DISCRETE INVERSE AND STATE ESTIMATION PROBLEMS

With Geophysical Fluid Applications

The problems of making inferences about the natural world from noisy observations and imperfect theories occur in almost all scientific disciplines. This book addresses these problems using examples taken from geophysical fluid dynamics. It focuses on discrete formulations, both static and time-varying, known variously as inverse, state estimation or data assimilation problems. Starting with fundamental algebraic and statistical ideas, the book guides the reader through a range of inference tools including the singular value decomposition, Gauss–Markov and minimum variance estimates, Kalman filters and related smoothers, and adjoint (Lagrange multiplier) methods. The final chapters discuss a variety of practical applications to geophysical flow problems.

Discrete Inverse and State Estimation Problems: With Geophysical Fluid Applications is an ideal introduction to the topic for graduate students and researchers in oceanography, meteorology, climate dynamics, geophysical fluid dynamics, and any field in which models are used to interpret observations. It is accessible to a wide scientific audience, as the only prerequisite is an understanding of linear algebra.

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To Walter Munk for decades of friendship and exciting collaboration.

Colour plate section appears between pages 182 and 183, and is also available for download in colour from www. cambridge.org/9781107406063

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Preface

This book is to a large extent the second edition of *The Ocean Circulation Inverse Problem*, but it differs from the original version in a number of ways. While teaching the basic material at MIT and elsewhere over the past ten years, it became clear that it was of interest to many students outside of physical oceanography – the audience for whom the book had been written. The oceanographic material, instead of being a motivating factor, was in practice an obstacle to understanding for students with no oceanic background. In the revision, therefore, I have tried to make the examples more generic and understandable, I hope, to anyone with even rudimentary experience with simple fluid flows.

Also many of the oceanographic applications of the methods, which were still novel and controversial at the time of writing, have become familiar and almost commonplace. The oceanography, now confined to the two last chapters, is thus focussed less on explaining why and how the calculations were done, and more on summarizing what has been accomplished. Furthermore, the time-dependent problem (here called "state estimation" to distinguish it from meteorological practice) has evolved rapidly in the oceanographic community from a hypothetical methodology to one that is clearly practical and in ever-growing use.

The focus is, however, on the basic concepts and not on the practical numerical engineering required to use the ideas on the very large problems encountered with real fluids. Anyone attempting to model the global ocean or atmosphere or equivalent large scale system must confront issues of data storage, code parallelization, truncation errors, grid refinement, and the like. Almost none of these important problems are taken up here. Before constructive approaches to the practical problems can be found, one must understand the fundamental ideas. An analogy is the need to understand the implications of Maxwell's equations for electromagnetic phenomena before one undertakes to build a high fidelity receiver. The effective engineering of an electronic instrument can only be helped by good understanding of how one works in principle, albeit the details of making one work in practice can be quite different.

In the interests of keeping the book as short as possible, I have, however, omitted some of the more interesting theoretical material of the original version, but which readers can find in the wider literature on control theory. It is assumed that the reader has a familiarity at the introductory level with matrices and vectors, although everything is ultimately defined in Chapter 2.

Finally, I have tried to correct the dismaying number of typographical and other errors in the previous book, but have surely introduced others. Reports of errors of any type will be gratefully received.

I thank the students and colleagues who over the years have suggested corrections, modifications, and clarifications. My time and energies have been supported financially by the National Aeronautics and Space Administration, and the National Science Foundation through grants and contracts, as well as by the Massachusetts Institute of Technology through the Cecil and Ida Green Professorship.

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Errata

A correction list compiled by the author can be found here: http://ocean.mit.edu/~cwunsch