

Index

- $(S^k E)_{\text{reg}}$, 177
 $(S_\pi V)_0$, 270
 1-generic tensor, 105
 1_A -generic tensor, 105
 2_A -generic, 114
 $Ch_d(V)$, Chow variety, 48
 $Flat_r^{k,d-k}(S^d V)$, 155
 G -module, 8
 G -module map, 28
 $G(k, V)$, 31
 $G(k, V)$, Grassmannian, 48
 G/P
 homogeneous variety, 95
 $GL(V)$, 8
 G_T , 79
 G_X , group preserving X , 9
 $I(X)$, 9
 $J(C, p)$, 164
 $J(X, L)$, 164
 $MJ(X, Y)$, 209
 M_{poly} , 270
 $N_x^* X$, conormal space, 156
 O , 4
 $PGL(U)$, 80
 $S^d V$, 29
 $S_{\overline{\pi}} V$, 222
 $S_\pi V$, 222
 $Seg(\mathbb{P}A_1 \times \cdots \times \mathbb{P}A_n)$, Segre variety, 47
 $Spec(R)$, 284
 $Sub_k(S^d V)$
 equations of, 233
 $Sub_k(S^n V)$, 154
 $T/\tilde{A}$, 123
 T_{STR} , 60
 $T_{WState,k}$, 138
 T_{WState} , 22
 $T_{q,CW}$, 68
 $T_{q,cw}$, 67
 V^* , dual vector space, 18
 X -border rank, 94
 $X^\vee$, dual variety of X , 165
 $Abel_A$, 106
 $\mathbb{C}[X]$, 52
 $\mathbb{P}V$, 31
 $B_{\rho,\mathbf{a}'}(T)$, 123
 Diag_A , 107
 $\text{Diag}_{\text{End}(B)}$, 107
 $\mathfrak{S}_n$, permutation group, 8
 $\mathfrak{S}_n \wr \mathfrak{S}_d$, 271
 Γ_n^E , 175
 Γ_S , symmetry group of decomposition, 79
 $\text{HilbP}_Z(X)$, Hilbert polynomial of projective
 scheme X , 292
 $\Lambda^d V$, 29
 $M_{(1)}^{\oplus t}$, 56
 $M_{(2)}^{\text{red}}$, 21
 $\Sigma\Pi\Sigma$ circuit, 188
 $\mathbf{R}_S$, 154
 $\mathcal{Q}$, 123
 $\mathcal{S}$, bundle on Grassmannian, 123
 $\mathcal{S}$, rank decomposition, 79
 dc , determinantal complexity, 15
 edc
 equivariant determinantal complexity,
 197
 $\text{esc}(P)$, 198
 $\mathfrak{g}$, 222
 $\mathfrak{gl}(V)$, 222
 $\mathfrak{k}_r(X)$, cactus variety, 291
 $\text{lct}_{X,x}(y)$, local cactus rank, 297
 ω : exponent of matrix multiplication, 5
 $\omega_{\text{prac},k}$, practical exponent of matrix mult.,
 77
 $\overline{\text{dc}}$, border determinantal complexity, 16
 Pad , 234
 π' , 222

- cr_X , cactus rank, 291
- $\sigma_r(X)$, secant variety of X , 94
- $\tau(C)$, tangential variety, 165
- $\text{HilbF}_k(X)$, Hilbert function of projective scheme X , 292
- Proj, 291
- End Abel_A , 107
- $\text{End}_S(V)$, commutator of S in $\text{End}(V)$, 239
- $\text{Zeros}(P)$, 144
- $\text{Zeros}(P)_{\text{Jac},k}$, 158
- $\mathbf{R}_S$, 154
- $\bar{\mathbf{R}}(T)$, 64
- VNP, 148
- VP, 148
- $\mathbf{VP}_e$, 151
- $\wedge$, 30
- $e_{n,N}$ elementary symmetric function, 149
- $k_{\pi,\mu,v}$, Kronecker coefficients, 250
- $l(\pi)$, 222
- α , 4
- s -rank, 76
- $v_d(\mathbb{P}V)$, Veronese variety, 48
- ABP, 191
- abstract secant variety, 95
- affine linear projection, 14
- affine scheme, 284, 290
- affine variety, 47
- algebra, 71
- algebraic branching program, 191
- algebraic Peter-Weyl theorem, 243
- algebraic variety, 10
- apolar ideal, 292
- apolar scheme, 292
- Apolarity Lemma, 292
- areole
 - greater, 126
- arithmetic circuit, 147
- asymptotic rank, 64, 142
- benchmarks, 14
- Bezout's theorem, 293
- big O notation, 4
- bilinear map, 6
- Black-List map, 272
- border determinantal complexity, 16
- border rank, 10, 22, 24
 - symmetric, 154
- Borel subgroup, 128, 223
- Bott-Borel-Weil theorem, 251
- Brill's equations, 279
- Burnside's theorem, 239
- cactus border rank, 291
- cactus rank, 291
- cactus variety, 291
- Cartan Lemma, 37
- catalecticants, 153
- Cauchy formula, 237, 246
- centralizer, 109, 239
- character of representation, 243
- characterized by symmetries, 16
- Chow variety, 48
 - Brill's equations for, 279
 - equations of, 265
- circuit
 - depth of, 183
 - depth three, 188
 - diagonal depth-3, 185
 - shallow, 183
- circuit size, 147
- class function, 243
- combinatorial restriction, 46
- combinatorial value, 66
- commutator, 239
- complete
 - for complexity class, 14
- complete flag, 112
- complete for complexity class, 149
- complete intersection, 211
- completely reducible module, 238
- compressible tensor, 121
- compression generic, 121
- concise polynomial, 154
- concise tensor, 25
- cone, 154, 164, 233
- conjugate partition, 222
- conormal space, 156
- convenient transpose symmetry, 85
- Coppersmith-Winograd tensor, 68
- degeneracy value, 64
- degeneration
 - toric, 62
- degeneration of tensor, 56
- degree
 - of projective scheme, 293
 - of variety, 52
- depth of circuit, 183
- depth three circuit, 188
- depth three powering circuit, 185
- desingularization, 234
- determinant, 14
- determinantal complexity, 15
- determinantal equations, 33
- DFT, 72
- diagonal depth-3 circuit, 185
- dimension
 - of projective scheme, 292
- dimension of variety, 49
- Discrete Fourier Transform, 72
- dual variety, 165
 - reflexivity theorem, 166
- dual vector space, 18

- easy Coppersmith-Winograd tensor, 67
- elementary symmetric function, 149
- equivariant determinantal expression, 197
- equivariant map, 28
- Euclidean closed, 23
- exponent of matrix multiplication, 5
- exterior algebra, 31
- fanin
 - unbounded, 185
- Fekete's Lemma, 65
- finite map, 276
- flattenings, 153
- formula, 151
 - quasi-polynomial, 151
- framed pinning, 81
- gate, 147
- GCT, 15
- GCT useful, 233
- general point, 19
- generalized W-state, 138
- generating function
 - for elementary symmetric polynomials, 150
 - for power sum symmetric polynomials, 150
- Geometric Complexity Theory, 15
- GIT-quotient, 220
- Gorenstein, 137, 157
- Grassmannian, 31, 48
- greater areole, 126
- Grochow-Moore decomposition, 83
- group
 - preserving a set, 9
- group algebra, 71
- hard for complexity class, 149
- hay in a haystack problem, 17
- Hermite-Hadamard-Howe map, 265
- highest weight, 223
- highest weight vector, 223
- Hilbert flattening, 228
- Hilbert function, 211
 - of projective scheme, 292
- Hilbert polynomial, 292
 - of projective scheme, 292
- hitting set, 218
- homogeneous variety, 95
- Howe-Young duality functor, 256
- hypersurface, 144
- ideal
 - of a set, 9
- immanants, 238
- incidence correspondence, 122
- incidence graph, 88
- inheritance, 232
- input
 - to circuit, 147
- integral, 276
- interlace, 225
- invariance under a group action, 9
- invariant
 - under group action, 8
- irreducible group action, 8
- irreducible variety, 19
- isomorphic G -modules, 29
- isotypic component, 29
- Jacobian ideal, 158
- Jacobian variety, 158
- Kempf-Weyman desingularization, 233
- Koszul flattening, 33
 - for polynomial, 227
- Kronecker coefficients, 250
 - symmetric, 250
- Latin square, 275
- leading monomial, 212
- length
 - of zero-dimensional scheme, 290
- lex-segment ideal, 211
- Lie algebra, 222
- lifting map
 - outer degree, 252
- linear map
 - rank, 19
 - rank one, 6
- Littlewood Richardson Rule, 225
- local cactus rank, 297
- local cactus variety, 298
- matrix coefficient basis
 - of group algebra, 72
- matrix coefficients, 242
- maximal torus, 223
- maximally compressible tensor, 134
- maximally symmetric compressible tensor, 134
- membership test, 231
- method of partial derivatives, 153
- module, 8
 - completely reducible, 238
 - semi-simple, 238
 - simple, 238
- module homomorphism, 28
- module map, 28
- monodromy loops, 231
- multisymmetric polynomials, 285
- multilinear depth 3 circuit, 203

- multilinear polynomial, 201
- multiplicative join, 209
- multiplicity
 - of irreducible submodule, 29
- nilradical, 139
- normalization
 - of a curve, 53
- objective function, 93
- obstruction
 - occurrence, 253
 - orbit occurrence, 249
- occurrence obstruction, 253
- orbit closure, 16, 56
- orbit occurrence obstruction, 249
- outer degree lifting map, 252
- padded polynomial, 233
- Pan's decomposition, 82
- parabolic group, 128
- Pascal determinant, 262
- permanent, 12
- permutation group, 8
- Peter-Weyl theorem
 - algebraic, 243
- Pieri formula, 224
- pinning set, 81
- polynomial
 - multilinear, 201
- polynomial reduction, 148
- power sum function, 150
- projective scheme, 291
- projective space, 31
- projective variety, 47
- pullback, 43
- quasi-polynomial size formula, 151
- quotient bundle
 - tautological, 123
- radical of an algebra, 139
- raising operator, 223
- rank
 - of linear map, 6
 - of matrix, 6
 - symmetric, 154
- rank decomposition
 - symmetry group of, 79
- rank of linear map, 19
- rank/border rank ratio, 141
- rational map, 51
- reordering isomorphism, 20
- read once oblivious ABP, 197
- reducible variety, 19
- reduction
 - polynomial, 148
- reflexivity theorem for dual varieties, 166
- regular endomorphism, 109
- regular function
 - on affine variety, 52
- regular map, 52
- regular semi-simple, 109
- regular weight space, 177, 256
- representation
 - of group, 8
- restricted model, 13, 183
- restriction value, 65
- resultant, 173
- Reynolds operator, 200
- ROABP, 197
- scheme, 158
 - zero-dimensional, 290
- Schur's Lemma, 29
- secant variety, 94
 - abstract, 95
- Segre variety, 47
- semi-simple module, 238
- shallow circuit, 183
- Shannon entropy, 205
- simple module, 238
- singular point, 49
 - of scheme, 158
- size
 - of circuit, 147
- skew-symmetric tensor, 29
- smooth point, 49
- standard basis
 - of group algebra, 72
- standard pinning, 81
- Stirling's formula, 205
- Strassen's tensor, 60
- structure tensor of an algebra, 71
- subspace bundle
 - tautological, 123
- subspace variety, 154
- substitution method, 42
- sum-product polynomial, 260
- support of scheme, 297
- symmetric algebra, 31
- symmetric border rank, 154
- symmetric function, 149
- symmetric Kronecker coefficients, 250
- symmetric polynomial, 149
- symmetric product, 30
- symmetric rank, 154
- symmetric subspace variety
 - equations of, 233
- symmetric tensor, 29
- symmetric vector polynomials, 285
- symmetry group
 - of tensor, 79
- syzygies, 211

- tangential variety, 165
- tensor
 - 1-generic, 105
 - 1_A -generic, 105
 - combinatorial restriction, 66
 - compressible, 121
 - compression generic, 121
 - concise, 25
 - degeneration of, 56
 - maximally compressible, 134
 - restricts to, 65
 - skew-symmetric, 29
 - symmetric, 29
 - symmetry group of, 79
- tensor algebra, 31
- tensor product, 20
- tensor rank, 3, 6, 7
- Terracini's lemma, 94
- toric degeneration, 62
- torus
 - maximal, 223
- trace test, 231
- transpose-like invariant decomposition, 88
- triple product property, 74
- unbalanced dimensions, 24
- unit tensor, 56
- value
 - combinatorial, 66
 - degeneracy, 64
 - restriction, 65
- variety
 - algebraic, 10
 - dimension of, 49
 - homogeneous, 95
 - irreducible, 19
 - of padded polynomials, 234
- vector bundle, 123
- Veronese variety, 48
- volume
 - of tensor, 61
- W-state, 22
- Waring decomposition, 80
- wedge product, 30
- weight space
 - regular, 177
- weight vector, 223
- weight, highest, 223
- wreath product, 271
- Young diagram, 222
- Young flattening, 227
- Zariski closed, 10, 23
- Zariski closure, 10