

# 1 Draw a diagram

There is an old saying: 'A picture is worth a thousand words'.

So, if a diagram is not provided then draw one. It might be helpful, and could give you some ideas about how to solve the problem.

Draw a decent diagram. A sketch is probably fine, but it needs to look like the thing it describes. If there is a triangle in the problem, then your shape should be a triangle. If there is supposed to be a straight line then your line should be straight. The actual sizes of sides and angles are probably not important. Try to make your diagrams large and clear.

Label your diagram. If there is information provided in the question (such as the lengths of sides, or the sizes of angles) then write these on your diagram. This will often help when you are solving a problem.

Add new information that you work out. When you work out something new, add this to the diagram too.

So, in summary:

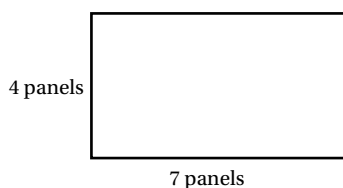
- draw a decent diagram
- annotate it
- add new information that you work out.

Here is a problem:

A farmer has 22 fence panels and wants to use them to make a rectangular enclosure for his sheep. How many different enclosures can he make?

A picture of a field with sheep and lambs in it will not be helpful here.

Instead, a diagram of a rectangle would be more useful.



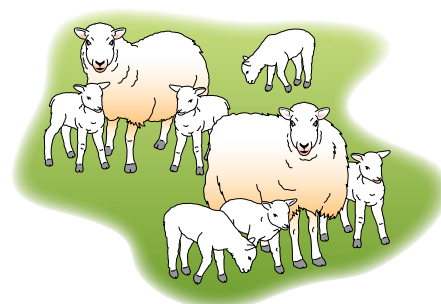
Now you can finish off this question.

The length and width of the rectangle add up to 11 (because the perimeter is 22 panels).

The dimensions that are possible are:

1 by 10      3 by 8      5 by 6  
2 by 9      4 by 7

So he can make 5 different enclosures.



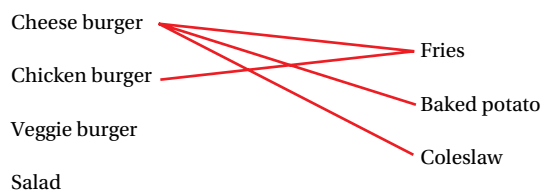
## GCSE Mathematics for AQA (Foundation)

It is fairly obvious what to do when a diagram has been provided as part of a question, but a diagram can sometimes be useful in other situations. Here is another example where drawing diagrams could help you.

At a fast food restaurant there is a ‘meal deal’ that involves first choosing one of the following: cheese burger, chicken burger, veggie burger or salad, and then ordering a side from the following list: fries, baked potato or coleslaw.

How many different meals could you have?

You could work systematically and create a list, but a diagram would also help.



The diagram above shows all the options and the lines show some of the possible combinations.

There are three lines coming from cheese burger. How many lines will come from chicken burger when the diagram is finished? Will this help you answer the question?

Alternatively, you could create a table like the one below to help you. What does each cell in this table represent? How does this table tell you, at a glance, how many meal possibilities there are?

	Fries	Baked potato	Coleslaw
Cheese burger			
Chicken burger			
Veggie burger			
Salad			

Each cell in the table is a possible meal deal combination. There are 12 cells in the table, therefore there are 12 different meals that you could have.

The following problems may be solved using more than one method; however, the worked solutions provided at the back of this book are based on the method introduced above.

1 Draw a diagram



Peter the factory manager planned to install a new hot drinks machine for the factory workers. He decided to fill it with tea as he thought tea was the most popular hot drink among the workers.

The workers did a survey to check what the preferred hot drink was among them. Each person surveyed could choose one drink from hot chocolate, tea and coffee.

Eight women wanted hot chocolate.

A total of 16 workers wanted tea, of which 7 were men.

10 men and 12 women chose coffee.

There were 25 men in total.

Was Peter correct?



Tip

What type of diagram would be useful here?



Here are the mock exam results (out of 100) in GCSE Mathematics and GCSE Statistics for a group of students (represented by letters A–L).

	A	B	C	D	E	F	G	H	I	J	K	L
Mathematics	68	62	50	54	86	65	74	60	54	39	48	
Statistics	73	68	54	62	87	71	32	65	59	57	53	70

Student L sat the GCSE Statistics examination and achieved a score of 70 but was ill during the GCSE Mathematics examination and could not complete the paper.

- Predict the Mathematics result for Student L.
- How good is your prediction likely to be?
- What is the statistical term for student G?



Tip

What type of diagram could you draw to help you see the link between the Maths and Statistics results?



A rectangle has length  $(2x + 3)$  cm and width  $(x - 1)$  cm.

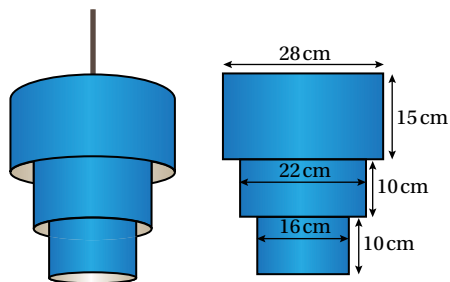
The perimeter of the rectangle is 70 cm.

- Write an expression for the perimeter of the rectangle.
- How long is the longest side?
- Work out the area of the rectangle.

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Ajit is making a lampshade, based on the design shown.  
 The lampshade consists of three cylinders covered in fabric.  
 How much fabric will Ajit need to make this lampshade?

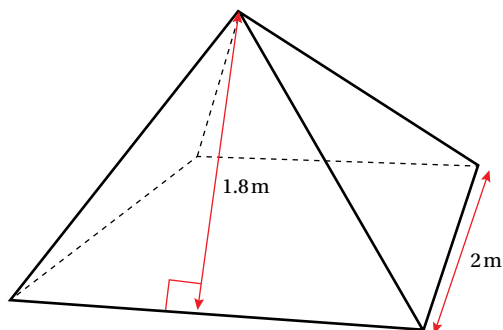


Tip

You need to calculate the surface area of each cylinder in the lampshade. Remember that each cylinder is covered with a rectangle of fabric.



How much fabric is required to construct a tent like the one shown below?  
 The top vertex is above the middle of the square base.



Tip

What shapes do you need? Draw them and label them.



Margaret's garden contains lavender bushes and geraniums.  
 There is a row of six lavender bushes, then a row of ten geraniums, then six lavenders, then ten geraniums, and so on.  
 The rows alternate and finish with a row of lavender.  
 There are 70 geranium plants.  
 How many lavender bushes are there?



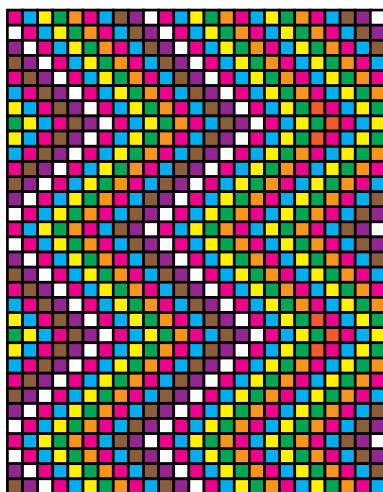
Two cafés record what types of drink they sell during a morning.

Type of drink	Frequency	
	Café A	Café B
Tea	28	15
Americano	22	8
Espresso	9	6
Latté	6	35
Cappuccino	7	26

- a Write a list of all of the types of statistical graph you know how to draw.
- b For each one say whether the graph is an appropriate way to represent this data.
- c Draw the graph you think is most appropriate and say what it tells you.



Granny Bessie is making a patchwork quilt with scraps of fabric.



Each patch is  $(2x - 3)$  cm long and  $(x + 3)$  cm wide.

- a There are 25 patches in each row. Write a possible expression for the width of the quilt.
- b There are 32 patches in each column. Write a possible expression for the length of the quilt.

The width of each patch is 8 cm.

- c Calculate the dimensions of the quilt. Give your answer in metres.



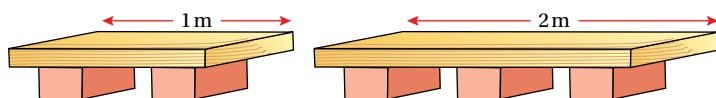
**Tip**

The diagram in the question is very detailed. Could a simpler diagram help?

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In the local cement factory, the cement bags are placed on pallets made of planks of wood and bricks as shown in the diagram below.



The number of bricks needed to make the pallet is calculated as 'one more than the length of the plank in metres'.

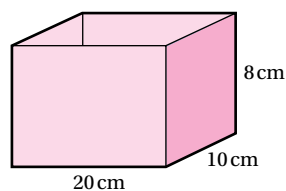
- What length of pallet uses five bricks?
- If a pallet is 3 m long, how many bricks are used in it?

The factory needs pallets with a total length of 15 m for the next batch of cement it makes. It has planks of wood that are 4 m long and 3 m long.

- What combinations of planks can they have?
- How many bricks would be needed for each combination?



Sophie has a sheet of coloured paper and she wants to use it to cover a box.



The box is in the shape of a cuboid but it does not have a lid.

The dimensions of the base of the box are 20 cm by 10 cm and it has a height of 8 cm.

- What is the surface area of the exterior five faces of the box?

The walls of the box are 5 mm thick. To secure the paper and make it much neater Sophie would like to cover the inside of the box as well, but not the inside of the base.

Her sheet of paper measures 45 cm by 55 cm.

- Is the sheet of paper big enough for Sophie to cover the outside and inside walls of her box?



Geoff and Ravinder often have badminton and squash matches. The probability of Geoff winning at badminton is 0.85 and the probability of Geoff winning at squash is 0.35.

- What is the probability that the next time they play both matches, Geoff wins both?
- What is the probability that Geoff loses at badminton but wins at squash?
- What is the probability that both Geoff and Ravinder win one match each?



The probability Leela catches the 6:30 am train to Brighton is 0.7. If she misses the train she will be late for work.

The probability the train will be late is 0.15. If the train is late she will be late for work.

What is the probability Leela will be on time for work on a particular day?



Tip

What type of diagram might help?



In a cycling race, hill climbs are rated according to a category system. A category 5 climb is the easiest: it has an average slope of 3 : 100 (for every 100 m horizontal distance you climb 3 m vertically).

- What is the angle of the slope in a category 5 climb?

The hardest category of climb has an average slope of 1 : 10.

- How much steeper in degrees is this climb compared to the category 5 climb?

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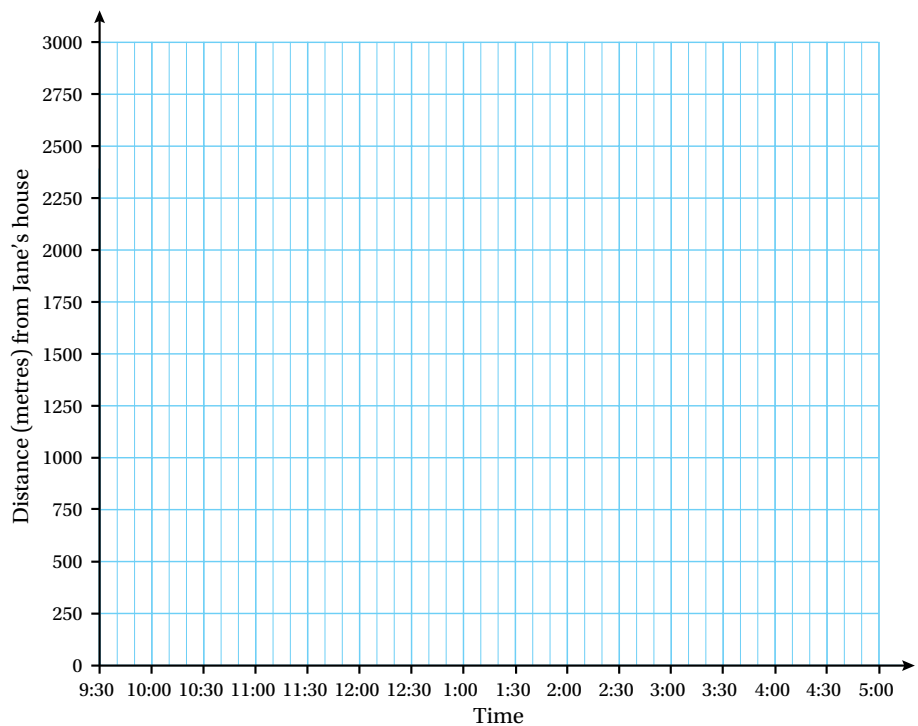
Jane left home at 9:30 am and walked at a speed of 5 kilometres per hour through the park until she reached the swings, which are 1250 metres from her house.

In the park she stopped to swing for 30 minutes.

After this, Jane continued for 1500 metres to Julie’s house. She arrived there at 11:30 am.

Jane stayed at Julie’s house until 3:00 pm. She then walked directly home, arriving at 4:40 pm.

- a Copy and complete the travel graph for Jane’s journey to and from Julie’s house.
- b At what speed did Jane walk for the second stage of her journey to Julie’s house?
- c What was Jane’s average speed on her return journey in the afternoon?

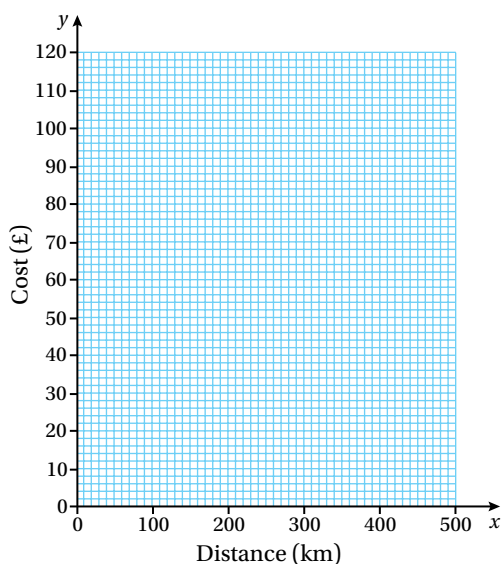


Marianne needs to make a long-distance journey. She is looking for the cheapest possible car hire.

Whacky Wheels has a standard charge of £35, then 15p for every kilometre driven.

Wheelies Rentals has a charge of 23p per kilometre travelled.

- a Draw the charges graph for both car hire companies.



**Tip**

In this question you can use the axes that are given to help you draw the diagram.



- b** Marianne thinks the return journey is 300 km.  
Which company would be cheaper to use?
- c** Unfortunately, Marianne had made a mistake and the return journey was 500 km.  
How much money would Marianne have saved by using the other hire company?



The vertices of a quadrilateral are A, B, C and D.

A has coordinates (2, 1).

$$\vec{AB} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \quad \vec{BC} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}, \quad \vec{AD} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$$

- a** Write a column vector for  $\vec{CD}$ .
- b** Compare  $\vec{CD}$  with  $\vec{AB}$ . What do you notice? Can you say why?
- c** What type of quadrilateral is ABCD?



**Tip**

Draw the shape on squared paper.

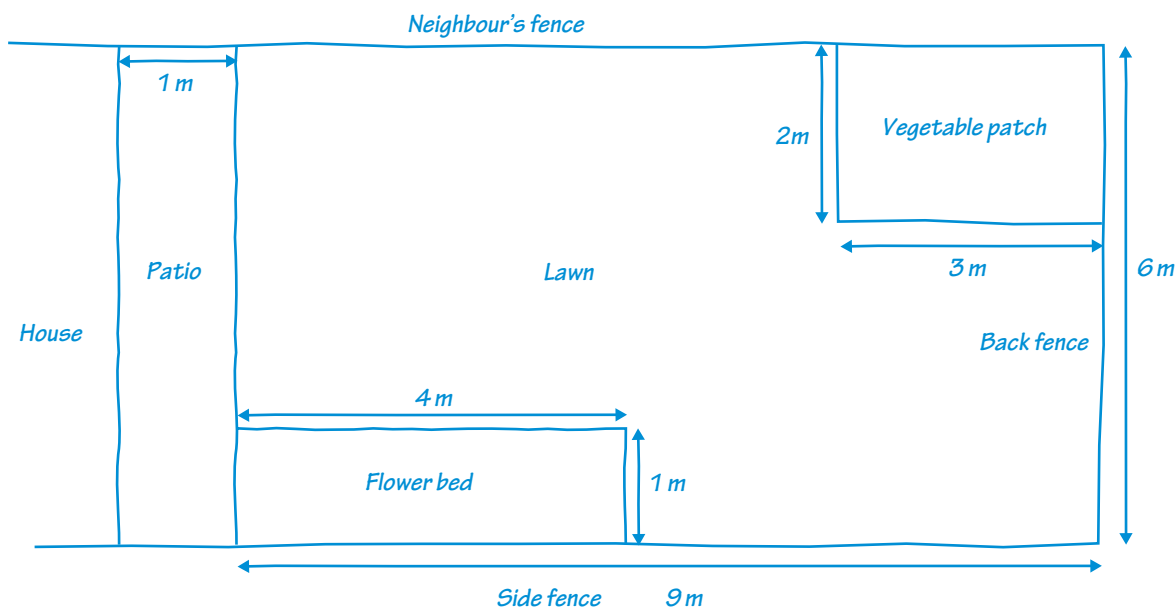


Ann-Marie wants to plant a cherry tree in her garden.  
 She knows that when it is fully grown it will have a diameter of 3 m.  
 Ann-Marie wants all of the fruit to fall on her lawn.  
 Here is a sketch, not drawn to scale, of Ann-Marie's garden.



**Tip**

This diagram isn't drawn to scale but your diagram should be.

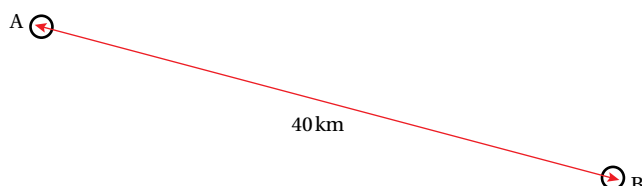


Where could the tree be planted?



The diagram represents two remote towns labelled A and B in the Scottish Highlands.

Diagram not to scale



The mountain rescue helicopters from both towns will rescue any casualty within a radius of 25 km of town A or town B.

The fire brigade from town B will travel to any accident scene closer to town B than town A.

Draw a diagram and shade the region that the helicopter and fire brigade will both cover.



A projector is placed 1 m from a screen.

When the projector is turned on, the image produced on the screen is only 20 cm high.

How far back should the projector be moved in order to produce an image that fills the screen, which is approximately 1.5 m in height? (Assume that no other adjustments are made to the projector.)



**a** Harriet has a challenge for her classmate, Janet. “I’m thinking of a triangle”, says Harriet:

- It has a right angle.
- It has one angle of  $40^\circ$ .
- It has one side that has a length of 5 cm.

“Can you draw my triangle, Janet?”

- i** Accurately construct a triangle that satisfies Harriet’s conditions.
- ii** Demonstrate that there is more than one triangle that Harriet could be thinking of.
- iii** How could Harriet alter her challenge so that only one triangle is possible?



**Tip**

To construct an accurate triangle you may need to use a ruler, protractor and a pair of compasses.



**Tip**

Before you start your accurate construction, make some sketches to show the positions of the sides and angles you are given.

**b** Janet then comes up with a challenge for Harriet: “I’m thinking of a triangle”:

- It has one side of length 4 cm.
- It has another side of length 7 cm.
- The angle in between these two sides is  $55^\circ$ .

“Can you draw my triangle, Harriet?”

- i** How many triangles satisfy Janet’s conditions? Give reasons for your answer.
- ii** Measure the length of the third side.



The probability of it raining on Tuesday is 0.15.

If it rains on Tuesday, the probability of it raining on Wednesday is 0.25.

If it does not rain on Tuesday, the probability of it raining on Wednesday is 0.45.

- a** Draw a tree diagram to show the possible outcomes.
- b** What is the probability it rains on both Tuesday and Wednesday?
- c** What is the probability it rains on only one of the days?



The owner of a new bookshop carried out a survey to find the most popular A-level courses.

A total of 200 students were asked if they were studying Chemistry, Physics or Maths.

43 of the students surveyed did not study any of these three subjects.

A total of 92 were studying Chemistry.

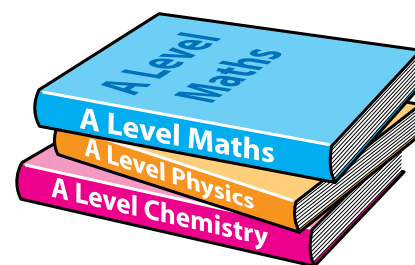
There were 23 students studying both Chemistry and Maths, but not Physics.

There were 19 students studying both Physics and Maths, but not Chemistry.

29 students were studying only Physics, and there were a total of 74 students who studied Physics.

53 students were studying two of these three subjects.

- a** Display the information in a Venn diagram.
- b** If one person was chosen at random, what is the probability that they studied only Maths?
- c** If one person was chosen at random, what is the probability that they studied at least two of the subjects?





Caroline and Janet swim a total of 45 lengths each. They start together and finish together.

Caroline and Janet swim at different speeds for different swimming strokes but take 30 minutes to finish their 45 lengths.

Caroline swims 45 lengths of breaststroke, completing each one at the same speed.

Janet swims 20 lengths of front crawl in the first 10 minutes, then the remaining 25 lengths in the rest of the time.

- a Is there a time when Caroline has swum more lengths than Janet?
- b How long does it take Caroline to swim 30 lengths?
- c What is Caroline's speed, in lengths per minute?



You can use the three transformations listed below.

A Reflect in the line  $y = x$ .

B Translate by vector  $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ .

C Enlarge by scale factor  $\frac{1}{2}$  about the point (2, 3).

- a Carry out all three transformations, in order, on a starting shape of your choice.
- b How does the resulting image change if the transformations are applied in reverse order  $C \rightarrow B \rightarrow A$ ?



### Tip

You will find this question easier if you actually try it out. Think about how you can make the question simpler by choosing particular shapes and certain side lengths that make the enlargement easier.