1 Draw a diagram

There is an old saying: 'A picture is worth a thousand words.'

So, if a diagram is not provided then draw one. It might be helpful, and could give you some ideas about how to solve the problem.

Draw a decent diagram. A sketch is probably fine, but it needs to look like the scenario it describes. If there is a triangle in the problem then your shape should be a triangle. If there is supposed to be a straight line then your line should be straight. The actual sizes of sides and angles are probably not important. Try to make your diagrams large and clear.

Annotate your diagram. If there is information provided in the question (such as the lengths of sides, or the sizes of angles) then write these on your diagram. This will often help when you are solving a problem.

Add new information that you work out. When you work out something new, add this to the diagram too.

So, in summary:

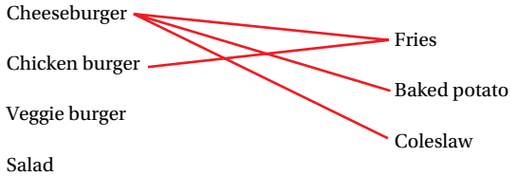
- draw a decent diagram
- label it
- add new information that you work out.

It is fairly obvious what to do when a diagram has been provided as part of a question, but a diagram can sometimes be useful in other situations. Here are two examples where drawing diagrams could help you.

At a fast food restaurant there is a 'meal deal' that involves first choosing one of the following: cheeseburger, chicken burger, veggie burger or salad, and then ordering a side from the following list: fries, baked potato or coleslaw.

How many different meals could you have?

You could work systematically and create a list, but a diagram would also help.



The diagram above shows all the options and the lines show some of the possible combinations.

There are three lines coming from cheeseburger. How many lines will come from chicken burger when the diagram is finished? Will this help you answer the question?

Alternatively, you could create a table like the one below to help you. What does each cell in this table represent? How does this table tell you, at a glance, how many meal possibilities there are?

	Fries	**Baked potato**	**Coleslaw**
Cheeseburger			
Chicken burger			
Veggie burger			
Salad			

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36
37	38	39	40	41	42
43	44	45	46	47	48
49	50	51	52	53	54
55	56	57	58	59	60
61	62	63	64	65	66
67	68	69	70	71	72
73	74	75	76	77	78
79	80	81	82	83	84
85	86	87	88	89	90
91	92	93	94	95	96

Here is another example:

Jilly says: 'If I write out numbers in rows of six, all of the prime numbers will either be in the top row, in the column that has 1 at the top or in the column that has 5 at the top.'

Can you tell if she is right?

You need to have some numbers to look at here so a diagram will be important.

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

Now you could start highlighting a few prime numbers.

This looks good so far. (It is worth noting that Jilly didn't say that every number in the top row would be prime, just that all of the primes would be in the top row or the 1 or 5 columns).

Now, why can't there be any prime numbers in the column with 2 at the top?

When you go down a row, it is the same as adding 6 to the number above. The 2 column goes: 2, 8, 14, 20, 26, and because you started with an even number and are adding an even number, these will always be even. So there can't be any more prime numbers in this column.

3
9
15
21
27
33

The column with 3 at the top has numbers that are all odd. But they are all multiples of 3.

The column with 4 at the top cannot have any prime numbers in it. Why not?

Neither can the column with 6 at the top.

This just leaves you with two columns and the top row for primes to go in. This means that Jilly is right.

The following problems may be solved using more than one method; however, the worked solutions provided at the back of this book are based on the method introduced above.

A rectangle has length $2x + 3$ and width $x - 1$.

a Write an expression for the perimeter of the rectangle.

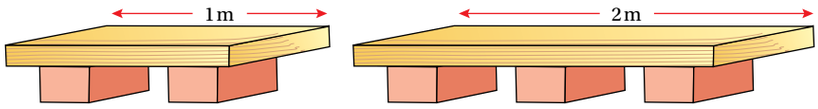
b Write an expression for the area of the rectangle.

The perimeter of the rectangle is 70 cm.

c How long is the longest side?

d What is the area of the rectangle?

In a cement factory, the cement bags are placed on pallets made of planks of wood and bricks.



The number of bricks needed to make the pallet is calculated as 'one more than the length of the plank in metres'.

a What length of pallet uses five bricks?

b If the pallet is 7 m long, how many bricks are used in it?

The factory needs pallets with a total length of 25 m for the next batch of cement. It has planks of wood that are 4 m long and 3 m long.

c What combinations of planks can they have?

d How many bricks would be needed for each combination?

Tip

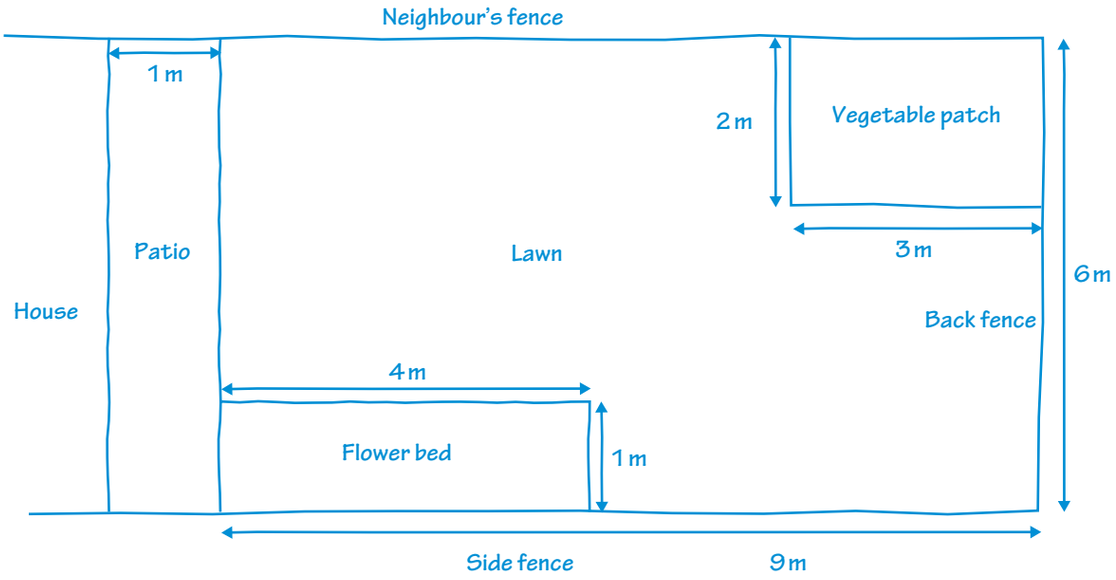
253 = 23 × 11

3

Ann-Marie wants to plant a cherry tree in her garden. When it is fully grown it will have a diameter of 3 m.

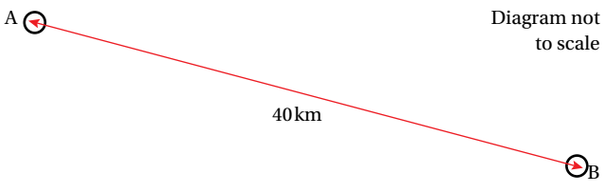
Ann-Marie wants all of the fruit to fall on her lawn.

Here is a sketch, not drawn to scale, of Ann-Marie's garden.



Where could the tree be planted?

4



The diagram represents two remote towns A and B.

The mountain rescue helicopters from both towns are dispatched to rescue any casualty within a radius of 25 km of town A or town B. The fire brigade from town B will travel to any accident scene closer to town B than town A.

Shade the region that the helicopters and town B's and fire brigade will both cover.

5

The probability Leela catches the 6.30 am train to Brighton is 0.7.

If she misses the train she will be late for work.

The probability the train will be late is 0.15.

If the train is late she will be late for work.

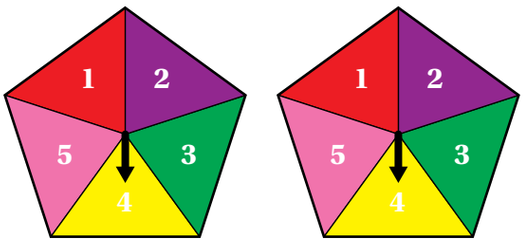
What is the probability Leela will be on time for work on a particular day?

Tip

What type of diagram might help here?

6

Two five-sided spinners are numbered 1 to 5. When the arrows are spun, your total score is calculated by adding the two numbers the spinners land on.



a Draw a suitable diagram to show all possible outcomes when spinning these spinners.

b What is the highest score you could get?

c What is probability of getting a total score of 8?

7

The vertices of a quadrilateral are A, B, C and D. A has coordinates (2, 1).

$\overrightarrow{AB} = \begin{pmatrix}2\\3\end{pmatrix}$, $\overrightarrow{BC} = \begin{pmatrix}4\\0\end{pmatrix}$, $\overrightarrow{AD} = \begin{pmatrix}4\\0\end{pmatrix}$

a Write a column vector for $\overrightarrow{CD}$.

b Compare $\overrightarrow{CD}$ with $\overrightarrow{AB}$. What do you notice? Can you explain?

c What type of quadrilateral is $ABCD$?

Tip

Draw the shape on squared paper.

8

A projector is placed 1 m from a screen. When the projector is turned on, the image produced is only 20 cm high.

How far back should the projector be moved in order to produce an image that fills the screen, which is approximately 1.5 m in height? (Assume that no other adjustments are made to the projector.)

9

Peter the factory manager planned to install a new hot drinks machine for the factory workers. He decided to fill it with tea because he thought tea was the most popular hot drink.

The workers did a survey to check their preferred hot drink. Each person could choose hot chocolate, tea or coffee.

8 women wanted hot chocolate. 16 workers wanted tea, of which 7 were men. 10 men and 12 women wanted coffee. There were 25 men in total.

Was Peter correct?

Tip

What type of diagram might be helpful?

10

Mr Rixson and Mrs Lloyd are the A Level Mathematics teachers at Swanend Hill School. They are extremely competitive and often have debates about their students' results.

The results from the latest assessments were as follows:

Class	Mean score	Median score	Range	IQR	Lowest score	Highest score
Rixson	7.5	7.5	9	4	3	12
Lloyd	9	7	17	14.5	1	18

Both Mr Rixson and Mrs Lloyd have 12 students in their A Level Mathematics classes.

Compare the results of the classes.

Tip

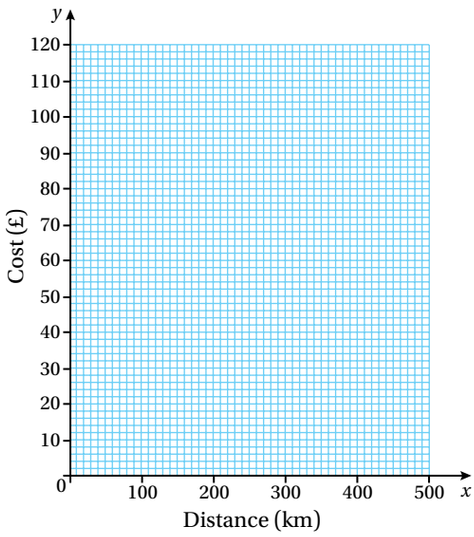
Draw a possible box plot for both classes.

Marianne needs to make a long-distance journey. She is looking for the cheapest car hire.

Whacky Wheels has a standard charge of £35, then 15p for every kilometre driven.

Wheelies Rentals has a charge of £23 per 100 km travelled.

a Complete the charges graph for both car hire companies.



Tip

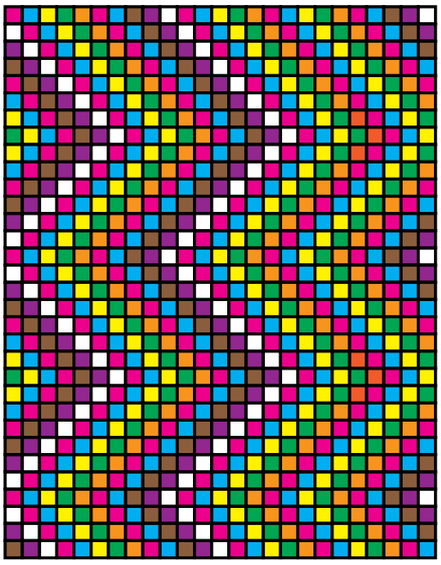
In this question you can use the axes that are given to help you draw the diagram.

b As Marianne thinks the return journey is 300 km, which company would be the cheaper to use?

c Marianne made a mistake in her route plan and the return journey was 500 km. Would Marianne have saved money by using the other hire company?

12 ★★☆

Granny Bessie is making a patchwork quilt with scraps of fabric.



Each patch is $(2x - 3)$ cm long and $(x + 3)$ cm deep.

a There are 25 patches in each row. Write a possible expression for the width of the quilt.

b There are 32 patches in each column. Write a possible expression for the length of the quilt.

c Write an expression for the area of the quilt, in the form $m(ax^2 + bx + c)$ where m is a constant.

The area of the quilt is $2.8\,m^2$.

d What are the dimensions of each patch? Give your answers in centimetres.

Tip

The diagram in the question is very detailed. Could a simpler diagram help?

Tip

$2.8\,m^2 = 28\,000\,cm^2$. It may be easier to work in centimetres.

13 ★★☆

A square-based food container has a capacity of $1440\,cm^3$.

The base of the container is of length x cm.

a Write an expression for the height of the container in terms of x.

The inside of the container (base and four sides) is to be lined with parchment paper, with no overlaps.

b What is the expression, in terms of x, for the total area of parchment paper needed?

c If the height of the container is 10 cm, what is the size of the base?

d What is the area of the parchment paper needed?

14

A rectangular swimming pool is surrounded by a path made of very decorative mosaic tiles. The width of the path is x.

The pool itself measures 35 m by 30 m.

a Write an expression for the area of the tile path in terms of x.

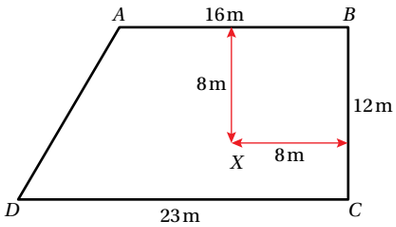
Having the tile path laid was very expensive.

It cost a total of £3196.80, at a rate of £30 per square metre.

b Find the width of the path to the nearest centimetre.

15

$ABCD$ is a field surrounded by fences AB, BC, CD and DA.



There is a dog tied to the spike X on a lead measuring 3 m.

There is a bull on an 8 m rope which is attached to the top of post A.

Find a route from corner D to corner B that would avoid both the bull and the dog.

16

Serrianne has taken up golf and goes to a golf range twice a week. She uses one bucket of balls each time. In every bucket of 25 balls there are always three yellow balls; the rest are white.

Serrianne hits one ball (chosen at random) at a time.

a What is the probability that the first three balls she uses will all be yellow?

b What is the probability that the first three balls she uses will all be white?

c Calculate the probability that the first three balls Serrianne uses are a mixture of two yellow and one white.

17 ★★☆

On his journey to work Abu must drive through two sets of traffic lights.

The probability of the first set being green is 0.7. If the first set is green the probability of the second set also being green is 0.8. But if the first set is not green the probability of the second set being green is 0.4.

a What is the probability that Abu does not have to stop on his journey to work tomorrow?

b What is the probability that Abu only has to stop once on his journey to work tomorrow?

18 ★★☆

Geoff and Ravinder are very competitive and often have badminton and squash matches. The probability of Geoff winning at badminton is 0.85 and the probability of Geoff winning at squash is 0.35.

a What is the probability that the next time they play both matches, Geoff wins both?

b What is the probability that Geoff loses at badminton but wins at squash?

c What is the probability that both boys win one match each?

19 ★★☆

On a commercial flight to Tanzania the passengers were questioned about their malaria precautions. Only 70% of the passengers had obtained and started a course of anti-malarial tablets. The chances of getting malaria are one in 200 if you take the tablets but one in 50 if you are not taking the tablets. What is the probability that one passenger selected randomly will catch malaria?

Tip

What type of diagram would be helpful?

20

A local bookshop was doing a survey on the most popular A Level courses to help decide how many revision guides to stock. 200 students were asked whether they studied Chemistry, Physics and Maths.

43 of the students surveyed did not study any of these three subjects.

92 were studying Chemistry.

23 were studying both Chemistry and Maths, but not Physics.

19 were studying both Physics and Maths, but not Chemistry.

29 were only studying Physics, and a total of 74 studied Physics.

53 of the students studied two of these three subjects.

a Display the information in an appropriate diagram.

b If one person was chosen at random, what is the probability they only studied maths?

c If one person was chosen at random, what is the probability they studied at least two of the subjects?

21

The point A has coordinates (2, 2).

$\overrightarrow{AB} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$

$\overrightarrow{BC} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$

a Find a possible pair of coordinates for D, if $ABCD$ is an isosceles trapezium.

b Write $\overrightarrow{AC}$ as a column vector.

c Find the coordinates of E, if $\overrightarrow{AE} = 4\overrightarrow{BC}$.

d Using these coordinates for E, write $\overrightarrow{BE}$ as a column vector.

Tip

Draw a sketch of the shape.

22

Eve took a photo of her mum on holiday. Later, when Eve looks at the picture, she sees that her mum seems to be the same height as a hill in the background.

Eve stood approximately 3 m away from her mum when she took the photo, and they were approximately 2 km away from the hill. Eve's mum is 15 cm taller than Eve.

How tall is the hill?

23 ✪✪☆

When enlarging photographs, the increase in width and length must be directly proportional to each other or the photos will be distorted.

A photo has a width of 40 cm and a length of 55 cm.

a An enlargement has a width of 112 cm. Find its length.

b Another enlargement has a length of 148.5 cm. What must its width be?

c A third photo with a length of 15 cm and an area of 127.5 cm^2 is enlarged to a poster with a width of 25.5 cm. What is the area of this poster?

24 ✪✪☆

a Harriet has a challenge for her classmate Janet:

'I'm thinking of a triangle with a right angle. It has one angle of 40° and one side length of 5 cm.'

i Accurately construct a triangle that satisfies Harriet's conditions.

ii Demonstrate that there is more than one triangle that fulfils her conditions.

iii How could Harriet alter her challenge so that only one triangle is possible?

b Janet comes up with a challenge for Harriet:

'My triangle has one side of length 4 cm and another that is 7 cm. The angle in between these two sides is 55°.'

i How many triangles satisfy Janet's conditions? Explain your answer..

ii Find the length of the third side.

Tip

To construct an accurate triangle you may need to use a ruler, protractor and a pair of compasses.

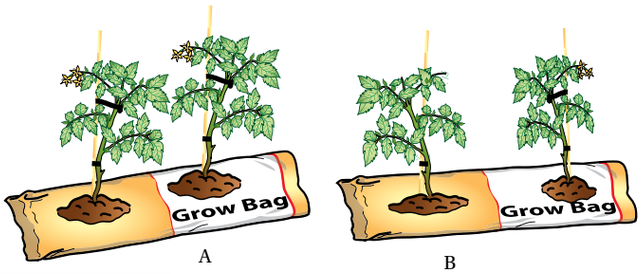
Tip

Before you start your accurate construction, make some sketches to show the positions of the sides and angles you are given.

25 ✪✪☆

Elspeth has an allotment. She is testing out two different types of grow bags for her tomato seedlings, which she gets to a healthy stage and then sells to her neighbours.

She has planted 20 seedlings in each bag. After three weeks she has recorded the heights of the plants as follows:



A B

	Shortest height (cm)	Lower quartile (cm)	Median (cm)	IQR (cm)	Biggest height (cm)
Grow Bag A	5	12	15	8	22
Grow Bag B	8	13	15	5	25

a Compare the data.

b In your opinion, which type of grow bag is the best: A or B? Explain your choice.

26 ✪✪☆

Caroline and Janet do some swimming every day. They swim a total of 45 lengths each. They always start together but never finish together. They swim at different speeds for different swimming strokes.

Caroline always swims 45 lengths of breaststroke in 30 minutes, completing each one at the same speed.

Janet always does 30 lengths of front crawl in the first 12 minutes, then the remaining 15 lengths at a speed of one length per 40 seconds.

a After 10 minutes, how many lengths has Caroline completed?

b How long does it take Janet to complete her final 15 lengths?

c What is Caroline's speed in lengths per minute?

d How long must Janet wait for Caroline to finish?

e Roughly, on average, how many lengths does Janet swim each minute?

f If Caroline continued swimming for another 10 minutes, in theory how many lengths should she complete in total? Explain why this figure might not be correct.

27 ✪✪✪

Two of the vertices of an equilateral triangle are located at points with coordinates (0, 0) and (6, 0).

a Work out the possible coordinates of the third vertex.

b Instead, if two of the vertices of an equilateral triangle are located at (–3, 2) and (5, –4), what is its area?

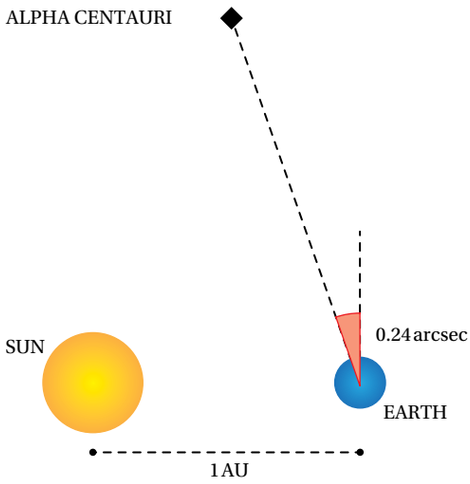
28 ✪✪✪

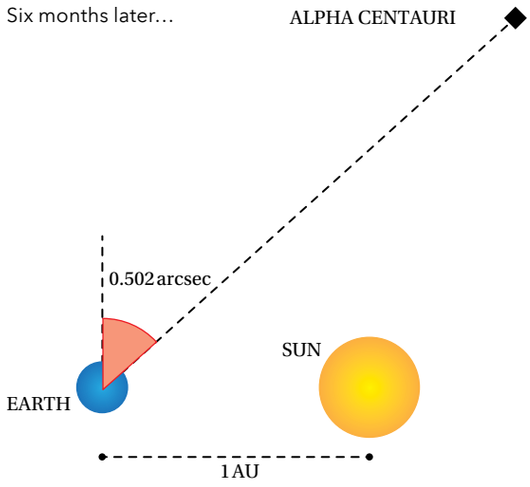
Two astronomers want to calculate the distance to one of our closest stars, Alpha Centauri. In order to do so they take two angle measurements, six months apart. The two angles measured by the astronomers are shown in the diagrams on the right.

1 Astronomical Unit (AU) $\approx 1.5 \times 10^8$ km

1 arcsec $= (\frac{1}{3600})°$

Use the astronomers' measurements to calculate an approximate distance to Alpha Centauri.





29 ✪✪✪

You are 'armed' with the three transformations listed below:

A Reflect in the line $y = x$.

B Translate by $\begin{pmatrix}1\\0\end{pmatrix}$.

C Enlarge by scale factor $\frac{1}{2}$ about the point (2, 3).

Carry out all three transformations, in order, to a starting shape of your choice.

a How does the resulting image change if the transformations are applied in reverse order? **C** ⟶ **B** ⟶ **A**

b How many different final images could be produced by changing the order in which the three transformations are applied?

Tip

You will find this question easier if you try it out. Think about how you can make it simpler by choosing (irregular) shapes and side lengths that make the enlargement easier.