \simeq 10^{-47}\,{\rm GeV}^4$. If vacuum energy with an energy density of the order of $\rho_{\rm vac} \simeq 10^{74}\,{\rm GeV}^4$ was present in the past, the Universe would have entered an eternal stage of cosmic acceleration already in the very early Universe. This is of course problematic because the success of the big bang cosmology based on the presence of radiation and matter epochs is completely destroyed. Hence the problem of the large vacuum energy density was known long before the discovery of dark energy in 1998.

If the cosmological constant is responsible for the present cosmic acceleration, we need to find a mechanism to obtain the tiny value of Λ consistent with observations. A lot of efforts have been made in this direction under the framework of particle physics. For example, the recent development of string theory shows that it is possible to construct de Sitter vacua by compactifying extra dimensions in the presence of fluxes with an account of non-perturbative corrections [24]. The fact that there is a huge number of different choices of fluxes gives rise to the so-called "string landscape" with more than 10^{500} vacua [25]. Some scientists argued that only the vacuum whose energy density is of the order of the present cosmological density can sustain life or complexity and this explains why we live in a low-Λ world. This anthropic argument is, to say the least, highly controversial.

If the origin of dark energy is not the cosmological constant, one may seek for some alternative models to explain the cosmic acceleration today. Basically there are two approaches to construct models of dark energy other than the cosmological constant.

The first approach is to modify the right-hand side (r.h.s.) of the Einstein equations given in Eq. (2.8) by considering specific forms of the energy-momentum tensor $T_{\mu\nu}$ with a negative pressure. The representative models that belong to this class are the so-called cosmon or quintessence [26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41], k-essence [42, 43, 44], and perfect fluid models [45, 46]. The quintessence makes use of scalar fields with slowly varying potentials, whereas in k-essence it is the scalar-field kinetic energy that drives the acceleration. The perfect fluid models are based on a perfect fluid with a specific equation of state such as the Chaplygin gas model [45] and its generalizations [46]. There have been many attempts to construct scalar-field models of dark energy based on particle physics (see Refs. [47, 48, 49, 50, 51, 52] for early works). In the context of inflation, since the associated energy scale is high, it is natural for scalar fields to be responsible for the acceleration of the Universe. The situation is different for dark energy – its energy scale is too low compared to typical scales appearing in particle physics. Moreover, the field potentials need to be sufficiently flat so that the field evolves slowly enough to drive the present cosmic acceleration. This demands that the field mass is extremely light ($m_\phi \simeq 10^{-33}\,\mathrm{eV}$) relative to typical mass scales appearing in particle physics. It would be expected that this light scalar field should mediate long-range forces with ordinary matter [36]. Such couplings need to be suppressed in order to be consistent with a number of local gravity experiments. In spite of the above-mentioned difficulties it is not hopeless to construct viable scalar-field dark energy models in the framework of particle physics.

The second approach for the construction of dark energy models is to modify the left-hand side (l.h.s.) of the Einstein equations (2.8). The representative models that belong to this class (that we denote "modified gravity") are the so-called $f(R)$ gravity [53, 54, 55], scalar-tensor theories [56, 57, 58, 59, 60], and braneworld models [61, 62]. The cosmological constant scenario (in other words, the "Λ-Cold-Dark-Matter (ΛCDM) model") corresponds to the Lagrangian density $f(R) = R - 2\Lambda$, where R is the Ricci scalar. A possible modification of the ΛCDM is described by a non-linear Lagrangian density f in terms of R, which is called

$f(R)$ gravity. Scalar-tensor theories correspond to theories in which the Ricci scalar R couples to a scalar field ϕ with a coupling of the form $F(\phi)R$. They include Brans–Dicke theory [63] and dilaton gravity [64] as specific cases. In the braneworld models proposed by Dvali, Gabadadze, and Porrati (DGP) [61] the late-time acceleration of the Universe can be realized as a result of the gravitational leakage from a 3-dimensional surface (3-brane) to a 5-th extra dimension on Hubble distances. Generally we require that modified gravity models satisfy local gravity constraints as well as conditions for the cosmic acceleration preceded by the matter-dominated epoch. In this sense modified gravity models are typically more strongly constrained than modified matter models from gravitational experiments and cosmological observations.

It is important to realize however that the two approaches, which we denote as modified matter and modified gravity, are not fundamentally different, at least if for a moment we do not consider their quantum field implications. From the viewpoint of classical General Relativity (which is all that matters for most of cosmology), one can always rephrase one into the other by defining a suitable conserved energy-momentum tensor that equals the Einstein tensor.

In order to distinguish this variety of models of dark energy, it is important to place constraints by using observational data such as SN Ia, CMB, and large-scale structure (LSS). Usually the equation of state of dark energy, $w_{\rm DE} \equiv P_{\rm DE}/\rho_{\rm DE}$, where $P_{\rm DE}$ is the pressure and $\rho_{\rm DE}$ is the energy density, is a good measure to describe the property of dark energy at background level. In the case of the cosmological constant we have $P_{\rm DE} = -\rho_{\rm DE}$ and hence $w_{\rm DE} = -1$. In other models of dark energy the equation of state $w_{\rm DE}$ generally varies in time. Perhaps the first task of dark energy research is to detect deviations from the value $w_{\rm DE} = -1$ in order to find whether dark energy can be identified with the cosmological constant or not.

The SN Ia observations have provided information of the cosmic expansion history around the redshift $z \lesssim 2$ by the measurement of luminosity distances of the sources. The presence of dark energy leads to a shift of the position of acoustic peaks in CMB anisotropies as well as a modification of the large-scale CMB spectrum through the so-called integrated Sachs–Wolfe effect. Although the CMB data alone are not sufficient to place strong constraints on dark energy, the combined analysis of SN Ia and CMB can provide tight bounds on the equation of state $w_{\rm DE}$ and the present energy fraction $\Omega_{\rm DE}^{(0)}$ of dark energy [15]. The distribution of large-scale clustering of galaxies in the sky also provides additional information on the properties of dark energy [65, 66, 67]. In 2005 the detection of a peak of baryon acoustic oscillations (BAO) was reported by Eisenstein *et al.* [68] at the average redshift $z = 0.35$ from the observations of luminous red galaxies in the Sloan Digital Sky Survey. This has also given us another independent test of dark energy. From the combined analysis of SN Ia, CMB, and BAO, the WMAP group [15] obtained the bound $-1.097 < w_{\rm DE} < -0.858$ at the 95% confidence level assuming a constant equation of state. The cosmological constant ($w_{\rm DE} = -1$) is well consistent with the current observational data while some dark energy models have been already excluded from observations.

In future observations it is expected that other observational data such as weak gravitational lensing and gamma ray bursts will shed light on the nature of dark energy. Confirming ΛCDM or detecting deviations from it would be an extremely important step towards understanding the origin of dark energy.

Units and conventions

Throughout this book we use units such that $c = \hbar = k_{\rm B} = 1$, where c is the speed of light, $\hbar$ is reduced Planck's constant, and $k_{\rm B}$ is Boltzmann's constant. We reinsert these symbols when the discussion needs it. In these units everything can be expressed in terms of a single unit, e.g., time, length, or energy. The gravitational constant G is related to the Planck mass $m_{\rm pl} = 1.2211 \times 10^{19}$ GeV via $G = 1/m_{\rm pl}^2$ and the reduced Planck mass $M_{\rm pl} = 2.4357 \times 10^{18}$ GeV via $\kappa^2 \equiv 8\pi G = 1/M_{\rm pl}^2$, respectively. We adopt the metric signature $(-, +, +, +)$. We list frequently used symbols after the Table of Contents.

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Expansion history of the Universe

Standard hot big bang cosmology is based on the cosmological principle, which states that the Universe is homogeneous and isotropic at least on large scales. This is supported by a number of observations, such as the CMB photons coming from different parts of the sky with almost the same temperature. The past cosmic expansion history is recovered by solving the Einstein equations in the background of the homogeneous and isotropic Universe. Of course we observe inhomogeneities and irregularities in the local region of the Universe such as stars and galaxies. These inhomogeneities have grown in time through gravitational instability from a matter distribution that was more homogeneous in the past. Then the inhomogeneities can be regarded as small perturbations evolving on the background (homogeneous) Universe.

In this chapter we provide the basic tools to understand the expansion history of the Universe. We also introduce a number of cosmic distances often used to put observational constraints on dark energy.

2.1 Friedmann equations

The line-element that describes a 4-dimensional homogeneous and isotropic spacetime is called Friedmann–Lemaître–Robertson–Walker (FLRW) spacetime and is given by

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -\mathrm{d}t^2 + a^2(t)\mathrm{d}\sigma^2 \,, \tag{2.1}$$

where $g_{\mu\nu}$ is a metric tensor, $a(t)$ is a scale factor with cosmic time t, and $\mathrm{d}\sigma^2$ is the time-independent metric of the 3-dimensional space with a constant curvature K:

$$\mathrm{d}\sigma^2 = \gamma_{ij}\mathrm{d}x^i \mathrm{d}x^j = \frac{\mathrm{d}r^2}{1 - Kr^2} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\, \mathrm{d}\phi^2) \,. \tag{2.2}$$

Here $K = +1, -1, 0$ correspond to closed, open, and flat geometries, respectively. We have used polar coordinates $(x^1, x^2, x^3) = (r, \theta, \phi)$ with $\gamma_{11} = (1 - Kr^2)^{-1}$, $\gamma_{22} = r^2$, and $\gamma_{33} = r^2 \sin^2\theta$. In Eq. (2.1) the Greek indices μ and ν run from 0 to 3, whereas in Eq. (2.2) the Latin indices i and j run from 1 to 3; the same convention applies to the whole book except when indicated otherwise. We follow Einstein's convention that the terms with same upper and lower indices are summed over. See the book of Weinberg [69] for the derivation of the metric (2.1) from a maximally symmetric spacetime. In addition to the cosmic time t, we also introduce the conformal time η defined by

$$\eta \equiv \int a^{-1}\, \mathrm{d}t\,. \tag{2.3}$$

The dynamical equations of motion in the expanding Universe can be derived from the Einstein equations by the following steps. From the metric $g_{\mu\nu}$ we obtain the Christoffel symbol:

$$\Gamma^{\mu}_{\nu\lambda} = \frac{1}{2} g^{\mu\alpha}(g_{\alpha\nu,\lambda} + g_{\alpha\lambda,\nu} - g_{\nu\lambda,\alpha})\,, \tag{2.4}$$

where $g_{\alpha\nu,\lambda} \equiv \partial g_{\alpha\nu}/\partial x^{\lambda}$. Note that $g_{\alpha\nu}$ satisfies the relation $g^{\mu\alpha} g_{\alpha\nu} = \delta^{\mu}_{\nu}$, where δ^{μ}_{ν} is Kronecker's delta ($\delta^{\mu}_{\nu} = 1$ for $\mu = \nu$ and $\delta^{\mu}_{\nu} = 0$ for $\mu \neq \nu$). The Ricci tensor is defined by

$$R_{\mu\nu} = \Gamma^{\alpha}_{\mu\nu,\alpha} - \Gamma^{\alpha}_{\mu\alpha,\nu} + \Gamma^{\alpha}_{\mu\nu}\Gamma^{\beta}_{\alpha\beta} - \Gamma^{\alpha}_{\mu\beta}\Gamma^{\beta}_{\alpha\nu}\,. \tag{2.5}$$

The contraction of the Ricci tensor gives the Ricci scalar (scalar curvature)

$$R = g^{\mu\nu} R_{\mu\nu}\,. \tag{2.6}$$

We can then evaluate the Einstein tensor

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R\,. \tag{2.7}$$

The cosmological dynamics can be obtained by solving the Einstein equations

$$G^{\mu}_{\nu} = 8\pi G T^{\mu}_{\nu}\,, \tag{2.8}$$

where T^{μ}_{ν} is the energy-momentum tensor of matter components. The l.h.s. of Eq. (2.8) characterizes the geometry of spacetime, whereas the r.h.s. describes energies and momenta of matter components. In the cosmological setting the cosmic expansion rate is determined by specifying the properties of matter in the Universe.

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For the FLRW metric (2.1) the non-vanishing components of Christoffel symbols are

$$\Gamma^0_{ij} = a^2 H \gamma_{ij} \,, \quad \Gamma^i_{0j} = \Gamma^i_{j0} = H\delta^i_j, \tag{2.9}$$

$$\Gamma^1_{11} = \frac{Kr}{1-Kr^2} \,, \quad \Gamma^1_{22} = -r(1-Kr^2) \,, \quad \Gamma^1_{33} = -r(1-Kr^2)\sin^2\theta, \tag{2.10}$$

$$\Gamma^2_{33} = -\sin\theta\cos\theta \,, \quad \Gamma^2_{12} = \Gamma^2_{21} = \Gamma^3_{13} = \Gamma^3_{31} = \frac{1}{r} \,, \quad \Gamma^3_{23} = \Gamma^3_{32} = \cot\theta, \tag{2.11}$$

where

$$H \equiv \dot{a}/a. \tag{2.12}$$

A dot represents a derivative with respect to cosmic time t. The quantity H, called the Hubble parameter, describes the expansion rate of the Universe. The Christoffel symbols given in Eqs. (2.10) and (2.11) correspond to those for the three-dimensional metric (2.2) with the curvature K.

From Eqs. (2.5) and (2.6) the Ricci tensor and the scalar curvature are

$$R_{00} = -3\left(H^2 + \dot{H}\right) \,, \quad R_{0i} = R_{i0} = 0 \,, \quad R_{ij} = a^2\left(3H^2 + \dot{H} + 2K/a^2\right)\gamma_{ij} \,, \tag{2.13}$$

$$R = 6\left(2H^2 + \dot{H} + K/a^2\right) . \tag{2.14}$$

From Eq. (2.7) together with the relation $G^\mu_\nu = g^{\mu\alpha}G_{\alpha\nu}$, the Einstein tensor is

$$G^0_0 = -3\left(H^2 + K/a^2\right) \,, \quad G^0_i = G^i_0 = 0 \,, \quad G^i_j = -\left(3H^2 + 2\dot{H} + K/a^2\right)\delta^i_j \,. \tag{2.15}$$

In the FLRW spacetime the energy-momentum tensor of the background matter is restricted to take the perfect fluid form:

$$T^\mu_\nu = (\rho + P)u^\mu u_\nu + P\delta^\mu_\nu \,, \tag{2.16}$$

where $u^\mu = (1, 0, 0, 0)$ is the four-velocity of the fluid in comoving coordinates, and ρ and P are functions of t. The (00) and (ij) components of T^μ_ν are $T^0_0 = -\rho$ and $T^i_j = P\delta^i_j$. Then ρ and P have the meaning of an energy density and a pressure, respectively. Since we are using the unit $c = 1$, the density ρ is not particularly distinguished from the energy density ρc^2. From the (00) and (ii) components of the Einstein equations (2.8) we obtain

$$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2} \,, \tag{2.17}$$

$$3H^2 + 2\dot{H} = -8\pi G P - \frac{K}{a^2} \,. \tag{2.18}$$

Eliminating the term K/a^2 gives

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)\,. \tag{2.19}$$

Multiplying Eq. (2.17) by a^2, differentiating and using Eq. (2.19), we find

$$\dot{\rho} + 3H(\rho + P) = 0\,. \tag{2.20}$$

The Einstein tensor satisfies the Bianchi identities

$$\nabla_\mu G^\mu_\nu \equiv \frac{\partial G^\mu_\nu}{\partial x^\mu} + \Gamma^\mu_{\alpha\mu} G^\alpha_\nu - \Gamma^\alpha_{\nu\mu} G^\mu_\alpha = 0\,, \tag{2.21}$$

where ∇_μ denotes the covariant derivative. Sometimes we use also the symbol "$_{;\mu}$" to represent the covariant derivative. From the Einstein equations (2.8) it follows that $\nabla_\mu T^\mu_\nu = 0$, which gives the same equation as (2.20) in the FLRW background (see problem 2.1). Hence Eq. (2.20) is called the conservation or continuity equation.

Equation (2.17) can be written in the form:

$$\Omega_M + \Omega_K = 1\,, \tag{2.22}$$

where

$$\Omega_M \equiv \frac{8\pi G\rho}{3H^2}\,, \qquad \Omega_K \equiv -\frac{K}{(aH)^2}\,. \tag{2.23}$$

We often refer to the present values of the density parameters. For relativistic particles, non-relativistic matter, dark energy, and curvature, we have, respectively

$$\Omega_r^{(0)} = \frac{8\pi G\rho_r^{(0)}}{3H_0^2}\,, \quad \Omega_m^{(0)} = \frac{8\pi G\rho_m^{(0)}}{3H_0^2}\,, \quad \Omega_{\rm DE}^{(0)} = \frac{8\pi G\rho_{\rm DE}^{(0)}}{3H_0^2}\,, \quad \Omega_K^{(0)} = -\frac{K}{(a_0 H_0)^2}\,. \tag{2.24}$$

When we wish to identify the electromagnetic radiation, rather than all the relativistic particles, we use the subscript γ. When we need to distinguish between (cold) dark matter and baryons we use the subscripts c and b, respectively.[1] When we refer to the cosmological constant, we also use the subscript Λ instead of DE. Finally, sometimes we use M to denote a generic matter component.

If the expansion of the Universe is decelerated (i.e. $\ddot{a} < 0$) then the curvature term $|\Omega_K|$ continues to increase (because the term $aH\,(=\dot{a})$ decreases), apart from the case where the Universe is exactly flat ($K = 0$) from the very beginning. The WMAP 5-year data [15] constrain the curvature of the present Universe to be $-0.0175 < \Omega_K^{(0)} < 0.0085$ at the 95% confidence level. We need a phase of

[1] See Section 2.3 for the definition of cold dark matter.