

Contents

	<i>Preface</i>	page vii
1	Probability and measure	1
	1.1 Do probabilists need measure theory?	1
	1.2 Continuity of additive set functions	1
	1.3 Independent events	8
	1.4 Simple random walk	10
2	Measures and distribution functions	12
	2.1 σ -finite measures	12
	2.2 Generated σ -fields and π -systems	15
	2.3 Distribution functions	17
3	Measurable functions/random variables	19
	3.1 Measurable real-valued functions	19
	3.2 Lebesgue- and Borel-measurable functions	20
	3.3 Stability properties	21
	3.4 Random variables and independence	25
4	Integration and expectation	30
	4.1 Integrals of positive measurable functions	31
	4.2 The vector space \mathcal{L}^1 of integrable functions	36
	4.3 Riemann v. Lebesgue integrals	40
	4.4 Product measures	42
	4.5 Calculating expectations	44
5	L^p-spaces and conditional expectation	49
	5.1 L^p as a Banach space	49
	5.2 Orthogonal projections in L^2	54
	5.3 Properties of conditional expectation	56

vi	<i>Contents</i>	
6	Discrete-time martingales	60
	6.1 Discrete filtrations and martingales	60
	6.2 The Doob decomposition	63
	6.3 Discrete stochastic integrals	65
	6.4 Doob's inequalities	67
	6.5 Martingale convergence	68
	6.6 The Radon–Nikodym Theorem	74
7	Brownian Motion	78
	7.1 Processes, paths and martingales	78
	7.2 Convergence of scaled random walks	81
	7.3 BM: construction and properties	83
	7.4 Martingale properties of BM	88
	7.5 Variation of BM	92
8	Stochastic integrals	95
	8.1 The Itô integral	95
	8.2 The integral as a martingale	101
	8.3 Itô processes and the Itô formula	104
	8.4 The Black–Scholes model in finance	108
	8.5 Martingale calculus	111
	<i>References</i>	118
	<i>Index</i>	119