Modern General Relativity

Black Holes, Gravitational Waves, and Cosmology

Einstein's general theory of relativity is widely considered to be one of the most elegant and successful scientific theories ever developed, and it is increasingly being taught in a simplified form at advanced undergraduate level within both physics and mathematics departments. Due to the increasing interest in gravitational physics, in both the academic and the public sphere, driven largely by widely-publicised developments such as the recent observations of gravitational waves, general relativity is also one of the most popular scientific topics pursued through self-study. *Modern General Relativity* introduces the reader to the general theory of relativity using an example-based approach, before describing some of its most important applications in cosmology and astrophysics, such as gamma-ray bursts, neutron stars, black holes, and gravitational waves. With hundreds of worked examples, explanatory boxes, and end-of-chapter problems, this textbook provides a solid foundation for understanding one of the towering achievements of twentieth-century physics.

Mike Guidry is Professor of Physics and Astronomy at the University of Tennessee, Knoxville. His current research is focused on the development of new algorithms to solve large sets of differential equations, and applications of Lie algebras to strongly-correlated electronic systems. He has written five textbooks and authored more than 120 journal publications on a broad variety of topics. He previously held the role of Lead Technology Developer for several major college textbooks in introductory physics, astronomy, biology, genetics, and microbiology. He has won multiple teaching awards and is responsible for a variety of important science outreach initiatives.

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For
Jo Ann

Brief Contents

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Preface

This book contains material used in an astronomy course on general relativity, black holes, gravitational waves, and cosmology that I teach regularly at the University of Tennessee for advanced undergraduates and beginning graduate students. The goal of the course and of the book is to provide an introduction that is topically current and accessible to a reader with some physics but minimal astronomy, astrophysics, and advanced mathematics background.

The reader is expected to have physics experience commensurate with that of a third or fourth year undergraduate physics major in a US university, and to be familiar with the material typically covered in an introductory descriptive course in astronomy. Readers are assumed to be conversant with special relativity and quantum mechanics only at the level typically covered in first or second year introductions to modern physics. Mathematically I assume the reader to be familiar with basic algebra, geometry, calculus, and differential equations, and the rudiments of matrices. I introduce sufficient differential geometry and tensor calculus to understand the topics to be covered as an integral part of the presentation. Given the target audience I opt for practicality over beauty, emphasizing the physicist's utilitarian "engineering" approach to these topics. However, I try to provide at various places a glimpse of the more elegant but abstract formulation that mathematicians would favor. Specifically, I have tried to present tensors both from the mathematical point of view as maps from vectors to the real numbers, and from the practical physics perspective in terms of transformation properties.

Our approach will be to build from the familiar to the unfamiliar, introducing central concepts first in euclidean (flat) space in two and three dimensions, where familiar mathematics will be cast in a form more useful for what follows. Then these ideas will be extended to still-flat but now 4-dimensional "pseudo-euclidean" spacetime, deriving in the process the theory of special relativity. Finally these concepts will be extended to 4-dimensional curved spacetime and, with the aid of the equivalence principle, used to formulate the theory of general relativity in terms of the Einstein field equations. With these tools in hand, some of the most interesting problems in astrophysics and cosmology will be addressed: spherical and rotating black holes, quantum black holes, the modern view of cosmology, dark matter, dark energy, and gravitational waves.

To aid in comprehension, a number of worked examples and supplementary information boxes are scattered throughout each chapter. These serve two general functions: to illustrate how to do some essential tasks, like conversion between geometrized ($G = c = 1$) units and normal units, or to set in context and provide broader perspective, like a concise overview of group theory as a mathematical and conceptual framework for dealing with symmetries.

A total of 273 problems of varying complexity and difficulty may be found at the ends of the chapters, each chosen to familiarize the reader with basic concepts, illustrate important points, fill in details, or prove assertions made in the text. Where appropriate, when teaching this material I encourage the use of programming tools such as MatLab, Mathematica, or Maple, or more formal programming languages like C/C++ or Java in solving problems. While very helpful, none of these tools is essential for working the problems. The solutions for all 273 problems are available at www.cambridge.org/guidryGR as a PDF file in typeset book format for instructors, and a subset of 107 problem solutions is available to students in the same format. Those problems with solutions available to students are marked by the symbol *** at the end of the problem.

A book dealing with astrophysics at an intermediate level must make a no-win decision concerning units. It is desirable to standardize units and in introductory astronomy it makes some sense to use the SI (MKS) system of units. However, professionals in the field routinely employ the CGS (centimeter–gram–second) system, or natural units that are defined such that fundamental constants like the speed of light, gravitational constant, Planck's constant, or Boltzmann constant take the value of one. Since one of the purposes of the present material is to address the significance of general relativity for cutting-edge research in astronomy and astrophysics, and to encourage students to use and explore the corresponding literature, I have adopted a policy of generally using the CGS system, or natural units.

Many papers referenced in this book are published in journals with limited free public access. To help ensure broad availability to these references for readers I have included for journal articles information allowing free access through the preprint server *arXiv* or the *ADS Astronomy Abstract Service*. More details may be found at the beginning of the References in Ref. [1], and instructions for using *arXiv* and *ADS* to retrieve articles are given in Appendix D.

Let me make some comments on my approach to teaching from this material. There is too much information to cover in full depth in a one-semester course, so some choices have to be made if only a single semester is available. I will suggest several possibilities that may be useful in guiding that choice. To be definite I will assume a traditional lecture format but my remarks should be adaptable to other teaching modes as well.

Survey track: This is my usual approach, where I try to cover most of the material, but with some only outlined, or assigned as reading and self-study. I would typically have the students read Chapters 1 and 2 as introduction, cover Chapters 3–9 in some depth since they provide the essential foundation, assign Chapter 10 as self-study, cover Chapters 10–13 in lecture, and lecture on only selected examples in Chapters 14–15. I would then assign Chapter 16 as self-study since it is introductory, cover Chapters 17–24 in as much depth as time permits, and assign Chapters 25–26 as reading.

Black hole track: Instructors wishing to emphasize black holes can assign Chapter 1 as reading, cover Chapter 2 or assign it as reading, depending on the background of the class, cover Chapters 3–13 in some depth, cover selected topics in Chapters 14–15 as evidence for the existence of black holes and their efficacy as power sources, and conclude with a brief introduction to gravitational waves with emphasis on black holes: Chapter 24 (with Chapters 22–23 as introduction if time permits).

Gravitational wave track: A course emphasizing gravitational waves can be constructed by assigning Chapters 1 and 2 as reading, covering in depth Chapters 3–13 as a background in general relativity, black holes, and neutron stars, and then concluding with gravitational waves from merging black holes and neutron stars in Chapters 22–24, and the discussion of tests of general relativity – particularly in the strong-field limit – in Chapter 25.

Cosmology track: Instructors wishing to emphasize cosmology can assign Chapter 1 as reading, cover Chapter 2 or assign it as reading, depending on the background of the class, cover Chapters 3–9 in some depth to provide essential foundation, cover Chapters 16–21 in depth, and finish with, or assign as reading, Chapter 26. Those wishing to at least mention gravitational waves can add Chapter 24, since it is relatively self-contained and highly observationally oriented.

Tuning a more-mathematical flavor: All of the problems at the ends of chapters have solutions available (often with some explanatory detail), and some of those problems cover material in more technical depth than the text itself. Therefore, it is possible to tune any of the above tracks in a more mathematical direction by incorporating into the classroom more-technical material from the exercise solutions.

For those wishing to teach from this book, several additional resources are available from the publisher at www.cambridge.org/guidryGR for instructors and for students:

1. *Instructor Solutions Manual for Modern General Relativity,* which is a PDF file typeset in the format of the book that presents the solutions for all 273 problems at the ends of chapters. This manual is available only to instructors.
2. *Student Solutions Manual for Modern General Relativity,* which is a PDF file typeset in the format of the book that contains the solutions for a subset of 107 of the 273 problems at the ends of chapters. This manual is available to students and instructors. As noted above, the problems contained in this solutions manual for students are marked by *** at the end of the problem in the text.
3. *Modern General Relativity Lecture Notes,* which presents a synopsis in PDF format appropriate for projection and presentation of the essential material in each chapter. Individual slides are organized in a presentation format suitable for teaching, with text formatted in larger fonts and in color. These are the slides that I use myself when teaching this material.

We conclude this list by noting that the inclusion of DOI or *arXiv* numbers for all journal references – which allows easy browser access through *arXiv* and *ADS* for most articles (see Appendix D) – may be viewed as an additional resource permitting creative literature-based projects to be assigned with minimal bother, if an instructor is so inclined.

Finally, I would like to extend my thanks to the many students and colleagues whose questions and comments sharpened this presentation, to Nicholas Gibbons, Ilaria Tassistro, Jon Billam, and Dominic Stock at Cambridge University Press for all their help in shepherding this book to publication, and especially to my wife Jo Ann for her patience and support over many years.