

# 1 Working with integers

In this chapter you will learn how to ...

- use formal written methods to calculate with positive and negative integers.
- perform operations in the correct order based on mathematical conventions.
- recognise inverse operations and use them to simplify and check calculations.



For more resources relating to this chapter, visit GCSE Mathematics Online.

## Using mathematics: real-life applications

Everyone uses numbers on a daily basis often without really thinking about them. Shopping, cooking, working out bills, paying for transport and measuring, all rely on a good understanding of numbers and calculation skills.



Tip

You probably already know most of the concepts in this chapter. They have been included so that you can revise them if you need to and check that you know them well.



"Number puzzles and games are very popular and there are mobile apps and games available for all age groups. I use an app with my GCSE classes where they have to work in the correct order to solve different number puzzles."

(Secondary school teacher)

## Before you start ...

KS3	You should be able to add, subtract, multiply and divide positive and negative numbers.	<b>1</b> Copy and complete each statement to make it true. Use only $<$ , $=$ or $>$ . <b>a</b> $2 + 3 \square 4 - 7$ <b>b</b> $-3 + 6 \square 4 - 7$ <b>c</b> $-1 - 4 \square 20 \div -4$ <b>d</b> $-6 \times 2 \square -7 - (-5)$
KS3	You should know the rules for working when more than one operation is involved in a calculation (BODMAS).	<b>2</b> Spot the mistake in each calculation and correct the answers. <b>a</b> $3 + 8 + 3 \times 4 = 56$ <b>b</b> $3 + 8 \times 3 + 4 = 37$ <b>c</b> $3 \times (8 + 3) \times 4 = 130$
KS3	You should understand that addition and subtraction, and multiplication and division are inverse operations.	<b>3</b> Identify the inverse operation by choosing the correct option. <b>a</b> $14 \times 4 = 56$ A $56 \times 4 = 14$ B $14 \div 4 = 56$ C $56 \div 4 = 14$ <b>b</b> $200 \div 10 = 20$ A $200 \div 20 = 10$ B $200 = 10 \times 20$ C $10 \times 200 = 2000$ <b>c</b> $27 + 53 = 80$ A $80 = 4 \times 20$ B $80 - 27 = 53$ C $80 + 27 = 107$




Find answers at: [cambridge.org/ukschools/gcemaths-studentbookanswers](http://cambridge.org/ukschools/gcemaths-studentbookanswers)

## Assess your starting point using the Launchpad

**STEP 1**

**1** Calculate without using a calculator and show your working.

**a**  $647 + 786$       **b**  $1406 - 289$   
**c**  $45 \times 19$       **d**  $414 \div 23$



**STEP 2**

**2** Choose the correct answer.


**a**  $9 \div (2 + 1) - 2$   
 A 9                  B  $3\frac{1}{2}$                   C 1                  D 0

**b**  $(3 \times 8) \div 4 + 8$   
 A 2                  B 30                  C 16                  D 14

**c**  $12 - 6 \times 2 + 11$   
 A 78                  B 23                  C 1                  D 11

**d**  $[5 \times (9 + 1)] - 3$   
 A 53                  B 47                  C 40                  D 43

**e**  $(6 + 5) \times 2 + (15 - 2 \times 3) - 6$   
 A 40                  B 20                  C 32                  D 25




**STEP 3**


**3** The perimeter of a square is equal to four times the length of a side.  
 If the perimeter is 128 cm, what is the length of a side?

**4** What should you add to 342 to get 550?


**5** A number divided by 45 is 30.  
 What is the number?




**GO TO**  
 Chapter review



**GO TO**  
**Section 1:**  
 Basic calculations



**GO TO**  
**Section 2:**  
 Order of operations



**GO TO**  
**Section 3:**  
 Inverse operations

## Section 1: Basic calculations

You will not always have a calculator so it is useful to know how to do calculations using mental and written strategies.


It is best to use a method that you are confident with and always **show your working**.

When a question asks you to find the:

- **sum**, you need to add
- **difference**, you need to subtract the smaller number from the larger number
- **product**, you need to multiply
- **quotient**, you need to divide.



## Tip

Some examination papers will not allow you to use your calculator. In this book the past paper questions that you are not allowed to use your calculator for are marked with a  symbol.

## WORK IT OUT 1.1

Look at these calculations carefully.

Discuss with a partner what methods these students have used to find the answer.

Which method would you use to do each of these calculations? Why?

①  $489 + 274$

$$\begin{array}{r} 400 + 200 \rightarrow 600 \\ 80 + 70 \rightarrow 150 \\ 9 + 4 \rightarrow 13 \\ \hline 763 \end{array}$$

②  $284 - 176$

$$\begin{array}{r} 284 \\ - 176 \\ \hline 108 \end{array}$$

③  $29 \times 17$

$$\begin{aligned} &= 30 \times 17 - 17 \\ &= 3 \times 170 - 17 \\ &= 510 - 17 \\ &= 493 \end{aligned}$$

④  $15 \times 62$

$$\begin{aligned} &= 30 \times 31 \quad 310 \\ &= 930 \quad 310 \\ &= 3 \times 310 \quad 310 \\ &= 930 \end{aligned}$$

⑤  $207 \times 47$

x	200	0	7
40	8000	0	280
7	1400	0	49
9400 + 0 + 329			
= 9729			

⑥  $2394 \div 42$

2394	$42 \times 10 = 420$
- 1680	$42 \times 20 = 840$
714	$42 \times 40 = 1680$
- 420	$42 \times 5 = 210$
294	$42 \times 2 = 84$
- 210	
84	
- 84	
0 57	



### Problem-solving strategies

The Problem-solving framework below outlines the steps that you can take to break down most problems to help you solve them.

Follow these steps each time you are faced with a problem to help you become more skilled at problem-solving and more able to self-check.

These are important skills both for your GCSE courses and for everyday life.

### Problem-solving framework

Sally had a budget of £60 to buy items.

Sally bought:

- a table for £32 and
- a bench for £18.

She spent £12 to repair them.

She then sold the two items for £69.

How much profit did she make?

Steps for solving problems	What you would do for this example
<b>Step 1:</b> Work out what you have to do. Start by reading the question carefully.	Find the profit.
<b>Step 2:</b> What information do you need?  Have you got it all?	Cost of items = £32 + £18 Cost of repairs = £12 Selling price = £69  Yes
<b>Step 3:</b> Is there any information that you don't need?	You don't need to know her budget. You just need to know how much she spent. Many problems contain extra information that you don't need to test your understanding.
<b>Step 4:</b> Decide what maths you can do.	Profit = selling price – cost
<b>Step 5:</b> Set out your solution clearly. Check your working and make sure your answer is reasonable.	Cost = £32 + £18 + £12 = £62 Profit = £69 – £62 = £7 Sally made £7 profit.
<b>Step 6:</b> Check that you have answered the question.	Yes. You needed to find the profit and you have found it.

## EXERCISE 1A

Solve these problems using written methods.

You **must** show your working.

- 1 A pack of pens cost £3.90 for three.  
Nola bought fifteen pens.
    - a i How much did she pay in total?
    - ii What is the cost per pen?
  - b How many packs of pens did Nola buy? Why do you need to know this?
  - c What operation would you do to find the total cost? Why?
  - d How would you work out the cost per pen?
  - e Does a price of £1.50 per pen seem reasonable?
- 2 A pair of jeans costs £34.  
A scarf costs £9.50.  
A top costs £20.  
Sandra saved £100 to buy these items.  
How much money did she have left?
- 3 How many 16-page brochures can you make from 1030 pages?
- 4 Jason can type 48 words per minute.
    - a How many words can he type in an hour and a half?
    - b Approximately how long would it take him to type 2000 words?
- 5 At the start of a year the population of Greenside Village was 56 309.  
During the year:  
617 people died,  
1835 babies were born,  
4087 people left the village  
and 3099 people moved into the village.  
What was the population at the end of the year?
- 6 The Amazon River is 6448 km long.  
The Nile River is 6670 km.  
The Severn River is 354 km long.
    - a How much longer is the Nile River than the Amazon River?
    - b How much shorter is the Severn River than the Amazon River?
- 7 What is the result when you add the sum of 132 and 99 to the product of 36 and 127?
- 8 Find the result when the difference between 8765 and 3087 is added to the result of 1206 divided by 18.



## Tip

You don't always need to write something for the first few steps in the Problem-solving framework, but you should still consider these steps mentally when approaching a problem in order to help you decide what to do. You should **always** show how you worked out the problem.



## Did you know?

The Severn is the longest river in the UK.



Find answers at: [cambridge.org/ukschools/gcsemaths-studentbookanswers](http://cambridge.org/ukschools/gcsemaths-studentbookanswers)



### Key vocabulary

**integers:** whole numbers in the set  $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$ ; when they have a negative or positive sign they can be referred to as **directed numbers**.



### Tip

You will be expected to work with negative and positive values in algebra, so it is important to make sure you can do this early on in your GCSE course.

## Working with positive and negative integers

When doing calculations involving positive and negative **integers**, you need to remember the following:






- Adding a negative number is the same as subtracting the number:  
 $4 + -3 = 1$
- Subtracting a negative number is the same as adding a positive number:  
 $5 - -3 = 8$
- Multiplying or dividing the same signs gives a positive answer:  
 $-4 \times -2 = 8$  and  $\frac{-4}{-2} = 2$
- Multiplying or dividing different signs gives a negative answer:  
 $4 \times -2 = -8$  and  $\frac{4}{-2} = -2$

### EXERCISE 1B

- What would you add to each number to get a result of 5?  
**a** 7      **b** 3      **c** -1      **d** -4      **e** -24
- What would you subtract from each number to get a result of -8?  
**a** 7      **b** 3      **c** -1      **d** -4      **e** -24
- 4 is multiplied by another number to get each result.  
 Work out what the other number is in each case.  
**a** 12      **b** -100      **c** -36      **d** 504      **e** 0
- By what would you divide -64 to get the following results?  
**a** 8      **b** -8      **c** 2      **d**  $-\frac{1}{2}$       **e** -256
- Here is a set of integers:  $\{-8, -6, -3, 1, 3, 7\}$   
 From the numbers in this set:
  - Write down two numbers with a difference of 9.
  - Write down three numbers with a sum of 1.
  - Write down two numbers whose product is -3.
  - Write down two numbers that, when divided, will give an answer of -6.
- One more than -6 is added to the product of 7 and 6 less than 3.  
 What is the result?
- Saleem has a container of wooden dowels.  
 Some are 5 cm long and some are 7 cm long.  
 If the dowels are joined end to end, investigate what lengths between 5 cm and 150 cm **cannot** be made.

## Section 2: Order of operations

Jose posted this calculation on his wall on social media.

	<b>JOSE:</b> $24 + 6 \div 2 - 1 \times 4 = ?$
COMMENT    ✓ LIKE    ↻ SHARE	
	<b>JOANNA:</b> 56
	<b>PETER:</b> 11
	<b>LUCIA:</b> 23
	<b>DIPAK:</b> 104

Which one of Jose's friends (if any) do you think is correct? Why?

There is a set of rules that tell you the order in which you need to work when there is more than one operation.

The order of operations is:

- 1 Do any operations in brackets first.
- 2 If there are any '**powers of**' or '**fractions of**' in the calculation, do them next.
- 3 Do division and multiplication next, working from left to right.
- 4 Do addition and subtraction last, working from left to right.

### Brackets

Brackets are used to group operations. For example:

$$(3 + 7) \times (30 \div 2)$$

When there is more than one set of brackets, work from the **innermost set** to the **outermost set**.

Different styles of bracket can be used to make it easier to identify each pair.

### WORKED EXAMPLE 1

Work out  $2((4 + 2) \times 2 - 3(1 - 3) - 10)$ .

$$2((4 + 2) \times 2 - 3(1 - 3) - 10)$$

Highlight the different pairs of brackets to help if you need to.

$$\begin{aligned} &2((4 + 2) \times 2 - 3(1 - 3) - 10) \\ &= 2((6) \times 2 - 3(-2) - 10) \\ &= 2(6 \times 2 - 3 \times -2 - 10) \end{aligned}$$

The red brackets are the innermost, so do the calculations inside these brackets first. There are two lots, so work from left to right. **Note** that you can leave  $-2$  inside brackets if you prefer because  $3(-2)$  is the same as  $3 \times -2$ .

$$\begin{aligned} &2(6 \times 2 - 3 \times -2 - 10) \\ &= 2(12 - -6 - 10) \\ &= 2(8) \\ &= 2 \times 8 \\ &= 16 \end{aligned}$$

Blue brackets are next. Do the multiplications first from left to right, then the subtractions from left to right.



### Tip

Many people remember these rules using the letters **BODMAS** (or sometimes **BIDMAS**).

**B**rackets

**O**f ('powers of' or 'fractions of'; in **BIDMAS** I stands for indices)

**D**ivide and/or **M**ultiply

**A**dd and/or **S**ubtract



**Tip**

$\frac{5-12}{3-8}$  is the same calculation as  $(5-12) \div (3-8)$ .



**Calculator tip**

Most modern calculators are programmed to use the correct order of operations. Check your calculator by entering  $2 + 3 \times 4$ . You should get 14.

If a calculation is written with brackets, you need to enter the brackets into the calculator to make sure it does these first.

For example, the following different types of brackets have been used below:

$( ), [ ], \{ \}$ .

$$\{2 - [4(2 - 7) - 4(3 + 8)] - 2\} \times 8$$

Other symbols can also be used to group operations.

For example:

Fraction bars:  $\frac{5-12}{3-8}$

Roots:  $\sqrt{16+9}$

These symbols are treated like brackets when you do a calculation.

**WORK IT OUT 1.2**

Which of the solutions is correct in each case?

Find the mistakes in the incorrect option.

	Option A	Option B
1	$7 \times 3 + 4$ $= 21 + 4$ $= 25$	$7 \times 3 + 4$ $= 7 \times 7$ $= 49$
2	$(10 - 4) \times (4 + 9)^2$ $= 6 \times 16 + 81$ $= 96 + 81$ $= 177$	$(10 - 4) \times (4 + 9)^2$ $= 6 \times (13)^2$ $= 6 \times 169$ $= 1014$
3	$45 - [20 \times (4 - 3)]$ $= 45 - [20 \times 1]$ $= 45 - 21$ $= 24$	$45 - [20 \times (4 - 3)]$ $= 45 - 20 \times 1$ $= 45 - 20$ $= 25$
4	$30 - 4 \div 2 + 2$ $= 26 \div 2 + 2$ $= 13 + 2$ $= 15$	$30 - 4 \div 2 + 2$ $= 30 - 2 + 2$ $= 30$
5	$\frac{18-4}{4-2}$ $= \frac{18}{2}$ $= 9$	$\frac{18-4}{4-2}$ $= \frac{14}{2}$ $= 7$
6	$\sqrt{36} \div 4 + 40 \div 4 + 1$ $= \sqrt{9} + 10 + 1$ $= 3 + 11$ $= 14$	$\sqrt{36} \div 4 + 40 \div 4 + 1$ $= \sqrt{9} + 40 \div 5$ $= 3 + 8$ $= 11$



## EXERCISE 1C

- 1 Check whether these answers are correct.

If not, work out the correct answer.

a  $12 \times 4 + 76 = 124$

b  $8 + 75 \times 8 = 698$

c  $12 \times 18 - 4 \times 23 = 124$

d  $(16 \div 4) \times (7 + 3 \times 4) = 76$

e  $(82 - 36) \times (2 + 6) = 16$

f  $(3 \times 7 - 4) - (4 + 6 \div 2) = 12$

- 2 Use the numbers listed to make each number sentence true.

a  $\square - \square \div \square = \square$       9, 11, 13, 18

b  $\square \div (\square - \square) - \square = \square$       1, 3, 8, 14, 16

c  $(\square + \square) - (\square - \square) = \square$       4, 5, 6, 9, 12

- 3 Insert brackets into each calculation, if necessary, to make it true.

a  $3 \times 4 + 6 = 30$

b  $25 - 15 \times 9 = 90$

c  $40 - 10 \times 3 = 90$

d  $14 - 9 \times 2 = 10$

e  $12 + 3 \div 5 = 3$

f  $19 - 9 \times 15 = 150$

g  $10 + 10 \div 6 - 2 = 5$

h  $3 + 8 \times 15 - 9 = 66$

i  $9 - 4 \times 7 + 2 = 45$

j  $10 - 4 \times 5 = 30$

k  $6 \div 3 + 3 \times 5 = 5$

l  $15 - 6 \div 2 = 12$

m  $1 + 4 \times 20 \div 5 = 20$

n  $8 + 5 - 3 \times 2 = 20$

o  $36 \div 3 \times 3 - 3 = 6$

p  $3 \times 4 - 2 \div 6 = 1$

q  $40 \div 4 + 1 = 11$

r  $6 + 2 \times 8 + 2 = 24$

- 4 Each  $\bigcirc$  represents an operation.

Fill in the missing operations to make these calculations true.

a  $12 \bigcirc (28 \bigcirc 24) = 3$

b  $88 \bigcirc 10 \bigcirc 8 = 8$

c  $40 \bigcirc 5 \bigcirc (7 \bigcirc 5) = 4$

d  $9 \bigcirc 15 \bigcirc (3 \bigcirc 2) = 12$

- 5 Calculate:

a  $\frac{7 \times \sqrt{16}}{2^3 + 7^{(2-1)}}$

b  $\frac{5^2 \times \sqrt{4}}{1 + 6^2 - 12}$

c  $\frac{2 + 3^2}{5^2 + 4 \times 10 - \sqrt{25}}$

d  $\frac{6^2 - 11}{2(17 + 2 \times 4)}$

e  $\frac{3^2 - 3}{2 \times \sqrt{81}}$

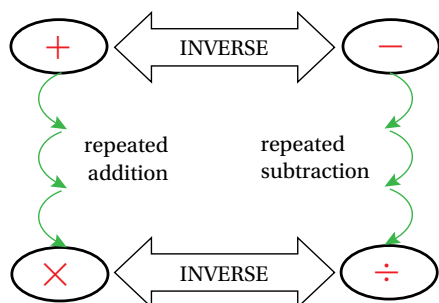
f  $\frac{3^2 - 5 + 6}{\sqrt{4} \times 5}$

g  $\frac{36 - 3 \times \sqrt{16}}{15 - 3^2 \div 3}$

h  $\frac{-30 + [18 \div (3 - 12) + 24]}{5 - 8 - 3^2}$

- 6 Work with a partner.

- Find a quick method for adding a set of consecutive whole numbers.
- Give a reason why your method works.
- Test your method on a set of consecutive negative integers.
- Does it work? Give a reason why or why not.

**Tip**

You will use inverse operations to solve equations and to deal with functions, so it is important that you understand how they work.

## Section 3: Inverse operations

The four operations, add, subtract, multiply and divide, are related to each other. Operations are inverses of each other if one undoes (cancels out) the effect of the other.

- Adding is the inverse of subtracting, for example, add 5 is undone by subtract 5.
- Multiplying is the inverse of dividing, for example, multiply by 2 is undone by divide by 2.
- Taking a square root is the inverse of squaring a number, for example,  $4^2$  is undone by  $\sqrt{16}$ .
- Taking the cube root is the inverse of cubing a number, for example,  $2^3$  is undone by  $\sqrt[3]{8}$ .

Inverse operations are useful for checking the results of your calculations.

For example, is  $4320 - 500 = 3820$  correct?

Check by adding 500 back to the result (that is, by doing the inverse operation) to see whether it gives you 4320.

$$3820 + 500 = 4320, \text{ so it is correct.}$$

When there is more than one operation involved, you have to reverse the order of the inverse operations to return to the starting number.

For example, is  $(50 + 62) \div 8 = 14$ ?

Check by working backwards and applying inverse operations:

$$14 \times 8 - 62 = 50, \text{ so it is correct.}$$

### EXERCISE 1D

- Use inverse operations to find the missing values in each of these calculations.
 

a $\square + 217 = 529$	b $\square + 388 = 490$	c $\square - 218 = 182$
d $121 \times \square = -605$	e $-6 \times \square = 870$	f $\square \div 40 = 5400$
- Use inverse operations to check these calculations.
 

a $45 \times 5 - 8 = 217$	b $14 + 5 \times 9 - 9 = 50$
c $(23 + 48) \times 4 = 284$	d $(412 - 128) \div 4 = 71$
- The formula for finding the area of a triangle is  $A = \frac{bh}{2}$ , where  $b$  is the base length and  $h$  is the height.
  - Find the height of a triangle with an area of  $54 \text{ cm}^2$  and a base of length 9 cm.
  - A triangle has an area of  $64 \text{ cm}^2$ .  
Find the height and the length of the base if the base is twice the height.
- Here is an expression:  $1 - \left(\frac{2}{3}(4 + 5) + 6\right) \times 7$ 
  - Calculate the value of the expression.
  - Keep the numbers in order (from 1 to 7) but change the operations as necessary to find:
    - the highest possible answer
    - the lowest possible answer.
  - Comment on how changing the operations affected your results.



### Checklist of learning and understanding

#### Basic calculations

- Written methods are important when you do not have a calculator.
- You can use any method as long as you show your working.
- Negative and positive numbers can be added, subtracted, multiplied and divided as long as you apply the rules to get the correct sign in the answer.

#### Order of operations

- In maths there is a conventional order for working when there is more than one operation:
  - Always work out brackets (or other grouping symbols) first,
  - then powers and fractions,
  - multiply and/or divide next,
  - then add and/or subtract.

#### Inverse operations

- An inverse operation undoes the previous operation:
- Addition and subtraction are the inverse of each other.
- Multiplication and division are the inverse of each other.
- Squaring and taking the square root are the inverse of each other.



### Chapter review



For additional questions on the topics in this chapter, visit [GCSE Mathematics Online](#).

- Choose the correct answer.
  - What is the first operation you would do in this calculation:  
 $4 \times [20 \div (5 - 3)] - 8 + 2$ ?  
 A +    B -    C  $\times$     D  $\div$
  - To make the statement  $5 - 3 \times 8 - 6 \div 2 = 2$  correct, you would need to insert brackets as follows:  
 A  $5 - (3 \times 8) - 6 \div 2 = 2$             B  $5 [- 3 \times (8 - 6)] \div 2 = 2$   
 C  $(5 - 3) \times 8 - 6 \div 2 = 2$             D  $(5 - 3) \times (8 - 6) \div 2 = 2$

- These are the solutions to a cross-number puzzle.  
 The clues are all calculations that involve using the correct order of operations.  
 Write a set of clues that would give these results. Use at least **two** operations for each clue.

- Use integers and operations to write ten different questions that give an answer of  $-17$ .

- On a page of a magazine there are three columns of text.  
 Each column contains 42 rows.  
 In each column row there is an average of 32 letters.  
 Approximately how many letters are there on a page?

	1	2	7					3	3					
4	1	4	8	5	6			6	1	9	7	4		
8	3	0		7								5		
	9	4	10	9			11	3	2	7		12	13	4
			2					1						8
			0				14	4	2	5		16	2	6
	17	4	1	18	7			1						0
19	2	3		20	7	21	2	0	4		22	9	0	
23	7	9	4			1				24	7	9		



Find answers at: [cambridge.org/ukschools/gcsemaths-studentbookanswers](http://cambridge.org/ukschools/gcsemaths-studentbookanswers)

- 5 A stadium has seats for 32 000 people.

There are 125 seats in a row.

How many rows are there in the stadium?



- 6 This grid follows two rules.



Rule 1: The sums of each row are equal.

Rule 2: The products of each column are equal.

			Sum of rows	
	5	32	80	117
	96	15	6	117
Product of columns	480	480	480	

The grid below follows the same two rules.

Work out the missing numbers.

			Sum of rows	
		5	6	
Product of columns	60			

(3 marks)

© AQA 2013

- 7 Two numbers have a sum of  $-15$  and a product of  $-100$ .

What are the numbers?

- 8 The sum of two numbers is 1, and their product is  $-20$ .

What are the numbers?

- 9 Jenna's bank account was overdrawn.

Then she deposited £1000.

Her new balance is £432.

By how much was her account overdrawn to start with?

- 10 You can use the formula  $F = 2C + 32$  to approximately convert temperatures from Celsius to Fahrenheit.

Find the approximate temperature in degrees Celsius when it is:

**a**  $68^{\circ}\text{F}$

**b**  $100^{\circ}\text{F}$