

## COMPUTATIONAL FLUID DYNAMICS

### *Second Edition*

This revised second edition of *Computational Fluid Dynamics* represents a significant improvement from the first edition. However, the original idea of including all computational fluid dynamics methods (FDM, FEM, FVM); all mesh generation schemes; and physical applications to turbulence, combustion, acoustics, radiative heat transfer, multiphase flow, electromagnetic flow, and general relativity is maintained. This unique approach sets this book apart from its competitors and allows the instructor to adopt this book as a text and choose only those subject areas of his or her interest.

The second edition includes new sections on finite element EBE-GMRES and a complete revision of the section on the flowfield-dependent variation (FDV) method, which demonstrates more detailed computational processes and includes additional example problems. For those instructors desiring a textbook that contains homework assignments, a variety of problems for FDM, FEM, and FVM are included in an appendix. To facilitate students and practitioners intending to develop a large-scale computer code, an example of FORTRAN code capable of solving compressible, incompressible, viscous, inviscid, 1-D, 2-D, and 3-D for all speed regimes using the flowfield-dependent variation method is available at <http://www.uah.edu/cfd>.

T. J. Chung is distinguished professor emeritus of mechanical and aerospace engineering at the University of Alabama in Huntsville. He has also authored *General Continuum Mechanics* and *Applied Continuum Mechanics*, both published by Cambridge University Press.

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*To my family*

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## *Second Edition*

**T. J. CHUNG**

University of Alabama in Huntsville



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T. J. Chung  
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## Preface to the First Edition

This book is intended for the beginner as well as for the practitioner in computational fluid dynamics (CFD). It includes two major computational methods, namely, finite difference methods (FDM) and finite element methods (FEM) as applied to the numerical solution of fluid dynamics and heat transfer problems. An equal emphasis on both methods is attempted. Such an effort responds to the need that advantages and disadvantages of these two major computational methods be documented and consolidated into a single volume. This is important for a balanced education in the university and for the researcher in industrial applications.

Finite volume methods (FVM), which have been used extensively in recent years, can be formulated from either FDM or FEM. FDM is basically designed for structured grids in general, but is applicable also to unstructured grids by means of FVM. New ideas on formulations and strategies for CFD in terms of FDM, FEM, and FVM continue to emerge, as evidenced in recent journal publications. The reader will find the new developments interesting and beneficial to his or her area of applications. However, the subject material is often inaccessible due to barriers caused by different training backgrounds. Therefore, in this book, the relationship among all currently available computational methods is clarified and brought to a proper perspective.

To the uninitiated beginner, this book will serve as a convenient guide toward the desired destination. To the practitioner, however, preferences and biases built over the years can be relaxed and redeveloped toward other possible options. Having studied all methods available, the reader may then be able to pursue the most reasonable directions to follow, depending on the specific physical problems of each reader's own field of interest. It is toward this flexibility that the present volume is addressed.

The book begins with Part One, Preliminaries, in which the basic principles of FDM, FEM, and FVM are illustrated by means of a simple differential equation, each leading to the identical exact solution. Most importantly, through these examples with step-by-step hand calculations, the concepts of FDM, FEM, and FVM can be easily understood in terms of their analogies and differences. The introduction (Chapter 1) is followed by the general forms of governing equations, boundary conditions, and initial conditions encountered in CFD (Chapter 2), prior to embarking on details of CFD methods.

Parts Two and Three cover FDM and FEM, respectively, including both historical developments and recent contributions. FDM formulations and solutions of various types of partial differential equations are discussed in Chapters 3 and 4, whereas

the counterparts for FEM are covered in Chapters 8 through 11. Incompressible and compressible flows are treated in Chapters 5 and 6 for FDM and in Chapters 12 through 14 for FEM, respectively. FVM is included in both Part Two (Chapter 7) and Part Three (Chapter 15) in accordance with its original point of departure. Historical developments are important for the beginner, whereas the recent contributions are included as they are required for advanced applications given in Part Five. Chapter 16, the last chapter in Part Three, discusses the detailed comparison between FDM and FEM and other methods in CFD.

Full-scale complex CFD projects cannot be successfully accomplished without automatic grid generation strategies. Both structured and unstructured grids are included. Adaptive methods, computing techniques, and parallel processing are also important aspects of the industrial CFD activities. These and other subjects are discussed in Part Four (Chapters 17 through 20).

Finally, Part Five (Chapters 21 through 27) covers various applications including turbulence, reacting flows and combustion, acoustics, combined mode radiative heat transfer, multiphase flows, electromagnetic fields, and relativistic astrophysical flows.

It is intended that as many methods of CFD as possible be included in this text. Subjects that are not available in other textbooks are given full coverage. Due to a limitation of space, however, details of some topics are reduced to a minimum by making a reference, for further elaboration, to the original sources.

This text has been classroom tested for many years at the University of Alabama in Huntsville. It is considered adequate for four semester courses with three credit hours each: CFD I (Chapters 1 through 4 and 8 through 11), CFD II (Chapters 5 through 7 and 12 through 16), CFD III (Chapters 17 through 20), and CFD IV (Chapters 21 through 27). In this way, the elementary topics for both FDM and FEM can be covered in CFD I with advanced materials for both FDM and FEM in CFD II. FVM via FDM and FVM via FEM are included in CFD I and CFD II, respectively. CFD III deals with grid generation and advanced computing techniques covered in Part IV. Finally, the various applications covered in Part V constitute CFD IV. Since it is difficult to study all subject areas in detail, each student may be given an option to choose one or two chapters for special term projects, more likely dictated by the expertise of the instructor, perhaps toward thesis or dissertation topics.

Instead of providing homework assignments at the end of each chapter, some selected problems are shown in Appendix E. An emphasis is placed on comparisons between FDM, FEM, and FVM. Through these exercises, it is hoped that the reader will gain appreciation for studying all available methods such that, in the end, advantages and disadvantages of each method may be identified toward making decisions on the most suitable choices for the problems at hand. Associated with Appendix E is a Web site <http://www.uah.edu/cfd> that provides code (FORTRAN 90) for solutions of some of the homework problems. The student may use this as a guide for programming with other languages such as C++ for the class assignments.

More than three decades have elapsed since the author's earlier book on FEM in CFD was published [McGraw-Hill, 1978]. Recent years have witnessed great progress in FEM, parallel with significant achievements in FDM. The author has personally experienced the advantage of studying both methods on an equal footing. The purpose

of this book is, therefore, to share the author's personal opinion with the reader, wishing that this idea may lead to further advancements in CFD in the future. It is hoped that all students in the university will be given an unbiased education in all areas of CFD. It is also hoped that the practitioners in industry will benefit from many alternatives that may impact their new directions of future research in CFD applications.

In completing this text, the author recalls with sincere gratitude a countless number of colleagues and students, both past and present. They have contributed to this book in many different ways.

My association with Tinsley Oden has been an inspiration, particularly during the early days of finite element research. Among many colleagues are S. T. Wu and Gerald Karr, who have shared useful discussions in CFD research over the past three decades.

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T. J. Chung

## Preface to the Revised Second Edition

This revised second edition of *Computational Fluid Dynamics* represents a significant improvement from the first edition. However, the original idea of including all computational fluid dynamics methods (FDM, FEM, FVM); all mesh generation schemes; and physical applications to turbulence, combustion, acoustics, radiative heat transfer, multiphase flow, electromagnetic flow, and general relativity is maintained. This unique approach sets this book apart from its competitors and allows the instructor to adopt this book as a text and choose only those subject areas of his or her interest.

The second edition includes new sections on finite element EBE-GMRES and a complete revision of the section on the flowfield-dependent variation (FDV) method, which demonstrates more detailed computational processes and includes additional example problems. For those instructors desiring a textbook that contains homework assignments, a variety of problems for FDM, FEM, and FVM are included in an appendix. To facilitate students and practitioners intending to develop a large-scale computer code, an example of FORTRAN code capable of solving compressible, incompressible, viscous, inviscid, 1-D, 2-D, and 3-D for all speed regimes using the flowfield-dependent variation method is available at <http://www.uah.edu/cfd>.