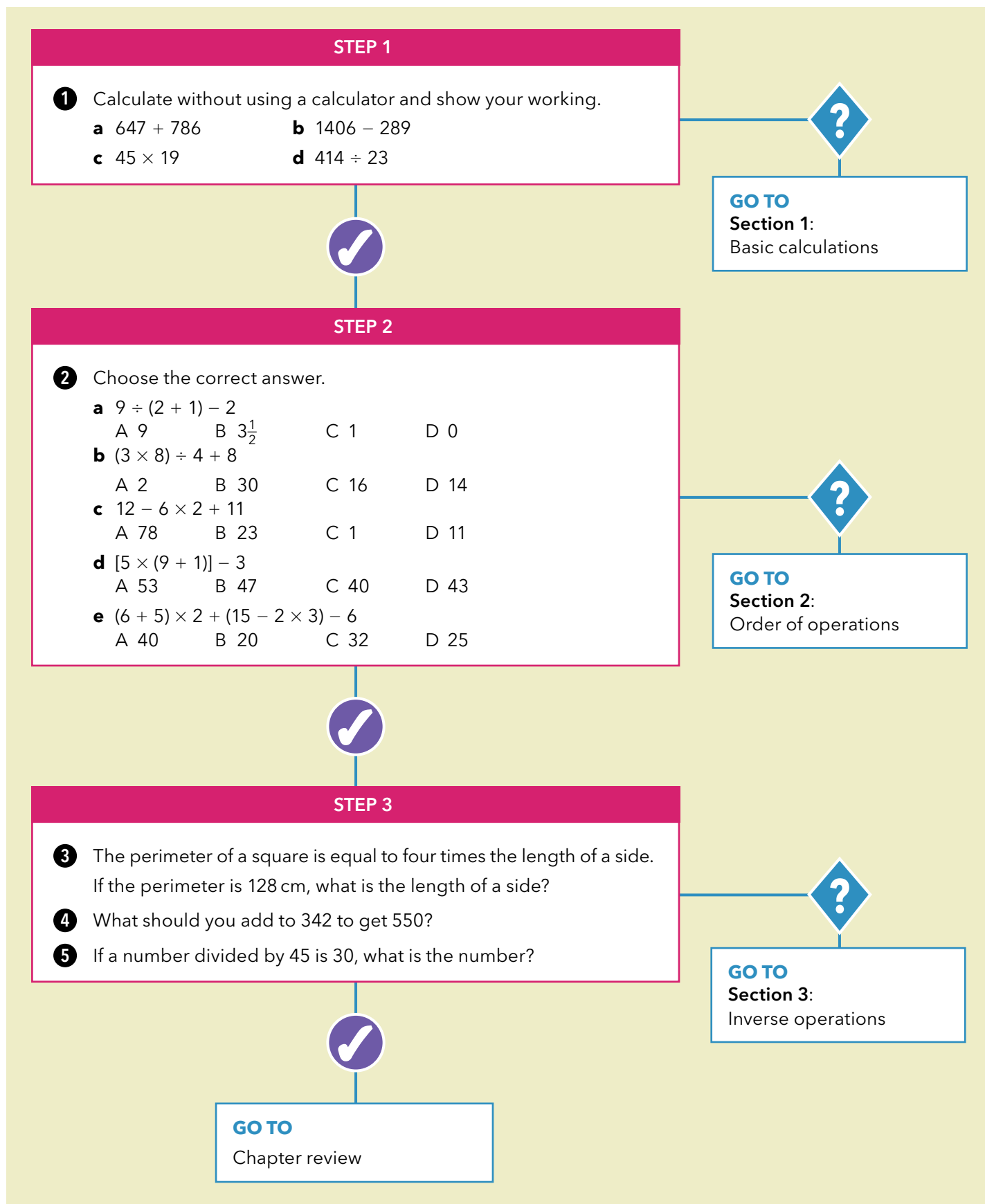


Assess your starting point using the Launchpad



Section 1: Basic calculations

You will not always have a calculator so it is useful to know how to do calculations using mental and written strategies.

It is best to use a method that you are confident with and always **show your working**.

Remember that when a question asks you to find the:

- **sum** you need to add.
- **difference** you need to subtract one number from another.
- **product** you need to multiply.
- **quotient** you need to divide.



Tip

These are the kind of skills that are tested in the non-calculator examination papers.

WORK IT OUT 1.1

Look at these calculations carefully.

Discuss with a partner what methods these students have used to find the answer.

Which method would you use to do each of these calculations? Why?

①	$489 + 274$	②	$284 - 176$
	$400 + 200 \rightarrow 600$		$\begin{array}{r} 284 \\ - 176 \\ \hline 108 \end{array}$
	$80 + 70 \rightarrow 150$		
	$9 + 4 \rightarrow 13$		
	$\underline{763}$		
③	29×17	④	15×62
	$\rightarrow 30 \times 17 - 17$		$= 30 \times 31 \quad 310$
	$\rightarrow 3 \times 170 - 17$		$= 930 \quad 310$
	$\rightarrow 510 - 17$		$\underline{310}$
	$\rightarrow 493$		$\underline{930}$
⑤	207×47	⑥	$2394 \div 42$
	$\begin{array}{r l} \times & 200 & 0 & 7 \\ \hline 40 & 8000 & 0 & 280 \\ 7 & 1400 & 0 & 49 \\ \hline & 9400 & + 0 & + 329 \\ & = 9729 & & \end{array}$		$\begin{array}{r} 2394 \\ - 1680 \quad (40) \\ \hline 714 \\ - 420 \quad (10) \\ \hline 294 \\ - 210 \quad (5) \\ \hline 84 \\ - 84 \quad (2) \\ \hline 0 \end{array} \quad \begin{array}{l} 42 \times 10 = 420 \\ 42 \times 20 = 840 \\ 42 \times 40 = 1680 \\ 42 \times 5 = 210 \\ 42 \times 2 = 84 \end{array}$



Find answers at: cambridge.org/ukschools/gcsemaths-studentbookanswers

Problem-solving strategies

Many of the written calculations that you have to do will be to solve word problems.

Everyone has their own method of solving problems but there are some useful strategies and techniques that you can use for problem solving.

The problem-solving framework below outlines the steps that you can take to break down most problems to help you solve them more easily.

If you follow these steps each time you are faced with a problem you will become more skilled at problem solving and more able to self-check.

These are important skills both for your GCSE courses and for everyday life.

Problem-solving framework

Sally buys, repairs and sells used furniture at a market.

Last week she bought a table for £32 and a bench for £18.

She spent £12 on wood, nails, varnish and glue to fix them up.

She then sold the two items on her stall for £69.

How much profit did she make on the two items?

Steps for approaching a problem-solving question	What you would do for this example
Step 1: Work out what you have to do. Start by reading the question carefully.	Find the profit on the two items.
Step 2: What information do you need? Have you got it all?	Cost of items = £32 + £18 Cost of repairs = £12 Selling price = £69
Step 3: Is there any information that you don't need?	In this problem you don't need to know what she spent money on for repairs. You just need to know how much she spent. Many problems contain extra information that you don't need to test your understanding.
Step 4: Decide what maths you can do.	Profit = selling price – cost You can add the costs and subtract them from the selling price.
Step 5: Set out your solution clearly. Check your working and make sure your answer is reasonable.	Cost = £32 + £18 + £12 = £62 Profit = £69 – £62 = £7 Sally made £7 profit.
Step 6: Check that you have answered the question.	Yes. You needed to find the profit and you have found it.

EXERCISE 1A

Solve these problems using written methods.
 Set out your solutions clearly to show the methods you chose.



Tip

You don't always need to write anything for the first few steps in the problem-solving framework, but you should try to mentally answer the questions as you read problems to decide what to do. You should always show how you worked to solve the problem.

- 1 After checking the prices at three different supermarkets, Nola found out that the cheapest pack of pens was £3.90 for three. She bought fifteen pens.
 How much did she pay in total and what does this work out to per pen?
 - a How many packs of pens did she buy? Why do you need to know this?
 - b What is the total cost of the fifteen pens?
 - c What is the cost of each pen?
- 2 Sandra bought a pair of jeans for £34, a scarf for £9.50 and a top for £20. If she had saved £100 to buy these items, how much money would she have left?
- 3 How many 16-page brochures can you make from 1030 pages?
- 4 Jason can type 48 words per minute.
 - a How many words can he type in an hour and a half?
 - b Approximately how long would it take him to type an article of 2000 words?
- 5 At the start of a year the population of Greenside Village was 56 309. During the year 617 people died, 1835 babies were born, 4087 people left the village and 3099 people moved into the village.
 What was the population at the end of the year?
- 6 The Amazon River is 6448 km long, the Nile River is 6670 km and the Severn is 354 km long.
 - a How much longer is the Nile than the Amazon?
 - b How much shorter is the Severn than the Amazon?
- 7 What is the combined sum of 132 plus 99 and the product of 36 and 127?
- 8 What is the result when the difference between 8765 and 3087 is added to the result of 1206 divided by 18?



Did you know?

The Severn is the longest river in the UK.



Find answers at: cambridge.org/ukschools/gcsemaths-studentbookanswers



Key vocabulary

integers: whole numbers belonging to the set $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$; they are sometimes called directed numbers because they have a negative or positive sign.



Tip

You will be expected to work with negative and positive values in algebra, so it is important to make sure you can do this early on in your GCSE course.

Working with negative and positive integers

When doing calculations involving positive and negative **integers**, you need to remember to apply the following rules:

- Adding a negative number is the same as subtracting the number:
 $4 + -3 = 1$.
- Subtracting a negative number is the same as adding a positive number:
 $5 - -3 = 8$.
- Multiplying or dividing the same signs gives a positive answer:
 $-4 \times -2 = 8$ and $\frac{-4}{-2} = 2$.
- Multiplying or dividing different signs gives a negative answer:
 $4 \times -2 = -8$ and $\frac{-4}{2} = -2$.

EXERCISE 1B

1 Calculate.

- | | | |
|--------------------------|-------------------------|--------------------------|
| a $12 - 5 + 8$ | b $-3 - 4 - 8$ | c $3 + 5 - 6$ |
| d $-2 - 8 + 5$ | e $14 - 3 - 9$ | f $9 - 3 - 4$ |
| g $-34 + 18 - 12$ | h $25 - 19 - 42$ | i $-9 - (-7)$ |
| j $-3 - (-10)$ | k $-4 - (-12)$ | l $8 - (-9)$ |
| m $9 - (-8)$ | n $-14 - (-14)$ | o $-3 - 8 - (-9)$ |

2 Calculate.

- | | | |
|-----------------------------------|----------------------------------|----------------------------------|
| a $-2 \times -4 \times -4$ | b $-4 \times 3 \times -6$ | c $-3 \times -4 \times 3$ |
| d $-4 \times -8 \times 3$ | e $3 \times 6 \times -4$ | f $12 \times 2 \times -3$ |
| g $1 \times -1 \times 10$ | h $-3 \times -8 \times 9$ | i $24 \div 3$ |
| j $-24 \div 3$ | k $-28 \div 2$ | l $-48 \div -6$ |
| m $-300 \div -10$ | n $400 \div -40$ | o $42 \div -7$ |
| p $-22 \div -22$ | q $-33 \div 11$ | r $-27 \div -3$ |
| s $45 \div -9$ | t $-64 \div -8$ | u $-81 \div 3$ |

3 Calculate.

- | | | |
|-----------------------------|----------------------------|---------------------------|
| a $\frac{-40}{5}$ | b $\frac{-28}{-4}$ | c $\frac{30}{-5}$ |
| d $\frac{12}{-2}$ | e $\frac{65}{-5}$ | f $\frac{-48}{-6}$ |
| g $\frac{-330}{-10}$ | h $\frac{-400}{40}$ | i $\frac{-63}{7}$ |
| j $\frac{-60}{-20}$ | k $\frac{60}{-6}$ | l $\frac{-36}{6}$ |

- 4 Apply the operations in the first row to the given number to complete each table.

a

	-10	$\times -2$	+ 4	$\div -2$	- 8	+ 1
-5						

b

	$\times -4$	$\div -5$	+ 8	- 3	$\times 2$	- 9
10						

c

	- 10	$\times -2$	+ 4	$\div -2$	- 8	+ 1
0						

- 5 Here are some bank transactions.

Calculate the new balance in each case.

- a Balance of £230, withdraw £100.
 b Balance of £250.50, withdraw £300.
 c Balance of -£450, deposit £900 then withdraw £300.
 d Balance of -£100, deposit £2000 then withdraw £550.

- 6 The opening of an oil well 5000 feet below sea level, caused a massive oil spill in the Gulf of Mexico in 2010.

The oil well itself extended to a depth of 13 000 feet.

Express the answers to these questions as directed numbers.

- a How deep was the deepest part of the oil well below the sea bed?
 b How far did oil travel from the bottom of the well to reach the surface of the water?
 c The oil company involved estimated that they were losing money at the rate of \$15 000 000 per day. Use an integer to express the money lost after:
 i one week. ii thirteen weeks.

- 7 Here is a set of integers.

$\{-8, -6, -3, 1, 3, 7\}$

- a Find two numbers with a difference of 9.
 b Find three numbers with a sum of 1.
 c Find two numbers whose product is -3.
 d Find two numbers which, when divided, will give an answer of -6.

- 8 One more than -6 is added to the product of 7 and six less than 3.
 What is the result?

- 9 The temperature in Inverness is 4 °C at 7 pm at night.
 By 1 am the same night, it has dropped by 12 degrees.

- a What is the temperature at 1 am?
 b What is the average hourly change in the temperature?
 c By noon the next day, the temperature is 7 °C.
 How many degrees warmer is this than it was at 1 am?








Tip

Feet is a standard unit of imperial measurement for length; the metric measurement for length is metres. You will learn more about metric measurements in Chapter 12.

Section 2: Order of operations

Jose posted this calculation on his wall on social media.

	JOSE: $24 + 6 \div 2 - 1 \times 4 = ?$
COMMENT ✓ LIKE ↻ SHARE	
	JOANNA: 56
	PETER: 11
	LUCIA: 23
	DIPAK: 104

Within minutes, his friends had posted four different answers.
 Which one (if any) do you think is correct? Why?



Tip

Many people remember these rules using the letters BIDMAS (or sometimes BODMAS).

Brackets

Indices (or **O**rders)

Divide and/or **M**ultiply

Add and/or **S**ubtract

There is a set of rules that tell you the order in which you need to work when there is more than one operation.

The order of operations is:

- 1 Do any operations in brackets first.
- 2 If there are any indices (powers or exponents) in the calculation, do them next.
- 3 Do division and multiplication next, working from left to right.
- 4 Do addition and subtraction last, working from left to right.

Brackets and symbols

Brackets are used to group operations. For example:

$$(3 + 7) \times (30 \div 2)$$

When there is more than one set of brackets, it is best to work from the **innermost set** to the **outermost set**.

WORKED EXAMPLE 1

Solve $2((4 + 2) \times 2 - 3(1 - 3) - 10)$

$$2((4 + 2) \times 2 - 3(1 - 3) - 10)$$

$$2((4 + 2) \times 2 - 3(1 - 3) - 10)$$

$$= 2(6 \times 2 - 3(-2) - 10)$$

$$= 2(6 \times 2 - 3 \times -2 - 10)$$

$$2(6 \times 2 - 3 \times -2 - 10)$$

$$= 2(12 - -6 - 10)$$

$$= 2(8)$$

$$= 2 \times 8$$

$$= 16$$

Highlight the different pairs of brackets to help if you need to.

The red brackets are the innermost, so do the calculations inside these ones first. There are two lots of red brackets, so work from left to right. **Note** that you can leave -2 inside brackets if you prefer because $3(-2)$ is the same as 3×-2 .

Black brackets are next. Do the multiplications first from left to right, then the subtractions from left to right.

Often a different style of bracket will be used to make it easier to identify each pair. For example, the following different types of brackets have been used below: $()$, $[\]$, $\{ \}$.

$$\{2 - [4(2 - 7) - 4(3 + 8)] - 2\} \times 8$$

Other symbols can also be used to group operations. For example:

Fraction bars: $\frac{5 - 12}{3 - 8}$

Roots: $\sqrt{16 + 9}$

These symbols are treated like brackets when you do a calculation.

WORK IT OUT 1.2

Which of the solutions is correct in each case?

Find the mistakes in the incorrect option.

	Option A	Option B
1	$7 \times 3 + 4$ $= 21 + 4$ $= 25$	$7 \times 3 + 4$ $= 7 \times 7$ $= 49$
2	$(10 - 4) \times (4 + 9)^2$ $= 6 \times 16 + 81$ $= 96 + 81$ $= 177$	$(10 - 4) \times (4 + 9)^2$ $= 6 \times (13)^2$ $= 6 \times 169$ $= 1014$
3	$45 - [20 \times (4 - 3)]$ $= 45 - [20 \times 1]$ $= 45 - 21$ $= 24$	$45 - [20 \times (4 - 3)]$ $= 45 - 20 \times 1$ $= 45 - 20$ $= 25$
4	$30 - 4 \div 2 + 2$ $= 26 \div 2 + 2$ $= 13 + 2$ $= 15$	$30 - 4 \div 2 + 2$ $= 30 - 2 + 2$ $= 30$
5	$\frac{18 - 4}{4 - 2}$ $= \frac{18}{2}$ $= 9$	$\frac{18 - 4}{4 - 2}$ $= \frac{14}{2}$ $= 7$
6	$\sqrt{36 \div 4} + 40 \div 4 + 1$ $= \sqrt{9} + 10 + 1$ $= 3 + 11$ $= 14$	$\sqrt{36 \div 4} + 40 \div 4 + 1$ $= \sqrt{9} + 40 \div 5$ $= 3 + 8$ $= 11$



Calculator tip

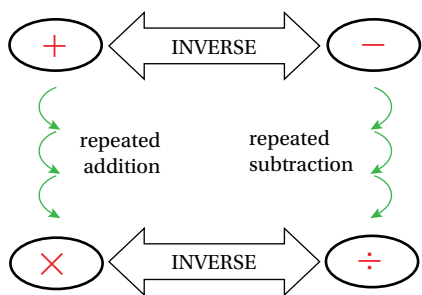
Most modern calculators are programmed to use the correct order of operations. Check your calculator by entering $2 + 3 \times 4$. You should get 14.

If the calculation has brackets, you need to enter the brackets into the calculator to make sure it does these first.

EXERCISE 1C

- 1** Calculate.
 Show the steps in your working.
- | | | |
|---------------------------------------|--------------------------------------|-------------------------------------|
| a $5 \times 10 + 3$ | b $5 \times (10 + 3)$ | c $2 + 10 \times 3$ |
| d $(2 + 10) \times 3$ | e $23 + 7 \times 2$ | f $6 \times 2 \div (3 + 3)$ |
| g $10 - 4 \times 5$ | h $12 + 6 \div 2 - 4$ | i $3 + 4 \times 5 - 10$ |
| j $18 \div 3 \times 5 - 3 + 2$ | k $5 - 3 \times 8 - 6 \div 2$ | l $7 + 8 \div 4 - 1$ |
| m $\frac{15 - 5}{2 \times 5}$ | n $(17 + 1) \div 9 + 2$ | o $\frac{16 - 4}{4 - 1}$ |
| p $17 + 3 \times 21$ | q $48 - (2 + 3) \times 2$ | r $12 \times 4 - 4 \times 8$ |
| s $15 + 30 \div 3 + 6$ | t $20 - 6 \div 3 + 3$ | u $10 - 4 \times 2 \div 2$ |
- 2** Check whether these answers are correct.
 If the answer is wrong, work out the correct answer.
- | | |
|---|---|
| a $12 \times 4 + 76 = 124$ | b $8 + 75 \times 8 = 698$ |
| c $12 \times 18 - 4 \times 23 = 124$ | d $(16 \div 4) \times (7 + 3 \times 4) = 76$ |
| e $(82 - 36) \times (2 + 6) = 16$ | f $(3 \times 7 - 4) - (4 + 6 \div 2) = 12$ |
- 3** Use the numbers listed to make each number sentence true.
- | | |
|---|-----------------|
| a $\square - \square \div \square = \square$ | 0, 2, 5, 10 |
| b $\square - \square \div \square = \square$ | 9, 11, 13, 18 |
| c $\square \div (\square - \square) - \square = \square$ | 1, 3, 8, 14, 16 |
| d $(\square + \square) - (\square - \square) = \square$ | 4, 5, 6, 9, 12 |

Section 3: Inverse operations



Operations are inverses of each other if one undoes (cancels out) the effect of the other.

- Adding is the inverse of subtracting, for example $+5$ is undone by -5 .
- Multiplying is the inverse of dividing, for example $\times 2$ is undone by $\div 2$.
- Taking a square root is the inverse of squaring a number, for example 4^2 is undone by $\sqrt{16}$.
- Taking the cube root is the inverse of cubing a number, for example 2^3 is undone by $\sqrt[3]{2}$.

When you add a number to its inverse the answer is always 0.

For example, $1 + -1 = 0$, -1 is known as the **additive inverse** of 1.

When you multiply a number by its inverse, the answer is always 1.

For example, $2 \times \frac{1}{2} = 1$

$\frac{1}{2}$ is known as the **multiplicative inverse** of 2.

The multiplicative inverse of a number is also called its **reciprocal**.

For example, $\frac{1}{3}$ is the reciprocal of 3.

Inverse operations are useful for checking the results of your calculations because carrying out the inverse operation gets you back to the number you started with.