

## 1

## Basics

The standard model of particle physics represents our deepest knowledge about what the world is made of. But this theory is far from unique and does not explain why the proton, neutron and electron masses are so finely tuned.

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## 1.1 NUCLEAR SIZE AND CONSTITUENTS

It is now well known that atoms of chemical elements consist of a nucleus surrounded by a cloud of electrons. Although it is generally appreciated that an atom is a very small entity, quantitative figures like  $\sim 10^{-10}$  m for its size are not easy to grasp. In Science, very large and small decimal numbers are conveniently expressed in terms of powers of 10, known as scientific notation. Some examples, spanning 36 orders of magnitude are given in the Table 1.1. Prefixes like milli, micro, and kilo may be familiar. It is useful to become familiar with the meaning of other prefixes. The drawback of scientific notation is that it desensitizes us to the enormous range of numbers that the scale of the universe demands. For example, the famous scientist Rutherford did an elegant and simple experiment to show that the size of the nucleus is not comparable to that of the atom, but about five orders of magnitude smaller:  $\sim 10^{-15}$  m. The difference in size given in scientific notation may not impress us until we realize that if the atom is magnified to the size of a cathedral, the nucleus will be no bigger than a housefly.

**Table 1.1** Decimal Fractions and Multiples

Fraction	Prefix	Symbol	Multiple	Prefix	Symbol
$10^{-3}$	milli	m	$10^3$	kilo	k
$10^{-6}$	micro	$\mu$	$10^6$	mega	M
$10^{-9}$	nano	n	$10^9$	giga	G
$10^{-12}$	pico	p	$10^{12}$	tera	T
$10^{-15}$	femto	f	$10^{15}$	peta	P
$10^{-18}$	atto	a	$10^{18}$	exa	E

The nucleus consists of two distinct particles: protons with an electrical charge (conventionally taken as positive), and neutrons with no net electrical charge. Protons and neutrons are collectively known as nucleons. A combination of protons and neutrons, stable or otherwise, is called, in general, a nuclide. Electrons have an equal and opposite electrical charge (taken as negative) to protons. In an electrically neutral atom, the number of protons in the nucleus is equal to that of electrons orbiting it. This is known as the proton number when referring to the nucleus, and atomic number ( $Z$ ) when referring to the atom as a whole. If an atom loses an electron, it will have a net positive charge and is called a positive ion, or cation; if it gains an electron, it will have a net negative charge and is called a negative ion, or anion.

## 1.2 FUNDAMENTAL FORCES

Electrons in an atom are bound to their nucleus by the electromagnetic force, which is attractive between unlike charges and repulsive between like charges. This raises the question as to how the protons are bound within the small nuclear volume despite their mutual repulsion. The answer is that a much stronger force, known as the strong or nuclear force, acts between nucleons. But as it is effective only within nuclear dimensions, two protons must first be brought close together against their electrostatic, or Coulomb repulsion, before the strong attractive force can hold them together. There is a third force, the weak nuclear force, with an effective range even smaller than that of the strong nuclear force. The only other force known in nature is the familiar gravitational force, which is many orders of magnitude weaker than the other three. Table 1.2 shows the differences in the strength and range of the four forces. The strong and weak forces do not extend beyond nuclear dimensions, whereas, the electromagnetic and gravitational forces reach out to the farthest corners of the universe. The four known forces separated and became individually distinct during the earliest moments of the universe (Chapter 3).

It was noted above that the strong force between protons balances their mutual electrostatic repulsion within nuclear dimensions. It should, therefore, be possible to form nuclei containing only protons. But all elements heavier than the lightest hydrogen contain protons and neutrons, in comparable number, in their nucleus. This implies that a combination of protons alone is not stable, and the pure attractive short-range nuclear force between protons and neutrons is additionally required for nuclear stability. It is also found that the number of neutrons for a given number of protons can vary slightly. Nuclides with the same number of protons, but different number of neutrons are called isotopes. The sum of the proton number ( $Z$ ) and neutron number ( $N$ ) is called mass number ( $A$ ). Nuclides with the same number of neutrons, but different number of protons are called isotones, and nuclides with the same number of nucleons ( $A$ ) are called isobars.

The symbol of an atom is just its chemical symbol without any explicit reference to its atomic number ( $Z$ ) and mass number ( $A$ ). For example, Cl for chlorine and Sm for samarium. The symbol for an isotope must necessarily include its mass number, usually as the left superscript. For example,  $^{35}\text{Cl}$  and  $^{147}\text{Sm}$ . The atomic number is understood from the chemical symbol. The use of left superscript to designate the mass number avoids the confusion with exponents, and retains the option to define the right superscript and subscript, as needed.

**Table 1.2** Characteristics of the Four Fundamental Forces

Force	Source	Strength	Range (cm)
<b>Strong</b>	Baryons, mesons	1	$10^{-13}$
<b>Electromagnetic</b>	Photons	$10^{-2}$	Infinity
<b>Weak</b>	Leptons, mesons	$10^{-5}$	$10^{-15}$
<b>Gravitational</b>	Mass/energy	$10^{-40}$	Infinity

### 1.3 NUCLEAR MASS

Like their sizes, masses of atoms are also extremely small: about 27 orders of magnitude smaller than the standard mass unit, the kilogram. As masses of individual atoms figure prominently in nuclear, atomic, and molecular physics, they are measured in a more convenient small mass unit, called the atomic mass unit (amu) or simply the unified mass unit (u). It is defined as a twelfth of the mass of the  $^{12}\text{C}$  atom, and is equal to  $1.6605389 \times 10^{-27}$  kg.

Mass of macroscopic objects is measured with analytical balances on the basis of the gravitational force on such objects. The smallest mass that can be determined in sophisticated analytical balances is about one microgram ( $10^{-9}$  kg), which is much greater than that of individual atoms. The analytical instrument commonly used to measure atomic masses is called a mass spectrometer (Chapter 6), which is based on the electromagnetic force (that is very much stronger than the gravitational force, but follows the same inverse square dependence on distance) on moving electrically charged atoms (ions). Mass spectrometers measure atomic masses relative to that of  $^{12}\text{C}$  atom (12 u). The values of atomic masses reported in the literature are not nuclear, but isotopic masses; they include the mass of the extra nuclear electrons in neutral atoms. This convention has some advantages in the treatment of nuclear reactions and energy relations. Atomic masses have now been determined with precisions varying between one and 10 parts per million; a few examples are given in the Table 1.3.

**Table 1.3** Nucleon and Nuclear Masses

Particle	Symbol	Mass (u)
<b>Proton</b>	p	1.007276
<b>Neutron</b>	n	1.008665
<b>Electron</b>	$e^-$	0.000549
<b>Hydrogen</b>	$^1\text{H}$	1.007825
<b>Helium</b>	$^4\text{He}$	4.001475
<b>Carbon</b>	$^{12}\text{C}$	12.00000

### 1.4 EQUIVALENCE OF MASS AND ENERGY

One of the important consequences of Einstein's Special Theory of Relativity is the equivalence of mass and energy. The total energy content  $E$  of a system of mass  $M$  is given by the relation,  $E = Mc^2$ , where  $c$  is the velocity of light in vacuum ( $2.99976 \times 10^{10}$  cm.s $^{-1}$ ). The energy equivalent of 1u is:

$$E = 1.661 \times 10^{-27} \text{ Kg} \times (2.999 \times 10^{10} \text{ cm})^2 = 1.493 \times 10^{-23} \text{ erg}$$

An energy unit much more convenient in nuclear work than the erg is the electron volt (eV), the kilo electron volt (KeV), and the million electron volt (MeV). The electron volt is defined as the energy necessary to raise one electron through a potential difference of one volt, akin to the increase

of potential energy of a mass when raised in the gravitational field of the earth.  $1 \text{ eV} = 1.662 \times 10^{-12}$  erg, which gives  $1 \text{ u} = 931 \text{ MeV}$ . MeV is a large energy in the nuclear scale, but extremely small in the human scale. The kinetic energy of a small ant moving at a speed of a few mm per second is a few thousand MeVs.

**Table 1.4** Mass-energy Equivalence of Protons, Neutrons, and Electrons

Particle	Mass (u)	Energy (MeV)	Electric Charge (Coulomb)
Proton	1.007276	938.3	$+1.60 \times 10^{-19}$
Neutron	1.008665	939.6	0
Electron	0.000549	0.51	$-1.60 \times 10^{-19}$

Table 1.4 shows that almost the whole mass of an atom is concentrated in its nucleus even though it accounts for only a trillionth of the total volume of the atom. The extra nuclear electrons contribute very little to the total mass, but represent the volume of the atom.

## 1.5 PERIODIC TABLE

When Mendeleev developed his famous periodic table of elements in 1871, the structure of atoms was not known. So he organized the elements known in his time according to their atomic mass. It is now known that the chemical properties of elements depend very little on the composition, and, hence, the mass of their nuclei, but mainly on the configuration of the extra-nuclear electrons. The modern periodic table is based on the electronic structure described by quantum mechanics. According to quantum mechanical principles, each electron in a multi-electron atom is characterized by a unique set of four quantum numbers, namely principal, azimuthal, magnetic, and spin quantum numbers. The meaning and role of each quantum number is given in many text books (Aitkins, 1986). For our purpose, it is sufficient to consider that electrons are organized in ‘shells’. The shells can be grouped in broad categories called the first shell (also known as the K shell), the second (or the L shell) and so on. They are given numbers 1, 2, 3, 4, etc. Each such major shell contains subshells: only one subshell ( $1s$ ) in the first shell, two subshells ( $2s$ ,  $2p$ ) in the second, three subshells ( $3s$ ,  $3p$ ,  $3d$ ) in the third shell, four subshells ( $4s$ ,  $4p$ ,  $4d$ ,  $4f$ ) in the fourth, and so on. There is a limit to the maximum number of electrons that can be accommodated in any one of the subshells: 2 in  $s$ -type, 6 in  $p$ -type, 10 in  $d$ -type, and 14 in  $f$ -type. The energy levels of the subshells vary. The order of increasing energy for the subshells or orbitals is shown in Figure 1.1.

The diagram in Figure 1.1 is qualitatively correct for almost every neutral atom, and can be used to find the electron configuration of all, but a few elements in the periodic table shown in Figure 1.2. The modern periodic table in its long form consists of 18 vertical sequences of elements known as ‘groups’ and horizontal sequences called ‘periods’ (Mahan, 1975). Each period starts with an element which has one electron in an  $s$ -orbital. The first period is only two elements (H and He) long, since the  $1s$ -orbital or subshell can accommodate only two electrons. The third electron in lithium (Li) must enter the  $2s$ -orbital, and the second period begins. Since the  $2s$  and  $2p$  orbitals or subshells of the second shell can accommodate a total of eight ( $2+6$ ) electrons, eight elements enter the table before the  $2s$  and  $2p$  orbitals are filled in the element neon. The third period is also eight elements long and ends when the  $3s$  and  $3p$  orbitals or subshells are filled in argon.



six electrons in krypton. In the fifth period, the  $5s$ -,  $4p$ - and  $5p$ -orbitals are filled in succession, as in the fourth period. The sixth period is different in that after the  $6s$ -orbital is filled, and one  $5d$ -electron enters to form the element lanthanum, the  $4f$ -orbitals are the next available in order of increasing energy. Since the  $f$ -orbitals can accommodate a total of 14 electrons, 14 elements enter the table before any of the remaining  $5d$ -orbitals can be filled in. After the 14 so-called rare earth elements have entered the table, the last set of transition metals appears as the  $5d$ -orbitals are occupied. These in turn are followed by six elements required to fill the  $6p$ -orbitals, and the sixth period ends with radon. The seventh period starts by filling the  $7s$ -orbital and after one  $6d$ -electron appears, subsequent electrons enter the  $5f$ -orbitals. Thus, the periodic table ends with the actinide series, a group of 14 elements analogous in properties and electronic structure to the rare earths. To keep the periodic table from becoming excessively long, the elements which follow lanthanum and 14 elements after actinides are placed in separate rows at the bottom of the table. Figure 1.2 also shows that the elements in which  $s$ -,  $p$ -,  $d$ -, and  $f$ -orbitals are being filled are grouped naturally in the long form of the table. The eight families of the  $s$  and  $p$  blocks are often called representative elements and those in the  $d$  block are called transition elements, while members of the  $f$  block are known as the inner transition elements.

Group 18 elements are the so called noble gases. Their unreactive nature is due to the high stability arising from full  $s$ - and  $p$ -orbitals in their outermost shell. Group 1 elements have a single valence electron outside the noble gas configuration and exhibit metallic properties. Group 17 elements have one electron short of noble gas configuration and precede the noble gases. They exhibit nonmetallic properties (halogens). Elements in groups from 2 to 17 show progressive gradation of properties between the two extreme metallic and non-metallic groups.

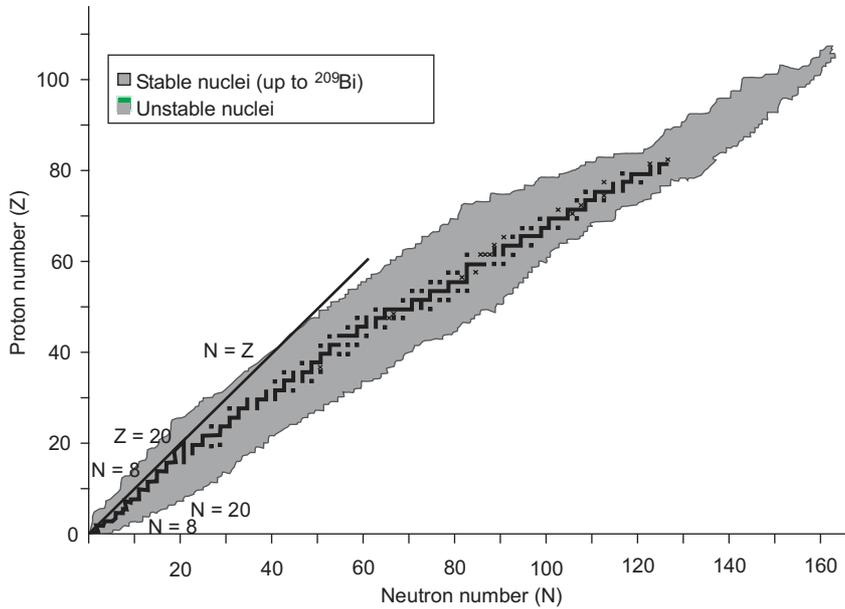
The elements in the same column of the periodic table have, for the most part, the same configuration for their valence (outermost) electrons, and, as is well known, they have similar chemical properties. Horizontal ‘chemical similarity’ also exists, such as among the rare earth and transition metals. These elements, which are chemically similar, differ only by the number of electrons in a particular type of orbital, such as  $3d$  and  $4f$ . In addition to these general correlations between electron configurations and chemical properties, there are many more detailed correlations which are of direct interest to chemists and geochemists.

The periodic table shows how the electronic structure of elements almost exclusively governs their chemical properties. But electrons can be configured only around bare nuclides with a stable or metastable combination of protons and neutrons. The elements technetium (Tc) ( $Z = 43$ ) and promethium (Pm) ( $Z = 61$ ), and transuranic elements ( $Z = 93$  and above) do not occur naturally at present on the earth. This implies the absence of nuclides with these proton numbers. When bare transuranic nuclides were artificially produced, they readily attracted the appropriate number of electrons from the surrounding to form the remaining elements of the actinide group. As each isotope of a natural element requires a specific nuclide composition, the number of different nuclides to form the known 90 chemical elements is much more, about 265.

## 1.6 NUCLEAR COMPOSITION AND STABILITY

The distribution of protons and neutrons in the stable, or nearly stable, 265 nuclides is shown in Figure 1.3, known as the chart of nuclides (Langmuir and Broecker, 2012). In this chart horizontal

rows of nuclides have the same  $Z$ , but different  $N$  to represent the different isotopes of a single element. Vertical lines of nuclides have the same  $N$ , but different  $Z$  known as isotones. Diagonal lines contain nuclides with the same nucleon number, known as isobars. The 265 stable nuclides conform to a narrow band across the chart, called, appropriately, the path of stability. Nuclides with low atomic numbers have approximately equal numbers of protons and neutrons, while heavier nuclides have many more neutrons than protons to stabilize them against the increasing electrostatic repulsion between protons confined to the small nuclear volume. The heaviest stable nuclide has  $Z = 83$  and  $N = 126$  corresponding to the isotope  $^{209}\text{Bi}$  of bismuth. Nuclei heavier than  $^{209}\text{Bi}$  are not stable with time and undergo radioactive decay (Chapter 2), but nuclides with  $Z = 90$  (thorium) and  $Z = 92$  (uranium), respectively, have extremely long lifetimes (Chapter 3) to survive to the present day.



**Figure 1.3** The chart of the nuclides. Stable nuclides (the band of stability) are indicated by the solid black boxes. Radioactive nuclides that decay back to the band of stability at variable rates are indicated by the gray field (Langmuir and Broecker, 2012)

A nuclear origin for the relative stability among the 265 nuclides is indicated by the far higher number of nuclides with even  $Z$  and  $N$ , as given in Table 1.5.

**Table 1.5** Distribution of Stable Nuclides with Odd/Even  $A$ ,  $Z$ , and  $N$

$A$	$Z$	$N$	Stable Nuclides	Long-lived Nuclides
Even	Even	Even	161	11
Odd	Even	Odd	55	3
Odd	Odd	Even	50	3
Even	Odd	Odd	4	5

More than half (160) of the stable nuclides have even  $Z$  and even  $N$ . Stable nuclides with even  $Z$  and odd  $N$ , or vice versa are much less common ( $\sim 50$ ). In great contrast, stable nuclides with both  $Z$  and  $N$  odd are extremely rare (4), namely  ${}^2\text{H}$  of hydrogen,  ${}^6\text{Li}$ ,  ${}^{10}\text{B}$  of boron, and  ${}^{14}\text{N}$  of nitrogen. Why are nuclides with even  $Z$  and even  $N$  more stable and hence abundant, as shown above? Whenever nucleons, either protons or neutrons, can pair with their spins in opposite directions, the forces holding them seem to be stronger and the nuclide more stable. This is very similar to the stability of atoms with paired electrons in each orbital. In fact, similar to the much higher stability (chemical inertness) of noble gas atoms with filled orbitals, nuclides with particular numbers (2, 8, 20, 28, 50, 82, 126) of either  $Z$  or  $N$  are exceptionally stable. These are called magic numbers.

A very large number of nuclides ( $>2,000$ ) have been artificially produced in laboratories. All these nuclides fall away on either side of, or beyond,  ${}^{209}\text{Bi}$  along the path of stability, and invariably are unstable. Given sufficient time, all unstable nuclides (both natural and artificial) will transform or decay in one or more steps to stable nuclides in the path of stability. This spontaneous time-dependent process is called radioactivity (Chapter 2), and the unstable nuclides are called radioactive nuclides, radionuclides, or radioisotopes.

The presence, at present, of a few long-lived radionuclides, like  ${}^{238}\text{U}$  of uranium, in nature suggests that some of the relatively short-lived nuclides (on either side of the path of stability) might also have been produced with them at some time in the past, but have all decayed to extinction since. The present occurrence of some of these short-lived nuclides at present must then be due to their continuous production, either from the series or chain decay of long-lived nuclides, or nuclear reactions due to cosmic rays or their secondary particles. The still surviving long-lived nuclides, short-lived nuclides supported by the former, and a few of the short-lived nuclides that have become extinct in materials available to us at present form the basis of the extensive field of radiogenic isotope geology.

## 1.7 NUCLEAR BINDING ENERGY

The general principle that the lowest energy configuration of an isolated system (large or small) is the most stable accounts for the stability of some combinations of nucleons relative to a system of the same number of free or separate nucleons. We can get some insight into the energy with which nucleons are bound to a nucleus like helium (with two protons and two neutrons) as follows; Nuclear Binding Energy (BE) is the energy required to separate a nuclide into its constituent nucleons and, hence, a measure of its stability.

Mass of  ${}^4\text{He}$ : 4.0026 u

Mass of  $2p + 2n + 2e$ :  $2.04652 + 2.01733 + 0.001098 = 4.03298$  u

Mass difference (mass defect) = 0.00304 u or 28.3 MeV

Binding Energy (BE) per nucleon =  $28.3/4 = 7.1$  MeV

The BE per nucleon vs mass number is shown in Figure 1.4. The distinct peak at  ${}^4\text{He}$  of helium (7.1 MeV) represents a very stable nuclear package. The BE increases rapidly from  $\sim 1$  MeV for low masses to a maximum of 8.7 MeV for  $A \sim 60$ , and decreases smoothly, thereafter. The BE

is insufficient to stabilize nuclei heavier than  $^{209}\text{Bi}$ . They decay spontaneously to rearrange their nucleon composition.

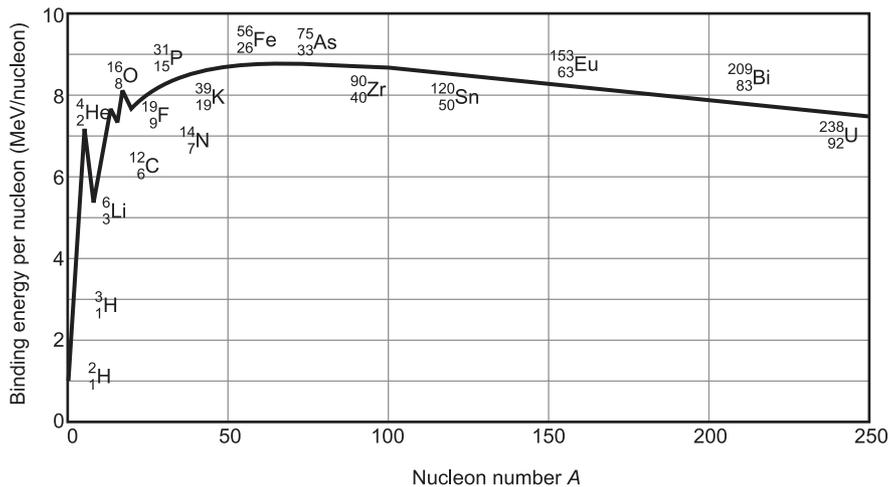


Figure 1.4 Binding energy/nucleon vs mass number diagram

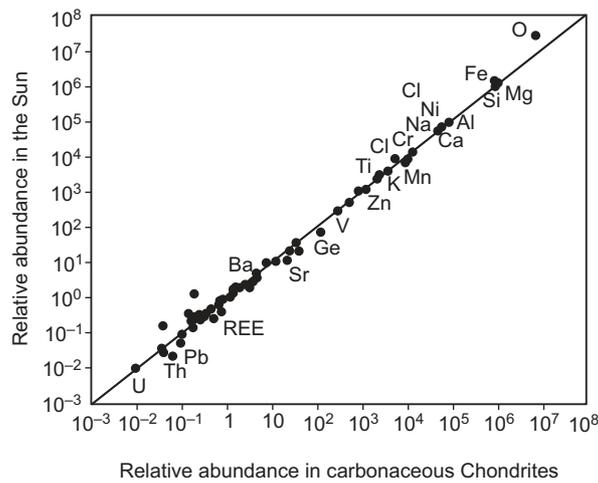
We have so far seen that naturally occurring elements can be systematized, both in terms of their chemical properties (periodic table) and their nuclear attributes (chart of nuclides). We have not considered how, where, and when they were produced in the first place and their relative abundances in the pristine matter from which the solar system formed. We will consider the latter first and defer the former to a later chapter.

## 1.8 COSMIC ABUNDANCES

Our solar system is believed to have resulted from the gravitational collapse of a large cloud of gas and fine dust from the interstellar medium. As the sun is the bulk repository of this cloud of matter, its composition must be representative of that of the parent cloud. The white light emitted by the sun is absorbed at characteristic wavelengths by the chemical elements in its outer atmosphere (photosphere) in proportion to their relative abundances. Spectroscopic measurements of such absorption bands in the otherwise continuous solar spectrum can be converted to relative elemental abundances, but not with high precision and accuracy. Elemental abundances of distant stars in other galaxies are also obtained by this remote sensing method. By convention, the relative abundance of an element is stated as the number of atoms of that element for each one million atoms of silicon (Si). A tabulation of solar abundances is given by Ross and Aller (1976). Relative abundances in the photosphere (surficial part of the sun) are taken as representative of the embryonic sun, as nuclear reactions taking place in the core of the sun (Chapter 3) are believed not to affect the original photospheric composition.

A tangible sample of the pristine solar system material can be directly and precisely analyzed in a laboratory for relative abundances of both elements and their isotopes. Tangible samples from the surficial parts of the earth and even the moon can hardly be representative of the whole earth or the

moon, much less of the primitive solar system. Pieces (large and small) of stones and iron (Fe) alloy have been falling from the sky from historical times. They were recognized to be of extraterrestrial origin only in the late nineteenth century. These are called meteorites and are described in some detail in Chapter 8 on the earliest history of the solar system. It is sufficient at this stage to note that the subgroup of meteorites, called the carbonaceous chondrites contain nonbiotic organic compounds, hydrous silicate minerals, and high temperature silicate minerals. They are thought to be the most primitive tangible or condensed solar system matter available for direct analysis in the laboratory.



**Figure 1.5** Comparison of solar and chondritic abundances with Si set at  $10^6$   
 (Data from Anders and Grevesse, 1989)

A comparison of elemental abundances relative to  $10^6$  atoms of Si between primitive carbonaceous chondrites and solar atmosphere is shown in Figure 1.5 (Anders and Grevesse, 1989). Note the logarithmic scale to cover many orders of magnitude difference in elemental abundances. A perfect agreement would be indicated by a tight alignment of the data to the diagonal line shown. The figure shows a good agreement for most elements, both abundant, like oxygen (O), magnesium (Mg), and Fe, and rare like thulium (Tm) and thorium (Th). The correspondence may be even better than indicated in view of the larger uncertainties in measuring the solar composition by optical spectroscopy. Significant exceptions are the relative depletion of Li and B in the solar photosphere and carbon (C) and N in carbonaceous chondrites, most likely due their volatility. As Li, B, and beryllium (Be) are systematically destroyed in the Sun, chondrites may record the composition of the ancient Sun even better than the present day Sun does. Other important inferences from this figure are:

1. Meteorites, particularly carbonaceous chondrites are extremely valuable links between cosmochemistry (strictly speaking solar system chemistry) and geochemistry.
2. The meteorites, more than the Sun, show that the pristine solar system material already contained all the 90 elements in the periodic table.
3. Similarity in the relative proportions of H and He in the solar photosphere with other stars indicate little mixing with the helium-enriched and hydrogen-depleted core of the sun (Chapter 3).