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978-1-107-40086-3 - From Measures to Ito Integrals

Ekkehard Kopp

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Preface

Undergraduate mathematics syllabi vary considerably in their coverage of measure-theoretic probability theory, so beginning graduates often find substantial gaps in their background when attending modules in advanced analysis, stochastic processes and applications. This text seeks to fill some of these gaps concisely. The exercises form an integral part of the text. The material arose from my experience of teaching AIMS students between 2004 and 2007, of which I retain many fond memories. The AIMS series format allows few explorations of byways; and the objective of arriving at a reasonably honest but concise account of the Itô integral decided most of the material. With motivation from elementary probability we discuss measures and integrals, leading via L^2 -theory and conditional expectation to discrete martingales and an outline proof of the Radon–Nikodym Theorem. The last two chapters introduce Brownian Motion and Itô integrals, with a brief look at martingale calculus. Here proofs of several key results are only sketched briefly or omitted. The Black–Scholes option pricing model provides the main application. None of the results presented are new; any remaining errors are mine.

Three happy, lively AIMS student cohorts suffered my attempts to introduce them to mathematical finance. My sincere thanks go to them, to the AIMS team for their support, to Alan Beardon for his encouragement, to two helpful reviewers, to Marek Capinski for wise advice and above all to my wife Margaret for her patient, loving support.