

## *Advanced Computational Fluid and Aerodynamics*

The advent of high performance computers has brought Computational Fluid Dynamics (CFD) to the forefront as a tool to analyze increasingly complex simulation scenarios in many fields. Computational aerodynamics problems are also increasingly moving towards being coupled, multi-physics and multi-scale with complex, moving geometries. The latter presents severe geometry handling and meshing challenges. Simulations also frequently use formal design optimization processes.

This book explains the evolution of CFD and provides a comprehensive overview of the plethora of tools and methods available for solving complex scenarios while exploring the future directions and possible outcomes.

Using numerous examples, illustrations and computational methods the author discusses:

- Turbulence Modeling
- Pre and Post Processing
- Coupled Solutions
- The Importance of Design Optimization
- Multi-physics Problems
- Reduced Order Models
- Large-Scale Computations and the Future of CFD

*Advanced Computational Fluid and Aerodynamics* is suitable for audiences engaged in computational fluid dynamics, including advanced undergraduates, researchers and industrial practitioners.

Paul G. Tucker is the Rank Professor at the University of Cambridge. He has written more than 300 journal, conference papers and technical reports. He has been a visiting researcher at NASA and is an associate editor of the *AIAA Journal*.

# *Advanced Computational Fluid and Aerodynamics*

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*To my family and Rosie the Leonberger – my constant and patient companion during writing*

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## *Preface*

In the past 25 years computers have become around a million times faster. This is allowing many examples where full flows or subzones involve the near-direct solution of the Navier-Stokes equations. Since these equations are remarkably exact, such simulations rival measurements. Hence, the Computational Fluid Dynamics (CFD) landscape is beginning to change dramatically. Eddy-resolving simulations should, in roughly the next 10 years, see substantial use in industry in niche areas. The use of eddy-resolving approaches moves CFD to being predictive rather than more postdictive.

CFD problems are increasingly moving towards being coupled, multi-physics and multi-scale with complex geometries. They also frequently use formal design optimization processes. This book attempts to meet this CFD evolution and give an overview of the plethora of methods available to the engineer. Unlike many other volumes, here numerical methods are restricted to just one chapter. This is partly motivated by the observation that even though a vast range of numerical methods exist, as with many other areas of CFD, such a Reynolds Averaged Navier-Stokes (RANS) and LES, just a few schemes/models see widespread use. Doubtless, many will regard this as a bold approach. However, it has enabled me to give more coverage to the areas of CFD knowledge that are needed to exploit it for aerodynamic design.

I am highly grateful to all the PhD students who have so kindly helped me with aspects of text preparation. Special thanks are due to Zaib Ali who, as ever, was a tremendous help with the text preparation. I am grateful for his careful and diligent work. Jiahuan Cui and Mahak Mahak and Richard Oriji also offered tremendous and kind help with the text preparation. I am also grateful to Richard Oriji, Hardeep Kalsi and Sanjeev Shanmuganathan for neatly drawing many of the schematics used. There are two exercises relating to writing compressible and incompressible flow solvers. Inspiration for the compressible was taken from the Cambridge University CFD course. Professor John Denton developed this course, and this inspiration is gratefully acknowledged. As stated by Confucius – I hear, I forget, I write, I remember, I do, I understand. Although time-consuming and challenging, the code-writing tasks are enlightening.

## Nomenclature

The nomenclature is set out as follows. First lowercase Roman letters are given, followed by uppercase. Then Greek lowercase, followed by uppercase symbols, are given. Then superscripts and subscripts are set out. Overbars are then listed, followed by special symbols and operators. Finally the abbreviations used in the text are summarized. Please note: to save space, symbols only used once locally in the text are generally not included in this nomenclature.

### Lowercase Roman

$a_{ij}$	Anisotropy tensor
$c$	Particle velocity, wave velocity, speed of sound or concentration
$c'$	Pseudo-acoustic speed
$c_p$	Specific heat capacity at constant pressure
$c_v$	Specific heat capacity at constant volume
$d$	Normal wall distance or displacement
$\tilde{d}$	Approximate wall distance function
$e$	Fluid internal energy due to molecular motion, fundamental charge
$f_w, f_{v1}, f_{v2}, f_d$	Functions in the Spalart-Allmaras turbulence model
$g$	Earth's acceleration due to gravity
$h$	Heat transfer coefficient or height
$i, j, k$	Array or grid point location identifiers
$k$	Thermal conductivity, turbulent kinetic energy, temporal weighting function component or variable to ensure that the acoustic wave speed is similar to the particle speed
$k_{ij}$	Coefficient in spring analogy
$l$	Turbulence length scale or smoothing length in SPH approach
$l_\mu, l_\varepsilon$	Turbulence model length scales
$m$	Particle mass
$\dot{m}$	Mass flow rate
$n$	Surface normal or direction cosine
$p$	Static pressure, or number of stages (Chapter 4)
$q$	Heat flux
$q_r$	Radiative heat flux
$q1 \dots q6$	Terms for transformation to curvilinear coordinate system
$r$	Local pressure gradient

xii *Nomenclature*

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$r, \theta, z$	Cylindrical polar spatial coordinates
$r_d$	Shielding function in delayed DES
$rms_\phi$	Normalised root mean square change
$s$	Entropy or streamwise coordinate
$s_l$	Laminar flame burning velocity
$\dot{s}_n$	Rate of change of species
$t$	Time
$t_r = t -  \mathbf{x} - \mathbf{y} /c$	Retarded time
$u$	Displacement
$u, v, w$	Instantaneous $x, y, z$ , velocity components
$w$	Wave number, velocity component, work done by a rotor
$x, y, z$	Spatial coordinates
$y_{1/2}$	Half width

*Uppercase Roman*

$A$	Area, global representation of spatial discretization, nodal coefficient, amplitude or Roe matrix element
$A_\mu, A_\varepsilon$	Turbulence model constants
$A_\omega$	Average cross-sectional area normal to vorticity vector
$C$	Courant number ( $u\Delta t/\Delta x$ ), objective, correlation function, constant or amplitude
$C_s$	Smagorinsky constant
$C_t$	Safety factor
$C_D$	Drag coefficient
$C_f$	Skin friction coefficient
$C_L$	Lift coefficient
$C_p$	Surface pressure coefficient
$D$	Time step to diffusion time scale ratio or diameter scale
$Da$	Damköhler number or damping function
$E$	Young's modulus, error, flux term or energy, constant in wall function or source term in $k$ - $\varepsilon$ model
$E_b$	Black body emission
$F$	General force term, strong conservation flux term, speed function, switching function in Menter SST model or function in level set equation
$FAR$	Free air ratio
$F_{i,j}$	View factor (ratio of the radiation received by surface $j$ to that emitted from surface $i$ )
$F_p, F_n$	Forces parallel and normal to blade passages, respectively
$[F_S]$	Force matrix
$F_{SST}$	Delayed DES function in Menter SST framework
$G$	Strong conservation flux term or filter kernel/operator
$GCI$	Grid convergence index
$Gr$	Grashof number
$H$	Strong conservation flux term or representation of step height
$I$	Prolongation operator



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*Nomenclature*

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$IQ$	Eddy-resolving simulation quality metric
$J$	Jacobian or radiosity
$Kn$	Knudsen number
$K$	Porosity, relaxation or acceleration parameter
$K_n, K_{vd}$	Body force model calibration constants
$[K_s]$	Stiffness matrix
$[K_f]$	Fluid system matrix
$L$	Length scale, linear turbulent stress component, wave operator or Laplacian
$\tilde{L}$	Free molecular path
$M$	Mach number
$N$	Number of mesh points or realizations
$\overline{NL}$	Non-linear turbulent stress component
$P$	Poisson's ratio or production term
$Pr = \mu c_p/k$	Prandtl number
$Q$	Volume flow rate or vorticity identification parameter
$Q1 \dots Q6$	Transformation terms
$R$	Gas constant, radius scale, residual or energy transfer term
$\mathbf{R}$	Reynolds stress tensor
$\tilde{R}$	Universal gas constant
$Re$	Reynolds number
$[R]$	Coupling matrix
$S$	Source or strain term
$S_{ij}$	Mean strain rate tensor
$Sc$	Schmidt number
$St$	Strouhal number
$T$	Temperature or time scale
$\mathbf{T}$	Matrix of eigenvectors
$T_{ij}$	Lighthill stress tensor
$TV$	Total variation
$U,$	Vector of conserved variables or reference velocity
$U, V, W$	Velocity components aligned with transformed coordinates
$U_c$	Bulk or convection velocity
$U_\tau$	Friction velocity
$U_\infty$	Free stream velocity
$V$	General velocity scale or volume
$Vol$	Cell volume
$Wf$	Weighting function

*Lowercase Greek*

$\alpha$	Grid expansion factor (Chapter 3), latency parameter in LNS model (Chapter 5), design variable, blade metal angle (Chapter 6), or weighting parameter in compact scheme or relaxation factor (Chapter 4).
$\beta$	Compressibility parameter, coefficient of thermal expansion or weighting parameter in compact scheme

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$\gamma = c_p/c_v$	Ratio of specific heats, weighting parameter in compact scheme or intermittency
$\delta$	Boundary layer thickness, grid spacing, step function or small number/perturbation
$\varepsilon$	Turbulence dissipation rate, smoothing parameter, strain in solid, small number, scaling parameter in level set related equations, (specified) error tolerance/level or emissivity
$\eta$	Parameter that defines time levels in discretized equations or transformed spatial variable
$\theta$	Angle
$\kappa$	von Karman constant or weighting parameter in MUSCL scheme (see Section 4.4.3.1)
$\lambda$	Temporal discretization control parameter, Eigen values, spectral radius of matrix, viscosity coefficient ( $-2\mu/3$ ), Lamé's coefficient or wave speed (in LES filter definition)
$\mu$	Dynamic viscosity, Lamé's coefficient
$\mu_t$	Turbulent viscosity
$\nu$	Kinematic viscosity
$\nu_t$	Turbulent kinematic viscosity
$\xi, \eta, \zeta$	General, transformed coordinates
$\rho$	Fluid density
$\sigma$	Normal stress, Diffusion Prandtl/Schmidt number, turbulence fluctuation scale or Stefan-Boltzmann constant
$\tau$	Transformed temporal coordinate, shear stress, pseudo time, time shift or relaxation time parameter
$\phi$	General variable, flux limiter, or distribution function in lattice Boltzmann method
$\psi$	Stream function, internal potential
$\omega$	Frequency (turbulence) or vorticity

*Uppercase Greek*

$\Gamma$	Diffusion coefficient, domain boundary or Jacobian matrix
$\Delta$	Filter width or space shift
$\Delta x, \Delta x, \Delta z$	Grid spacings
$\Delta t$	Time-step length
$\Lambda$	Adjoint variable, spectral radius or eigenvalues
$\Phi$	Mass fraction, general conserved variable or electric field
$\Psi$	Shock switch
$\Omega$	Angular velocity or vorticity

*Superscripts*

$eq$	Equilibrium value
$H$	High-order component
$L$	Low-order component
$n$	Time level

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*Nomenclature*

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<i>new</i>	Latest value
<i>old</i>	Previous value
<i>t</i>	Pertaining to tangential component
$\Delta X$	Variable computed with a coarser grid spacing
'	Perturbation or first derivate of variable or correction in the pressure correction equation (see Section 4.7.3)
"	Second derivative
+	Dimensionless distance in wall units
*	Approximate value in the pressure correction equation (Section 4.7.3) or distance in wall units

*Subscripts*

<i>amb</i>	Ambient value
<i>A</i>	Actual value in full scale system
<i>BD</i>	Pertaining to backwards difference scheme
<i>c</i>	Convective flux
<i>coll</i>	Pertaining to collisions
⊕	Centerline value
<i>CVF</i>	Pertaining to control volume face
<i>CFD</i>	Pertaining to CFD
<i>DB</i>	Pertaining to database
<i>DES</i>	Pertaining to the DES model
<i>fp</i>	Relating to a particular moving fluid particle
<i>g</i>	Pertaining to grid movement or flow translation
<i>HJ</i>	Pertaining to HJ equation
<i>i, j, k</i>	Array subscripts
<i>in</i>	Pertaining to inlet
<i>I1, I2</i>	Nodes that straddle a control volume face
<i>k</i>	Pertaining to turbulence kinetic energy
<i>KEP</i>	Pertaining to kinetic energy preserving scheme
<i>l</i>	Pertaining to a liquid
<i>LES</i>	Pertaining to LES model
<i>max</i>	Maximum value
<i>min</i>	Minimum value
<i>M</i>	Values in model
<i>NB</i>	Neighboring values
<i>o</i>	Reference value, or pertaining to offset
<i>out</i>	Pertaining to outlet
<i>p</i>	Pertaining to a particle or droplet
<i>P</i>	Central grid point
<i>PS</i>	Pressure surface
<i>ref</i>	Reference value
<i>rel</i>	Relative velocity component
<i>R</i>	Pertaining to radiation
<i>RANS</i>	Pertaining to RANS model

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<i>Roe</i>	Pertaining to Roe scheme
<i>s</i>	Pertaining to solid or sand grain roughness
<i>stat</i>	Pertaining to stationary coordinate system
<i>SGS</i>	Pertaining to the subgrid scale
<i>SS</i>	Pertaining to suction surface
<i>target</i>	Target value
<i>t</i>	Pertaining to turbulence
<i>up</i>	Pertaining to point of separation
<i>u, v, w</i>	Pertaining to listed velocity components
<i>v</i>	Pertaining to a vapour or viscous flux
<i>w, e, n, s, f, b</i>	Geographic grid point notation for control volume face
<i>W, E, N, S, F, B</i>	Geographic grid point notation for grid points
<i>x, y, z</i>	Pertaining to the <i>x</i> , <i>y</i> and <i>z</i> directions, respectively
<i>z, r, <math>\theta</math></i>	Pertaining to the axial, radial and tangential directions, respectively
$\phi$	Pertaining to the variable $\phi$
$\xi, \eta, \zeta$	General, transformed coordinates
$\Delta x, \Delta X$	Variables represented on coarse and fine grids
$\theta$	Stagnation property

*Overbars*

- ~ Dimensionless or smoothed variable
- Averaged or filtered value
- ^ Relating to undivided Laplacian

*Special Symbols/Operators*

$N(a, b)$	Normally distributed random number operator with mean <i>a</i> and standard deviation <i>b</i>
$NS(\phi), NS^s(\phi)$	Navier-Stokes and steady Navier-Stokes operator
$URANS(\phi)$	Unsteady RANS operator
$\delta(\varphi)$	Dirac delta function (see Chapter 7)
$\delta_{ij}$	Kronecker delta ( $\delta_{ij} = 1$ if $i = j$ and $\delta_{ij} = 0$ if $i \neq j$ )
$\varepsilon_{ijk}$	Alternating third-rank unit tensor
$  $	Modulus of quantity

## *Abbreviations*

ADI	Alternating Direct Implicit
ACTRAN	ACoustic TRANsmission
ALE	Arbitrary Lagrangian-Eulerian
AUSM	Advection Upstream Splitting Method
AVPI	Pressure correction scheme variant for unsteady flows
BASIC	Beginner's All-purpose Symbolic Instruction Code
BEM	Boundary Element Method
Bi-CGSTAB	Biconjugate Gradient Stabilized Method
BREP	Boundary Representation
BTD	Balanced Tensor Diffusivity (see Section 4.8.1.4)
CAA	Computational Aeroacoustics
CAD	Computer Aided Drawing
CFD	Computational Fluid Dynamics
CGNS	CFD General Notation System
CPR	Correction Procedure via Reconstruction
CVF	Control Volume Face
CSG	Construction Solid Geometry
CVS	Control Volume Surface
DES	Detached Eddy Simulation
DFT	Discrete Fourier Transform
DG	Discontinuous Galerkin
DNS	Direct Numerical Simulation
DOE	Design of Experiment
DRAGON	Direct Replacement of Arbitrary Grid Overlapping by Non-structured
DRP	Dispersion Relation Preserving
DSM	Deterministic Stress Model
DSMC	Direct Simulation Monte-Carlo
ENO	Essentially Non-Oscillator
ERCOFTAC	European Research Community On Flow Turbulence And Combustion
FD	Finite Difference
FE	Finite Element
FFT	Fast Fourier Transform

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FMM	Fast Multipole Method
FORTRAN	FORmula TRANslating System
FT	Forward Transition
FWH	Ffowcs-Williams and Hawkings
GA	Genetic Algorithm
GCI	Grid Convergence Index
GMRES	Generalized Minimum Residual
GPU	Graphical Processor Unit
HJ	Hamilton-Jacobi
HOT	High-Order Term
HPT	High-Pressure Turbine
ICE	Implicit Continuous-fluid Eulerian
IGES	International Graphics Exchange Standard
ILES	Implicit Large Eddy Simulation
KEP	Kinetic Energy Preserving
k-d	k-dimensional
LEE	Linear Euler Equation
LES	Large Eddy Simulation
LIC	Line Integral Convolution
LNS	Limited Numerical Scales
LNSE	Linear Navier-Stokes Equations
LPT	Low-Pressure Turbine
MD	Molecular Dynamics
MATLAB	MATrix LABoratory
MDICE	Multidisciplinary Computing Environment
MDO	Multidisciplinary Design Optimization
MEM	Maximum Entropy Method
MEMS	MicroElectroMechanical Systems
MILES	Monotone Integrated Large Eddy Simulation
MMS	Method of Manufactured Solutions
MRM	Multiple Reciprocity Method
MST	Mean Source Terms
MUSCL	Monotone Upstream-Centred Schemes for Conservation Laws
NACA	National Advisory Committee for Aeronautics
NAFEMS	National Finite Element Methods and Standards
	NLAS
	Non-Linear Acoustics Solver
NLDE	Non-Linear Disturbance Equation
NLES	Numerical Large Eddy Simulation
NLAS	Non-Linear Acoustics Solver
NSS	Nearest Surface Search
PANS	Partially Averaged Navier-Stokes
ParMETIS	Parallel Graph Partitioning and Fill-reducing Matrix Ordering
PPW	Points Per Wave
PISO	Pressure Implicit with Splitting of Operator

POD	Proper Orthogonal Decomposition
RANS	Reynolds Averaged Navier-Stokes
RK	Runge-Kutta Scheme
ROM	Reduced Order Model
RPM	Recursive Projection Methods
RSM	Response Surface Methods or Reynolds Stress Model
RT	Reverse Transition
SARC	SA with Rotation or/and Curvature
SAS	Scale Adaptive-Simulation
SIMPLE	Semi-Implicit Method for Pressure-Linked Equations
SIMPLER	Semi-Implicit Method for Pressure-Linked Equations-Revised
SIMPLEC	Semi-Implicit Method for Pressure Linked Equations-Consistent
SIMPLE*	Further SIMPLE (see above) scheme variant
SIMPLE2	Further SIMPLE (see above) scheme variant
SIP	Strongly Implicit Procedure
SPH	Smooth Particle Hydrodynamics
SST	Shear Stress Transport
SD	Spectral Difference
SV	Spectral Volume
TDMA	Tri-Diagonal Matrix Algorithm
T-S	Tollmien-Schlichting
TSL	Thin Shear Layer
TVD	Total Variation Diminishing
ULIC	Unsteady Line Integral Convolution
UMIST	University of Manchester Institute of Science and Technology
URANS	Unsteady Reynolds Averaged Navier-Stokes
VLES	Very Large Eddy Simulation
WALE	Wall Adapting Local Eddy-Viscosity
WENO	Weighted Essentially Non-Oscillatory