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978-1-107-40606-3 - Discrete Inverse and State Estimation Problems: With Geophysical Fluid Applications

Carl Wunsch

Index

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Index

- 4DVAR, *see* Lagrange multipliers; adjoint method
- acoustic tomography, *see* tomography, acoustic
- adaptive estimation, *see* estimation
- adjoint, 64
- and least-squares, 118
 - and Riccati equation, 227
 - and sensitivity, 62, 119, 218
 - and SVD, 113
 - in terminal constraint problem, 218
 - interpretation, 68
 - method, 239
 - non-linear, 239
 - of differential operators, 146
 - time-dependent models, 214
- adjoint method
- Kalman filter equivalent, 216
- adjoint model, 218, 272
- advection-diffusion equation, *see* partial differential equations, 340
- Aitken–Lagrange, *see* interpolation
- algorithmic differentiation, *see* automatic differentiation
- aliasing, *see* sampling
- autocorrelation, 51
- automatic differentiation (AD), 189, 241
- chain rule, 242
 - examples, 250
 - reverse mode, 245
- autoregressive process (AR), 183
- average, *see* mean
- basis functions, 132
- kriging, 170
- basis, definition, 20
- β -spiral, 328
- BLUE – best linear unbiased estimator, 129
- box model, 58, 98, 99, 111, 209, 279, 280
- and representers, 225
 - and terminal control, 220
- inversion
- adding properties, 304
- branching statements – in computer code, 245
- canonical form, 129, 266
- Gaussian, 39
 - in finite differences, 326
 - in least-squares, 43
 - linear programming, 167
 - linear programming dual, 168
 - time-dependent model, 183
- chain rule, 242, 243
- characteristic functions, 41
- chi-square, *see* probability density
- Cholesky decomposition, 40, 53
- column scaling, 56, 67, 105
- columnar flows, 318
- combinatorial optimization, 176
- companion matrix, *see* matrix
- completing the square (matrix), 29
- constraints
- exact, 118
- control, 193
- control theory, 214
- deadbeat control, 220
 - open loop, 218
 - terminal constraint problem, 218
- control vector
- in global ocean state estimation, 345
- controllability, 232
- convolution theorem, 41
- correlation coefficient, 35
- cost function, *see* objective function
- covariances
- singular forms, 160
- data assimilation, 180
- meteorological applications, 356
 - meteorological practice, 260
 - nudging, 261
- data ranking, 111
- data sets
- in global ocean state estimation, 345
- deadbeat control, 220, 227
- degrees-of-freedom, 41

Cambridge University Press

978-1-107-40606-3 - Discrete Inverse and State Estimation Problems: With Geophysical
Fluid Applications

Carl Wunsch

Index

[More information](#)

368

design matrix, 190
 diag (operator), 23
 difference equation, 183
 differentiability
 of model, 235
 differential
 least-squares, 45
 differential equations, 248
 discretized, 6
 heat equation, 4
 mass–spring oscillator, 6
 direct models, *see* forward models
 Dirichlet problem, *see* partial differential equations
 dispersion, *see* statistics
 doubling algorithm, 266, 268, 274
 dual, *see* adjoint
 estimation and control, 230
 filters and adjoints, 229
 Duffing equation, 237
 dye, *see* tracers
 dynamic method, 284
 discrete representation, 289

 Eckart–Young–Mirsky theorem, 169,
 269
 eigenvectors, *see* matrix, 72, 83
 as POPs, 73
 right and left, 152, 270
 Ekman flux, 292
 Ekman transport, 286
 empirical orthogonal functions (EOFs), 169
 applications, 333
 relation to singular value decomposition, 169
 ensemble filter, 256
 ensemble methods
 ensemble generation, 258
 ergodic hypothesis, 324
 estimation
 adaptive, 237, 273
 Gauss–Markov, 125
 initial conditions, 228
 maximum-likelihood, 145
 mixing prior and posterior values, 136

 Florida Straits flow, 290
 Fokker–Planck equation, 260
 forward models, 17, 248
 Fourier transform, 41, 71, 271

 Gauss–Markov estimation
 and least-squares, 129
 geostrophic box model, 302
 mapping
 derivatives, 164
 higher dimensional, 161
 one-dimensional, 158
 mean value, 135
 mean value determination, 133
 oceanographic application, 302
 of linear algebraic equations, 129
 realistic oceanographic, 313

Index

recursive, 140, 148
 theorem, 127
 Gaussian probability density, 32
 multivariable, 37
 genetic algorithms, 176
 geostrophic balance, 282
 discrete representation, 289
 geostrophic box model, 290
 geostrophy, 279
 gradient, 27
 Gram–Schmidt process, 22
 Green function, 188, 223, 229
 Green functions, 146
 adjoint, relationship, 124

H definition, 46
 hard constraints, 68
 heat (temperature) transport, 348
 heat equation, *see* differential equations; partial
 differential equations
 Hessian, 27, 114, 175

 idempotent, 92
 ill-conditioning, 94, 213
 ill-posedness, 3, 94, 222, 248
 inequality constraints, 164
 linear programming, 166
 initial condition estimation, 228
 interpolation, 158
 Aitken–Lagrange, 158
 by Gauss–Markov estimation, 158
 by Gauss–Markov estimator, 158
 linear, 158
 inverse, 25
 left and right, 105
 Moore–Penrose, 64, 105
 inverse methods, 4, 7, 340
 alleged failure in oceanography, 331
 inverse problems, 8
 non-linear, 335

 Jacobian, 27, 245
 in multivariable Gaussian, 38
 in probability density, 37
 joint frequency function, *see* multivariable probability
 density

 Kalman filter, 140, 195, 197
 and objective mapping approximation, 263
 backwards in time, 229
 computational load, 203
 ensemble methods, 256
 extended, 234
 gain, 262
 gain in general circulation model, 353
 in global ocean state estimation, 351
 linearized, 234
 nudging approximation, 262
 reduced-state methods, 267
 static limit, 203
 steady-state approximation, 264

Cambridge University Press

978-1-107-40606-3 - Discrete Inverse and State Estimation Problems: With Geophysical
Fluid Applications

Carl Wunsch

Index

[More information](#)

Index

369

- Kolmogorov equation, 260
- kriging, 170
- Kuhn–Tucker–Karush theorem, 165
- Lagrange multipliers, 58, 61, 82, 123
 - 4DVAR, 261
 - and time-dependent models, 214
 - global estimates, 342
 - linear programming, relation to, 168
- Laplace equation, *see* partial differential equations
- Laplace–Poisson equation, *see* partial differential equations
- least-squares, 43, 59
 - basic objective function, 45
 - canonical form, 43
 - column scaling, 56, 67
 - combinatorial methods, 176
 - Lagrange multipliers, 58
 - non-linear, 171
 - non-negative, 164
 - normal equations, 45, 46
 - quadratic fit, 47
 - recursive, 136, 138, 148
 - as Kalman filter, 196, 197
 - recursive solution, 229
 - relation to singular value decomposition, 102
 - row scaling, 52
 - smoothest solution, 56, 66
 - tapered, 55, 82
 - tapered and weighted
 - relation to singular value decomposition, 114
 - resolution and variance, 117
 - total inversion, 172
 - total least-squares, 171
 - trade-off parameter, 54
 - uncertainty of solution, 47
 - underdetermined, 58
 - weighted, 52
- left and right eigenvectors, *see* eigenvectors
- linear algebraic equations
 - by Gauss–Markov estimation, 129
- linear programming, 166
 - canonical form, 167
 - dual form, 168
 - in oceanography, 326
- linear simultaneous equations, *see* simultaneous equations
- log-normal probability density, 259
- mass–spring oscillator, *see* differential equations, 183, 198, 213, 246
 - and ensemble filter, 258
 - and Kalman filter, 200, 265
 - coupled oscillators, 265
 - non-linear, 236
- matrix
 - Cholesky decomposition, 40
 - companion matrix, 184
 - completing square, 29
 - decomposition, symmetric, 78
 - definition, 20
 - design, 190
 - differential, 28
 - differentiation, 27
 - eigenvectors, 25, 72, 83
 - Hessian, 27
 - idempotent, 46, 92
 - identities, 27
 - identity, 23
 - inverse, 23, 25
 - left and right, 70
 - inverse, Moore–Penrose, 64
 - inverse, pseudo, 105
 - inversion lemma, 28
 - Jordan form, 83
 - left, right inverse, 23
 - multiplication, definition, 23
 - nullspace, 74
 - observation, 190
 - positive-definite, 27
 - properties, 19
 - resolution, 70, 110
 - resolution, compact, 71
 - singular value decomposition, 89
 - state transition, 182
 - determining, 187
 - symmetric, defined, 23
 - transpose, 20
- matrix inversion lemma, 28
- maximum-likelihood estimation, 145
- mean, 30
 - from Kalman filter, 204
 - Gauss–Markov estimate of, 133
 - sample mean, 17
- minimization
 - constrained, 62
- minimum variance estimation, *see* Gauss–Markov estimation
- model
 - adjoint, 218
 - backwards in time, 185, 212
 - definition(s), 44
 - differentiability, 235
 - forward, 17, 248
 - non-linear
 - and extended Kalman filter, 234
 - straight-line, 182
 - time-dependent
 - canonical form, 183
 - whole-domain form, 187
 - time-dependent fluid, 180
 - validation, invalidation, 18, 49
- models
 - finite difference, 325
 - oceanic general circulation, 343
- moments
 - vector, 39
- Monte Carlo methods, 256
- Moore–Penrose inverse, *see* inverse

Cambridge University Press

978-1-107-40606-3 - Discrete Inverse and State Estimation Problems: With Geophysical
Fluid Applications

Carl Wunsch

Index

[More information](#)

370

multivariable probability density, 33
 change of variable, 36

Needler's formula, 285, 336

Neumann problem, 9, 101

non-Gaussian processes, 258

non-linear least-squares, *see* least-squares

non-normal systems, 154, 270

norm
 1-norm, 169
 Frobenius, 26
 matrix, 26
 spectral, 26
 vector norm, 20, 24

normal equations, 45, 82, 119

notation, 46, 195, 217, 289

nudging, 261

nullspace, 74, 88

numerical weather prediction (NWP), 260

Nyquist frequency, *see* sampling

objective function, 45, 194
 degrees-of-freedom, 49
 for Kalman filter, 197
 for smoother, 205

objective interpolation, *see* Gauss–Markov estimation

objective mapping, *see* Gauss–Markov estimation

observability, 232, 266

ocean circulation inverse problem, 279, 282

oceanography, 279
 β -spiral, 328
 equations of motion, 282
 Florida Current, 292
 geostrophic balance, 282
 Gulf Stream, 292
 heat (temperature) transport, 348
 Kalman filter
 global ocean state estimation, 351
 realistic estimation
 global, 342
 regional, 341
 realistic inversions, 311
 by linear programming, 326
 error estimates, 322
 finite difference models, 325
 global scale, 319
 initial skepticism, 318
 regional, 311
 reference-level, 284

Ockham's Razor, 77

operations research, 167

operator adjoints, 146

optimization
 combinatorial, 176

P -equation, 338

parameter estimation, 237

partial differential equations, 7, 122,
 181
 advection–diffusion, 280
 estimation with, 340

Index

 one-dimensional, 281
 two-dimensional, 281

advection–diffusion equation, 164
 and RTS smoother, 209

diffusion equation, 179

Dirichlet problem, 8

heat equation, 179

Laplace–Poisson equation, 7

mass–spring oscillator, 183,
 184
 and Kalman filter, 198

Neumann problem, 9, 101

P -equation, 338

wave equation, 178

particle filter, *see* ensemble methods

“PO”, 319

POPs, *see* principal oscillation patterns

positive-definite matrix, *see* matrix

predictable variance, *see* statistics

principal oscillation pattern (POP), 73,
 271

probability density, 29
 chi-square, 33, 41, 50
 χ^2 , 33, 41

property fluxes
 oceanographic, 309

quadratic form, 27

random processes and variables, 32,
 50
 characteristic function, 41
 functions of, 32
 sums, 32, 40
 vector, 37, 39
 covariance of, 39

rank
 and singular value decomposition, 103
 and tapering, 115
 by SVD, 90
 deficiency, 103
 determination, 105
 effective, 104

Rayleigh distribution, 37

recursive estimation, *see* Gauss–Markov estimation

recursive least-squares, *see* least-squares

reduced state
 state reduction, 267

reduced state methods, *see* Kalman filter

reference-level velocity, *see* oceanography,
 reference-level

regularization, 115

representers, 223
 in practice, 355

reservoir model, *see* box model

residuals
 test, 297

resolution
 tapered and weighted, 117

resolution matrix, 70, 110
 compact, 71

Cambridge University Press

978-1-107-40606-3 - Discrete Inverse and State Estimation Problems: With Geophysical
Fluid Applications

Carl Wunsch

Index

[More information](#)

Index

371

- Riccati equation, 234, 265
 - control form, 227
 - duality, 231
- right and left eigenvectors, 270
- row scaling, 52, 105, 107
- row weighting, *see* row scaling
- RTS smoother, 205, 208
 - in global ocean state estimation, 351
- salinity
 - in inversions, 306
 - practical salinity scale, 304
- salinity anomaly, 308
- sample variance, *see* statistics
- sampling, 155
 - aliasing, 156
 - Nyquist frequency, 156
- Shannon–Whitaker formula, *see* sampling
- simplex method, *see* linear programming
- simulated annealing, 176
- simultaneous equations, 24
 - as column sum, 25
 - as row sum, 25
 - determination (over-determined, under-determined, etc.), 25
- singular value decomposition (SVD), 89
 - and least-squares, 102
 - and total least-squares, 172
 - Eckart–Young–Mirsky theorem, 170
 - examples of, 93
 - geostrophic box inversion, 294
 - in differential equations, 100
 - oceanographic application, 297
 - resolution, 110
- singular values, 86
- singular vectors, 86
- smoother
 - alternate forms, 213
 - alternative forms, 214
 - backwards running Kalman filter, 212
 - limiting form, 208
- smoothing, 205
 - RTS algorithm, 208
- soft constraints, 68
- solvability condition, 75, 88
- spanning set, *see* vector
- standard form, models and data, 193
- state reduction, 267
- state transition matrix, 182
 - determining, 187
- state vector, 180
- stationarity in statistics, 42
- stationary
 - objective function, 61
- statistics, 29
 - autocorrelation, 51
 - bias, 30
 - correlation coefficient, 35
 - degrees-of-freedom, 41
 - dispersion, 31
 - Gaussian, 32
 - mean, 30
 - moment matrices, 39
 - moments, 30
 - multivariable probability density, 33
 - predictable variance, 35
 - regression, 36
 - sample mean, 138
 - sample variance, 31, 42
 - stationary process, 42
 - unpredictable variance, 35
- steady-state filter, *see* Kalman filter; Wiener filters and smoothers
- straight-line
 - and Kalman filter, 198
 - and RTS smoother, 208
- SVD, *see* singular value decomposition (SVD)
- tangent linear model, 235
- thermal wind equations, *see* dynamic method
- tomography, 95, 190
- tomography, acoustic, 11, 118, 354
- total inversion, method of, 172
- total least-squares, 171
- tracer
 - box model, 218
- tracer problem, 58, 220
 - box models, 14
- tracers, 280
 - box model, 10
 - in advection–diffusion equation, 209
 - time-dependent estimation, 341
- trade-off parameter, 54
- transpose, *see* matrix
- underdetermined systems, 58
- uniform distribution, 37
- unit solution, 188
- validation, verification, *see* model
- vec (operator), 9
- vector
 - basis, 20
 - definition, 20
 - expansion, eigenvector, 73
 - expansions, 69
 - gradient, 27
 - projection, 21
- vibrating string, 16
- w^* , 287
- wave equation, *see* partial differential equations
- white noise, 40, 47, 145
- whole domain form
 - model, 187
 - observations, 190, 191
- Wiener filters and smoothers, 214, 263
- wind stress
 - as boundary condition, 343