

## REVERSIBILITY AND STOCHASTIC NETWORKS

This classic in stochastic network modelling broke new ground when it was published in 1979, and it remains a superb introduction to reversibility and its applications.

The book concerns behaviour in equilibrium of vector stochastic processes or stochastic networks. When a stochastic network is reversible its analysis is greatly simplified, and the first chapter is devoted to a discussion of the concept of reversibility. The rest of the book focuses on the various applications of reversibility and the extent to which the assumption of reversibility can be relaxed without destroying the associated tractability. Now back in print for a new generation, this book makes enjoyable reading for anyone interested in stochastic processes thanks to the author's clear and easy-to-read style. Elementary probability is the only prerequisite and exercises are interspersed throughout.

'the exposition is clear and precise without being pedantic . . . an example of the art of the true applied mathematician, who can penetrate the essence of a real problem by applying the right mathematical tool in just the right place. This book, then, will be essential (and enjoyable) reading for any operational researcher.'

*J. F. C. Kingman*

*European Journal of Operational Research*

'*Reversibility and Stochastic Networks* is simply a timeless classic. Students in operations research, electrical engineering, management science, mathematics, etc. can benefit tremendously by reading the technically deep and elegantly presented material in this book.'

*Nick Bambos, Stanford University*

'Its wealth of ideas is so rich and (as in much of Kelly's work) it starts with very elementary ideas which in his hands are built up until suddenly you have something that is very valuable. . . . It is a book that should be on the shelf of anyone working on stochastic networks.'

*Onno Boxma, EURANDOM and Eindhoven University of Technology*

# REVERSIBILITY AND STOCHASTIC NETWORKS

F. P. KELLY



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## Preface to the reissued edition

Networks have long been natural models in the physical, biological and social sciences, and our modern interconnected society relies increasingly upon our communication and transport networks. In response, there has been an explosion of interest in networks as objects worthy of study in their own right. I am pleasantly surprised by the continuing interest in this book, written to give an elementary introduction to the behaviour in equilibrium of stochastic networks, and I am grateful to Cambridge University Press for their encouragement to reissue the volume.

Frank Kelly

Cambridge, Christmas 2010

## Preface

The main topic of this book is the study of the behaviour in equilibrium of vector stochastic processes, or stochastic networks. Such processes have a wide range of applications: to give some examples, the components of the vector may represent queue sizes in a queueing network, gene frequencies in a population, or the condition of fruit trees in an orchard. When a stochastic network is reversible its analysis is greatly simplified, and the first chapter is devoted to a discussion of the concept of reversibility. Two themes emerge from the remainder of the book: first, the various uses of reversibility, in the study of the output from a queue, the flow of current in a conductor, the age of an allele, or the equilibrium distribution of a polymerization process; second, the extent to which the assumption of reversibility can be relaxed without destroying the associated tractability.

The main prerequisite is an understanding of Markov processes at about the level of Feller's *Introduction to Probability Theory and Its Applications*, Volume I. In Section 1.1 the necessary material is very briefly reviewed, primarily to establish terminology and notation.

For their comments and advice I am indebted to many people, particularly Dave Aldous, Andrew Barbour, Dieter Koenig, Rolf Schassberger, and Geoff Watterson. I am especially grateful to Peter Whittle, whose lectures on reversibility first interested me in the subject and without whose encouragement the book would not have been written. Finally, my thanks go to Jackie Kelly for computing the graphs in the book and to Angie Ashton for typing the final draft.

Cambridge, Christmas 1978

FRANK KELLY

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