

1 Calculations

In this chapter you will learn how to ...

- use non-calculator methods to calculate with positive and negative numbers.
- perform operations in the correct order based on mathematical conventions.
- recognise inverse operations and use them to simplify and check calculations.



For more resources relating to this chapter, visit GCSE Mathematics Online.

Using mathematics: real-life applications

Everyone uses numbers on a daily basis often without really thinking about them. Shopping, cooking, working out bills, paying for transport and measuring all rely on a good understanding of numbers and calculation skills.



"Number puzzles and games are very popular and there are mobile apps and games available for all age groups. I use an app with my GCSE classes where they have to work in the correct order to solve different number puzzles."

(Secondary School Teacher)



Tip

You probably already know most of the concepts in this chapter. They have been included so that you can revise concepts if you need to and check that you know them well.

Before you start ...

KS3	You should be able to add, subtract, multiply and divide positive and negative numbers.	<p>1 Copy and complete each statement to make it true. Use only $<$, $=$ or $>$.</p> <p>a $2 + 3 \square 4 - 7$ b $-3 + 6 \square 4 - 7$</p> <p>c $-1 - 4 \square 20 \div -4$ d $-6 \times 2 \square -7 - (-5)$</p>
KS3	You should know the rules for working when more than one operation is involved in a calculation (BODMAS).	<p>2 Spot the mistake in each calculation and correct the answers.</p> <p>a $3 + 8 + 3 \times 4 = 56$ b $3 + 8 \times 3 + 4 = 37$</p> <p>c $3 \times (8 + 3) \times 4 = 130$</p>
KS3	You should understand that addition and subtraction, and multiplication and division are inverse operations.	<p>3 Identify the inverse operation by choosing the correct option.</p> <p>a $14 \times 4 = 56$</p> <p>A $56 \times 4 = 14$ B $14 \div 4 = 56$ C $56 \div 4 = 14$</p> <p>b $200 \div 10 = 20$</p> <p>A $200 \div 20 = 10$ B $200 = 10 \times 20$ C $10 \times 200 = 2000$</p> <p>c $27 + 53 = 80$</p> <p>A $80 = 4 \times 20$ B $80 - 27 = 53$ C $80 + 27 = 107$</p>



Find answers at: cambridge.org/ukschools/gcsemaths-studentbookanswers

Assess your starting point using the Launchpad

STEP 1

1 Calculate, without using a calculator, and show your working.

a $647 + 786$
b $1406 - 289$
c 45×19
d $414 \div 23$

GO TO
Section 1:
 Basic calculations

STEP 2

2 Choose the correct answer.

a $9 \div (2 + 1) - 2$
A 9 **B** $3\frac{1}{2}$ **C** 1 **D** 0

b $(3 \times 8) \div 4 + 8$
A 2 **B** 30 **C** 16 **D** 14

c $12 - 6 \times 2 + 11$
A 78 **B** 23 **C** 1 **D** 11

d $[5 \times (9 + 1)] - 3$
A 53 **B** 47 **C** 40 **D** 43

e $(6 + 5) \times 2 + (15 - 2 \times 3) - 6$
A 40 **B** 20 **C** 32 **D** 25

GO TO
Section 2:
 Order of operations

STEP 3

3 The perimeter of a square is equal to four times the length of a side. If the perimeter is 128 cm, what is the length of a side?

4 What should you add to 342 to get 550?

5 If a number divided by 45 is 30, what is the number?

GO TO
Section 3:
 Inverse operations

GO TO
 Chapter review

Section 1: Basic calculations

You will not always have a calculator so it is useful to know how to do calculations using mental and written strategies.

It is best to use a method that you are confident with and always **show your working**.

Remember that when a question asks you to find the:

- **sum** you need to add
- **difference** you need to subtract the smaller number from the larger number
- **product** you need to multiply
- **quotient** you need to divide.



Tip

Some examination papers will not allow you to use your calculator.

WORK IT OUT 1.1

Look at these calculations carefully.

Discuss with a partner what methods these students have used to find the answer.

Which method would you use to do each of these calculations? Why?

① $489 + 274$

$$\begin{array}{l} 400 + 200 \rightarrow 600 \\ 80 + 70 \rightarrow 150 \\ 9 + 4 \rightarrow 13 \\ \hline 763 \end{array}$$

② $284 - 176$

$$\begin{array}{r} 284 \\ - 176 \\ \hline 108 \end{array}$$

③ 29×17

$$\begin{array}{l} \rightarrow 30 \times 17 - 17 \\ \rightarrow 3 \times 170 - 17 \\ \rightarrow 510 - 17 \\ \rightarrow 493 \end{array}$$

④ 15×62

$$\begin{array}{l} = 30 \times 31 \quad 310 \\ = 930 \quad 310 \\ \hline 310 \\ 930 \end{array}$$

⑤ 207×47

x	200	0	7
40	8000	0	280
7	1400	0	49
	9400 + 0 + 329		
	= 9729		

⑥ $2394 \div 42$

2394	$42 \times 10 = 420$
$- 1680$	$42 \times 20 = 840$
714	$42 \times 40 = 1680$
$- 420$	$42 \times 5 = 210$
294	$42 \times 2 = 84$
$- 210$	
84	
$- 84$	
= 57	



Problem-solving strategies

There are some useful strategies and techniques that you can use to break down complex problems to help you solve them more easily.

If you follow these steps each time you are faced with a problem, you will become more confident at problem solving and more able to check that your answers are sensible.

These are important skills both for your GCSE courses and for everyday life.

Problem-solving framework

Sally buys, repairs and sells used furniture at a market.
 Last week she bought a table for £32 and a bench for £18.
 She spent £12 on wood, nails, varnish and glue to fix them up.
 She then sold the two items on her stall for £69.
 How much profit did she make on the two items?

Steps for solving problems	What you would do for this example
Step 1: Work out what you have to do. Start by reading the question carefully.	Find the profit on the two items.
Step 2: What information do you need? Have you got it all?	Cost of items = £32 + £18 Cost of repairs = £12 Selling price = £69
Step 3: Is there any information that you don't need?	In this problem you don't need to know what she spent the money on, you just need to know how much she spent. Many problems contain extra information that you don't need so as to test your understanding.
Step 4: Decide what maths you can do.	Profit = selling price – cost You can add the costs and subtract them from the selling price.
Step 5: Set out your solution clearly. Check your working and make sure your answer is reasonable.	Cost = £32 + £18 + £12 = £62 Profit = £69 – £62 = £7 Sally made £7 profit.
Step 6: Check that you have answered the question.	Yes. You needed to find the profit and you have found it.

EXERCISE 1A

Solve these problems using written methods.

Set out your solutions clearly to show the methods you chose.

- 1 Nola checked the prices of pens at three different supermarkets. She found that the cheapest pack of pens was £3.90 for three. She bought 15 pens.
 How much did she pay in total and how much did she pay for each pen?
 - a What two things are you asked to find here?
 - b How many packs of pens did she buy? Why do you need to know this?
 - c What operation would you do to find the total cost? Why?
 - d How would you work out the cost of each pen?
 - e Does a price of £1.30 for a pen seem reasonable to you?
- 2 Sandra bought a pair of jeans for £34, a scarf for £9.50 and a top for £20.
 If she had saved £100 to buy these items, how much money would she have left?
- 3 How many 16-page brochures can you make from 1030 pages?
- 4 Jason can type 48 words per minute.
 - a How many words can he type in an hour and a half?
 - b Approximately how long would it take him to type an article of 2000 words?
- 5 At the start of a year the population of Greenside Village was 56 309. During the year 617 people died, 1835 babies were born, 4087 people left the village and 3099 people moved into the village.
 What was the population at the end of the year?
- 6 The Amazon River is 6448 km long, the Nile River is 6670 km and the Severn is 354 km long.
 - a How much longer is the Nile than the Amazon?
 - b How much shorter is the Severn than the Amazon?
- 7 What is the combined sum of 132 and 99 plus the product of 36 and 127?
- 8 What is the result when the difference between 8765 and 3087 is added to the result of 1206 divided by 18?



Tip

You don't always need to write something for the first few steps in the problem-solving framework, but you should still consider these steps mentally when approaching a problem in order to help you decide what to do. You should **always** show your working fully.



Did you know?

The Severn is the longest river in the UK.



Find answers at: cambridge.org/ukschools/gcsemaths-studentbookanswers



Key vocabulary

integers: whole numbers belonging to the set $\{\dots -3, -2, -1, 0, 1, 2, 3, \dots\}$; they are sometimes called directed numbers because they have a negative or positive sign.



Tip

You will be expected to work with negative and positive values in algebra, so it is important to make sure you can do this early on in your GCSE course.

Working with negative and positive integers

When doing calculations involving positive and negative **integers**, you need to remember the following:

- Adding a negative number is the same as subtracting the number:
 $4 + -3 = 1$
- Subtracting a negative number is the same as adding a positive number:
 $5 - -3 = 8$

- Multiplying or dividing the same signs gives a positive answer:

$$-4 \times -2 = 8 \text{ and } \frac{-4}{-2} = 2$$

- Multiplying or dividing different signs gives a negative answer:

$$4 \times -2 = -8 \text{ and } \frac{-4}{2} = -2$$

EXERCISE 1B

- What would you add to each number to get a result of 5?
a 7 **b** 3 **c** -1 **d** -4 **e** -24
- What would you subtract from each number to get a result of -8?
a 7 **b** 3 **c** -1 **d** -4 **e** -24
- 4 is multiplied by another number to get each of the following results.
 Work out what the other number is in each case.
a 12 **b** -100 **c** -36 **d** 504 **e** 0
- By what would you divide -64 to get the following results?
a 8 **b** -8 **c** 2 **d** $-\frac{1}{2}$ **e** -256
- Here is a set of integers:
 $\{-8, -6, -3, 1, 3, 7\}$
 From the numbers in this set:
 - Find two numbers with a difference of 9.
 - Find three numbers with a sum of 1.
 - Find two numbers whose product is -3.
 - Find two numbers which, when divided, will give an answer of -6.
- One more than -6 is added to the product of 7 and 6 less than 3.
 What is the result?









7 Saleem has a container of wooden dowels.

Some are 5 cm long and some are 7 cm long.

If the dowels are joined end to end, investigate what lengths between 5 cm and 150 cm **cannot** be made.

Section 2: Order of operations

Jose posted this calculation on his wall on social media.

	JOSE: $24 + 6 \div 2 - 1 \times 4 = ?$
	COMMENT
	LIKE
	SHARE
	JOANNA: 56
	PETER: 11
	LUCIA: 23
	DIPAK: 104

Within minutes, his friends had posted four different answers.

Which one (if any) do you think is correct? Why?

There is a set of rules that tell you the order in which you need to work when there is more than one operation.

The order of operations is:

- 1 Do any operations in brackets first.
- 2 If there are any '**powers of**' or '**fractions of**' in the calculation, do them next.
- 3 Do division and multiplication next, working from left to right.
- 4 Do addition and subtraction last, working from left to right.

Brackets and other grouping symbols

Brackets are used to group operations. For example:

$$(3 + 7) \times (30 \div 2)$$

When there is more than one set of brackets, you work from the **innermost set** to the **outermost set**.



Tip

Many people remember these rules using the letters **BODMAS**. (or sometimes BIDMAS).

Brackets

Of ('powers of' or 'fractions of'; in BIDMAS, I stands for Indices)

Divide and/or **M**ultiply

Add and/or **S**ubtract.

WORKED EXAMPLE 1

Calculate $2((4 + 2) \times 2 - 3(1 - 3) - 10)$

$$2((4 + 2) \times 2 - 3(1 - 3) - 10)$$

Highlight the different pairs of brackets to help if you need to.

$$\begin{aligned} &2((4 + 2) \times 2 - 3(1 - 3) - 10) \\ &= 2(6 \times 2 - 3(-2) - 10) \\ &= 2(6 \times 2 - 3 \times -2 - 10) \end{aligned}$$

The red brackets are the innermost, so do the calculations inside these ones first. There are two lots of red brackets, so work from left to right. **Note** that you can leave -2 inside brackets if you prefer because $3(-2)$ is the same as 3×-2 .

$$\begin{aligned} &2(6 \times 2 - 3 \times -2 - 10) \\ &= 2(12 - -6 - 10) \\ &= 2(8) \\ &= 2 \times 8 \\ &= 16 \end{aligned}$$

Blue brackets are next. Do the multiplications first from left to right, then the subtractions from left to right.

Often a different style of bracket will be used to make it easier to identify each pair.

For example, the following different types of brackets have been used below: $()$, $[]$, $\{\}$.

$$\{2 - [4(2 - 7) - 4(3 + 8)] - 2\} \times 8$$

Other symbols can also be used to group operations.

For example:

fraction bars: $\frac{5 - 12}{3 - 8}$

roots: $\sqrt{16 + 9}$

These symbols are treated like brackets when you do a calculation.



Tip

$\frac{5 - 12}{3 - 8}$ is the same calculation as $(5 - 12) \div (3 - 8)$

WORK IT OUT 1.2

Which of the solutions is correct in each case?

Find the mistakes in the incorrect option.

	Option A	Option B
1	$7 \times 3 + 4$ $= 21 + 4$ $= 25$	$7 \times 3 + 4$ $= 7 \times 7$ $= 49$
2	$(10 - 4) \times (4 + 9)^2$ $= 6 \times 16 + 81$ $= 96 + 81$ $= 177$	$(10 - 4) \times (4 + 9)^2$ $= 6 \times (13)^2$ $= 6 \times 169$ $= 1014$
3	$45 - [20 \times (4 - 3)]$ $= 45 - [20 \times 1]$ $= 45 - 21$ $= 24$	$45 - [20 \times (4 - 3)]$ $= 45 - 20 \times 1$ $= 45 - 20$ $= 25$
4	$30 - 4 \div 2 + 2$ $= 26 \div 2 + 2$ $= 13 + 2$ $= 15$	$30 - 4 \div 2 + 2$ $= 30 - 2 + 2$ $= 30$
5	$\frac{18 - 4}{4 - 2}$ $= \frac{18}{2}$ $= 9$	$\frac{18 - 4}{4 - 2}$ $= \frac{14}{2}$ $= 7$
6	$\sqrt{36 \div 4} + 40 \div 4 + 1$ $= \sqrt{9} + 10 + 1$ $= 3 + 11$ $= 14$	$\sqrt{36 \div 4} + 40 \div 4 + 1$ $= \sqrt{9} + 40 \div 5$ $= 3 + 8$ $= 11$



Calculator tip

Most modern calculators are programmed to use the correct order of operations. Check your calculator by entering $2 + 3 \times 4$. You should get 14.

If the calculation has brackets, you need to enter the brackets into the calculator to make sure it does these first.

EXERCISE 1C

- 1 Check whether these answers are correct.

If the answer is wrong, work out the correct answer.

- a** $12 \times 4 + 76 = 124$ **b** $8 + 75 \times 8 = 698$
c $12 \times 18 - 4 \times 23 = 124$ **d** $(16 \div 4) \times (7 + 3 \times 4) = 76$
e $(82 - 36) \times (2 + 6) = 16$ **f** $(3 \times 7 - 4) - (4 + 6 \div 2) = 12$



Find answers at: cambridge.org/ukschools/gcsemaths-studentbookanswers

2 Use the numbers listed to make each number sentence true.

a $\square - \square \div \square = \square$ 9, 11, 13, 18

b $\square \div (\square - \square) - \square = \square$ 1, 3, 8, 14, 16

c $(\square + \square) - (\square - \square) = \square$ 4, 5, 6, 9, 12

3 Insert brackets into each calculation to make it true.

a $3 \times 4 + 6 = 30$ **b** $25 - 15 \times 9 = 90$ **c** $40 - 10 \times 3 = 90$

d $14 - 9 \times 2 = 10$ **e** $12 + 3 \div 5 = 3$ **f** $19 - 9 \times 15 = 150$

g $10 + 10 \div 6 - 2 = 5$ **h** $3 + 8 \times 15 - 9 = 66$ **i** $9 - 4 \times 7 + 2 = 45$

j $10 - 4 \times 5 = 30$ **k** $6 \div 3 + 3 \times 5 = 5$ **l** $15 - 6 \div 2 = 12$

m $1 + 4 \times 20 \div 5 = 20$ **n** $8 + 5 - 3 \times 2 = 20$ **o** $36 \div 3 \times 3 - 3 = 6$

p $3 \times 4 - 2 \div 6 = 1$ **q** $40 \div 4 + 1 = 11$ **r** $6 + 2 \times 8 + 2 = 24$

4 Each \bigcirc represents an operation.

Fill in the missing operations to make these statements true.

a $12 \bigcirc (28 \bigcirc 24) = 3$ **b** $88 \bigcirc 10 \bigcirc 8 = 8$

c $40 \bigcirc 5 \bigcirc (7 \bigcirc 5) = 4$ **d** $9 \bigcirc 15 \bigcirc (3 \bigcirc 2) = 12$

5 Calculate:

a $\frac{7 \times \sqrt{16}}{2^3 + 7^2 - 1}$

b $\frac{5^2 \times \sqrt{4}}{1 + 6^2 - 12}$

c $\frac{2 + 3^2}{5^2 + 4 \times 10 - \sqrt{25}}$

d $\frac{6^2 - 11}{2(17 + 2 \times 4)}$

e $\frac{3^2 - 3}{2 \times \sqrt{81}}$

f $\frac{3^2 - 5 + 6}{\sqrt{4} \times 5}$

g $\frac{36 - 3 \times \sqrt{16}}{15 - 3^2 \div 3}$

h $\frac{-30 + [18 \div (3 - 12) + 24]}{5 - 8 - 3^2}$

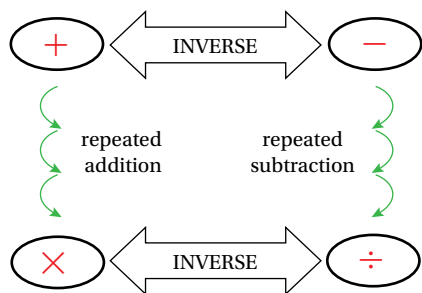
6 Work with a partner.

- a** Find a quick method for adding a set of consecutive whole numbers.
- b** Explain why your method works.
- c** Test your method on a set of consecutive negative integers.
- d** Does it work? Explain why or why not.

Section 3: Inverse operations

The four operations (add, subtract, multiply and divide) are related to each other. Operations are inverses of each other if one undoes (cancels out) the effect of the other.

- Adding is the inverse of subtracting, e.g. add 5 is undone by subtract 5.
- Multiplying is the inverse of dividing, e.g. multiply by 2 is undone by divide by 2.
- Taking a square root is the inverse of squaring a number, e.g. 4^2 is undone by $\sqrt{16}$.
- Taking the cube root is the inverse of cubing a number, e.g. 2^3 is undone by $\sqrt[3]{8}$.



Additive inverse

The additive inverse of 1 is -1 (add 1 is undone by subtract 1).

The additive inverse of -5 is 5 (subtract 5 is undone by add 5).

When you add a number to its inverse the answer is always 1.

Multiplicative inverse

The multiplicative inverse of 2 is $\frac{1}{2}$ (multiply by 2 is undone by divide by 2).

The multiplicative inverse of $\frac{1}{4}$ is 4 (divide by 4 is undone by multiply by 4).

When you multiply a number by its inverse, the answer is always 1.

Inverse operations are useful for checking the results of your calculations because carrying out the inverse operation gets you back to the number you started with.

For example,

is $4320 - 500 = 3820$ correct?

Check by adding 500 back to the result (i.e. doing the inverse operation) to see whether it gives you 4320.

$$3820 + 500 = 4320$$

When there is more than one operation involved, you have to reverse the order of the inverse operations to return to the starting number.

For example, is $(50 + 62) \div 8 = 14$?

Check by working backwards and applying inverse operations:

$$14 \times 8 - 62 = 50$$

**Tip**

The multiplicative inverse of a number is also called its **reciprocal**. For example, $\frac{1}{3}$ is the reciprocal of 3.

**Tip**

You will use inverse operations to solve equations and when you deal with functions, so it is important that you understand how they work.

EXERCISE 1D

- 1 Use inverse operations to find the missing values in each of these calculations.

a $\square + 217 = 529$

b $\square + 388 = 490$

c $\square - 218 = 182$

d $121 \times \square = -605$

e $-6 \times \square = 870$

f $\square \div 40 = 5400$

- 2 Use inverse operations to check these calculations.

a $45 \times 5 - 8 = 217$

b $14 + 5 \times 9 - 9 = 50$

c $(23 + 48) \times 4 = 284$

d $(412 - 128) \div 4 = 71$

- 3 The formula for finding the area of a triangle is $A = \frac{bh}{2}$ (where b = base length and h = height).

a Find the height of a triangle with an area of 54 cm^2 and a base length of 9 cm.

b A triangle has an area of 64 cm^2 .

Find the height and the base length if the base is twice the height.



- 4** Three boys each have 15 pence in their left pockets.
 They also have the same amount as each other in their right pockets.
 The total of all their money is 120 pence.
- Using R to represent the money in the right-hand pocket, write a sum to show how you can work out the total.
 - Use your sum to find out how much each boy has in their right pocket.
- 5** Here is an expression which includes different operations:
- $$1 - \left(\frac{2}{3} (4 + 5) + 6 \right) \times 7$$
- Calculate the value of the expression.
 - Keep the numbers in order (from 1 to 7) but change the operations as necessary to:
 - find the highest possible answer
 - find the lowest possible answer.
 - Comment on how changing the operations affected your results.



Checklist of learning and understanding

Basic calculations

- Written methods are important for when you do not have a calculator.
- You can use any method as long as you show your working.
- Negative and positive numbers can be added, subtracted, multiplied and divided as long as you apply the rules to get the correct sign in the answer.

Order of operations

- In maths there is a conventional order for working when there is more than one operation.
- Always work out brackets (or other grouping symbols) first, then powers. Multiply and/or divide next, then add and/or subtract.
- A useful memory aid for the order of operations is BODMAS.

Inverse operations

- An inverse operation undoes the previous operation.
- Addition is the inverse of subtraction.
- Multiplication is the inverse of division.
- Squaring is the inverse of taking the square root.



Chapter review



For additional questions on the topics in this chapter, visit GCSE Mathematics Online.

- 1 These are the solutions to a cross-number puzzle.
 The clues are all calculations that involve using the correct order of operations.
 Write a set of clues that would give these results.

	¹ 2	² 7					³ 3			
⁴ 1	4	8	⁵ 6			⁶ 1	9	⁷ 7	4	
⁸ 3	0		7						5	
	⁹ 4	¹⁰ 9			¹¹ 3	2	7		¹² 2	¹³ 4
		2			1					8
		0			¹⁴ 4	¹⁵ 2	5		¹⁶ 2	6
	¹⁷ 4	1	¹⁸ 7			1				0
¹⁹ 2	3		²⁰ 3	²¹ 2	0	4		²² 9	0	
²³ 7	9	4		1			²⁴ 7	9		

- 2 Use integers and operations to write ten different calculations that give an answer of -17 .
- 3 a Work out $2 \times (8 - 3)$ (1 mark)
 b Work out $32 + 4 \times 5$ (2 marks)

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- 4 On a page of a magazine there are three columns of text.
 Each column contains 42 rows.
 If there is an average of 32 letters per column row, approximately how many letters are there on a page?
- 5 A stadium has seats for 32 000 people. How many rows of 125 is this?
- 6 Two numbers have a sum of -15 and a product of -100 .
 What are the numbers?
- 7 The sum of two numbers is 1, but their product is -20 .
 What are the numbers?
- 8 Josie's bank account was overdrawn.
 She deposited £1000 and this brought her balance to £432.
 By how much was her account overdrawn to start with?
- 9 You can use the formula $F = 2C + 30$ to convert approximately temperatures from Celsius to Fahrenheit.
 Find the temperature in degrees Celsius when it is:
 a 68°F b 100°F



Find answers at: cambridge.org/ukschools/gcsemaths-studentbookanswers