PART I

STELLAR STRUCTURE

1 Some Properties of Stars

The fundamental building blocks of visible matter in the Universe are stars.[1] This chapter will discuss some of the basic properties of stars such as luminosity, radius, mass, color, and temperature as a prelude to a more detailed exploration of their structure and evolution. Let's begin with a discussion of their most obvious characteristic: that they are visible in our sky, and that even casual observation indicates that there is a substantial variation in brightness between different stars.

1.1 Luminosities and Magnitudes

The apparent brightness of a star is a combination of an intrinsic brightness, which is related to the internal structure of the star, and the effect of distance, since the intensity falls off as the square of the distance. To make much headway in understanding stars these two factors must be separated. This requires a direct or indirect measurement of the distance to the star, or comparison of stars that are known to be at equivalent distances (even if the distance itself is not known). Measuring the distance to stars is difficult and can be accomplished directly only for more nearby stars. The effect of the distance scale can be factored out if stars are compared that are members of physical (gravitationally bound) groupings called *clusters,* which come in two types: *open or galactic clusters* containing tens to hundreds of stars that are found preferentially in the plane of the galaxy, and *globular clusters* containing as many as hundreds of thousands of stars that are found preferentially in the galactic halo. Comparison of stars in a cluster makes it certain that they lie at almost the same distance. From the variation in brightness for stars in clusters, it is found that stellar luminosities L vary over some 10 orders of magnitude, $10^{-4}L_\odot < L < 10^6 L_\odot$, where $L_\odot$ represents the luminosity of the Sun.

1.1.1 Stellar Luminosities

A flux is defined to be the amount of energy crossing a unit surface area per unit time. The luminosity L of a star is the power required to sustain the total energy flux across a

[1] Chapters 1 and 2 review material normally covered in introductory astronomy courses. For readers without an introductory astronomy background they serve as an overview of concepts that will be important for later discussion. These chapters may be skipped if you are familiar with the basic properties of stars and with the relationship of luminosity to surface temperature for stars captured in the Hertzsprung–Russell diagram.

Box 1.1 Ejection of Mass by Stars

The presence of the solar wind in our own Solar System suggests that all stars lose at least some mass continuously. However, many stars appear to have periods of very large mass loss early in their lives (*T Tauri winds* from stars just settling to the main sequence and strong mass flows from young, massive main sequence stars) and late in their lives (*red giant winds*, *planetary nebula*, and related phenomena). In addition, explosions such as novae and supernovae associated with dead and dying stars eject mass into interstellar space, sometimes in large amounts.

Systematics of white dwarf populations give generic observational evidence that prior to the white dwarf stage many stars must undergo substantial mass loss [176]. In the solar neighborhood white dwarfs with accurately determined masses $\sim 0.4 M_\odot$ are found. Since the study of stellar evolution in clusters indicates that there has been insufficient time for stars formed with that little mass to have evolved to the white dwarf stage, these white dwarfs must have come from main sequence stars that have shed considerable mass since their formation. In addition, direct observation indicates the presence of white dwarfs in some clusters with masses less than the main sequence stars in the cluster, again indicating that they must have evolved from stars that underwent considerable mass loss in their evolution.

closed surface surrounding the star. It has units of energy per unit time and is a sum of three primary components, $L = L_\gamma + l_\nu + L_{\Delta m}$, which are associated with emission of photons, emission of neutrinos, and surface mass loss, respectively.

Photon emission: The total luminosity associated with the photon flux is L_γ. This flux is emitted primarily from the thin *photosphere* at the surface of the star; it is the principal luminosity source for most young stars, and is most often what is meant when speaking loosely of stellar luminosity.

Neutrino emission: The quantity L_ν is the total luminosity associated with neutrino emission from the star. Cooling by neutrino emission becomes important in massive stars late in their life and the energy of a core collapse supernova, which represents the death of a massive star, is radiated primarily in the form of neutrinos.

Surface mass loss: Most stars have mechanisms by which they lose mass from their surfaces (see Box 1.1). Since ejected matter must be lifted in a gravitational field, mass loss subtracts from the energy budget of the star and is a source of luminosity according to our general definition. The term $L_{\Delta m}$ accounts for this source.

1.1.2 Photon Luminosities

Henceforth, unless otherwise specified, by "luminosity" we will mean the photon luminosity. For a spherical star the luminosity is given by

$$L_\gamma = 4\pi R^2 \int_0^\infty F_\lambda \, d\lambda, \tag{1.1}$$

where F_λ is the net outgoing energy flux at wavelength λ and R is the radius. The corresponding flux f_λ detected at the surface of the Earth is reduced according to the inverse square law,

$$f_\lambda = (R/r)^2 F_\lambda, \tag{1.2}$$

where r is the distance of the star from Earth. (Generally, the observed flux at the surface of the Earth must be corrected for absorption in the interstellar medium and in the Earth's atmosphere; those corrections will be discussed below.) Therefore, $F_\lambda R^2 = f_\lambda r^2$, and a measurement of $f_\lambda r^2$ at all λ determines $F_\lambda R^2$ at all wavelengths and hence the photon luminosity L_γ through Eq. (1.1). The most difficult part is likely to be the determination of the distance r, unless the star is nearby so that trigonometric methods may be used to measure the distance with confidence.

1.1.3 Apparent Magnitudes

It is useful to express actual and apparent brightness in terms of logarithmic *magnitude scales*. Two general classes of magnitudes may be distinguished: *apparent magnitudes*, which are associated with the apparent brightness of objects in our sky, and *absolute magnitudes*, which define brightness with the dependence on distance scales factored out. Apparent magnitudes will be discussed in this section and absolute magnitudes will be discussed in Section 1.1.5 below.

The apparent magnitude m is defined such that for two stars labeled 1 and 2 with observed fluxes f_1 and f_2, respectively,

$$m_2 - m_1 = 2.5(\log f_1 - \log f_2) = 2.5 \log\left(\frac{f_1}{f_2}\right), \tag{1.3}$$

where log means the base-10 logarithm[2] and the normalization of the magnitude scale is discussed in Box 1.2. This definition implies that a difference of five orders of magnitude corresponds exactly to a factor of 100 in brightness, and that algebraically smaller magnitudes are associated with brighter objects. It is often useful to define a set of apparent magnitudes that are restricted to a limited range of frequencies (for example, by the use of telescopic filters; see Fig. 1.3 below). Some common ones are

1. The *visual magnitude* m_{v}, determined from the flux in the range of frequencies to which the human eye is sensitive (peaking in the yellow–green part of the spectrum).
2. The *blue-sensitive magnitude* m_{B}, which is the magnitude determined if the light is collected using a blue filter.
3. The *photovisual magnitude* m_{V}, which is the magnitude determined if the light is collected using a yellow filter to make the resulting magnitude correspond more closely to the visual magnitude defined above.
4. The *ultraviolet magnitude* m_{U}, which is determined using filters to emphasize the UV part of the spectrum.

Various other apparent magnitudes can be introduced by using other filters that emphasize different parts of the spectrum but the ones described above are common and representative. More will be said about such magnitudes when color indices are discussed in Section 1.3.

[2] In this book we use $\log \equiv \log_{10}$ to denote the base-10 logarithm and $\ln \equiv \log_e$ to denote the base-e or natural logarithm.

Box 1.2 Normalization of Magnitude Scales

The definition of individual apparent magnitudes in Eq. (1.3) requires setting the scale by an arbitrary choice. The scales for the magnitudes m_v and m_V are set conventionally so that modern visual magnitude scales coincide well with the subjective scales of ancient astronomy. The convention in use employs the magnitudes of a set of stars to fix a scale but practically it is such that a spectral class A0 star such as Vega has visual magnitude zero (spectral classes are discussed in Chapter 2). In the resulting scale, the brightest stars in the sky have apparent visual or photovisual magnitudes near zero. A representative set of apparent visual magnitudes for common objects is displayed in Table 1.1. The magnitude zeros for other apparent magnitude scales are set by similar conventions. For example, the blue-sensitive magnitude scale is defined so that $m_B = m_v$ for a spectral class A0 star.

Table 1.1 Some apparent visual magnitudes

Object	m_V
Sirius (brightest star)	-1.5
Venus at brightest	-4.4
Full Moon	-12.6
The Sun	-26.8
Faintest naked-eye stars	$+6-7$
Faintest object visible from Earth with largest conventional telescopes	$\sim +25$
Faintest object visible from the Hubble Space Telescope	$\sim +30$

The apparent magnitudes discussed above conflate intrinsic properties (energy output) with geometric effects. It is desirable to factor out the distance dependence to address issues of stellar structure. This is done formally by introducing *absolute magnitude scales*, which will be defined in the next section. Before doing that, it is convenient to introduce a unit called the *parsec* that is a preferred unit of distance for astronomers.

1.1.4 The Parsec Distance Unit

The apparent relative positions of stars on the celestial sphere shift by small amounts over a six-month period because of the parallax effect as the Earth goes around its orbit. The angle p defined in Fig. 1.1, which is equal to half the angular size of the Earth's orbit as viewed from the star, is called the *parallax angle*; it is related trigonometrically to the distance d to the star through

$$\tan p = \frac{1\ \text{AU}}{d}, \tag{1.4}$$

where the *astronomical unit* AU is the average separation of the Earth and Sun (the length of the Earth's semimajor orbital axis; 1 AU $\sim 1.5 \times 10^8$ km). A small-angle approximation

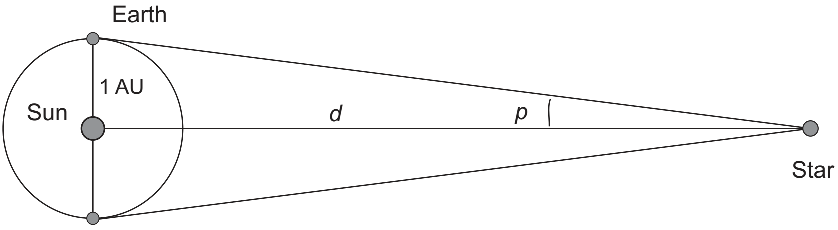

Fig. 1.1 The parallax angle p for a star as observed from Earth.

is justified and $p\,(\text{radians}) = 1\text{AU}/d$. Converting the angular measure to seconds of arc (1 degree = 3600 arcsec ≡ 3600″ and 1 radian = 2.06×10^5 arcsec), permits writing

$$\frac{d}{1\text{ AU}} = \frac{2.06 \times 10^5}{p''}, \tag{1.5}$$

where the notation indicates that p is to be given in seconds of arc.

The relationship between parallax angle and distance given by Eq. (1.5) suggests defining a natural distance unit equal to the distance at which a star would have a parallax angle of 1″. This unit is termed the *parsec* (from concatenating “parallax” and “seconds”), and is abbreviated by the symbol pc. With these units the distance in parsecs is just the inverse of the parallax angle in seconds of arc:

$$d\,(\text{pc}) = \frac{1}{p''}. \tag{1.6}$$

From this equation the relationship of the parsec to other common distance units is easily found. For example,

$$1\text{ pc} = 2.06 \times 10^5\text{ AU} = 3.09 \times 10^{18}\text{ cm} = 3.26\text{ ly},$$

where a lightyear (ly) is the distance light travels in a year. Parallax angles for even the nearest stars are tiny, as illustrated in Example 1.1.

Example 1.1 The nearby star α Centauri has a measured parallax of 0.742″ [2], corresponding to a distance of $d = 1/0.742 = 1.348$ pc, or 4.4 ly. To set this parallax angle in perspective, 1″ is the angle subtended by a 2-cm diameter coin at a distance of 4 km.

Parallax angles can be measured reliably down to about 0.01″ for ground-based telescopes without adaptive optics, so the traditional parallax method is useful for distance measurements out to about 100 pc (though the uncertainty becomes substantial for larger distances). Observations with the *Hipparcos satellite* could measure a parallax of 0.001″ and extended the parallax range to about 1000 pc, allowing determination of high-precision parallaxes for more than 100,000 new stars (and 2.5 million additional stars at low precision). More recently the European Space Agency *Gaia* mission was launched in 2013 with a goal of mapping precisely the position, brightness, and variations in brightness, color, velocities, and evidence for a companion for more than 10^9 stars by 2018, including parallax measurements for more than 200 million new stars. To enable this it has the capability to measure parallax angles as small as 5×10^{-6} arcseconds (the angle subtended by a thumbnail on the Moon as viewed from Earth). Beyond these distances, other less-direct methods must be employed.

1.1.5 Absolute Magnitudes

By convention, the absolute magnitude, denoted by M to distinguish it from the apparent magnitude m, is the apparent magnitude that a star would have if it were placed at a standard distance of 10 pc = 32.6 ly. Using previous expressions for the apparent magnitude, it is then easy to show (Problem 1.3) that the absolute and apparent magnitudes are related by

$$m - M = 5 \log \left(\frac{d}{10\,\text{pc}} \right), \tag{1.7}$$

where the quantity $m - M$ is termed the *distance modulus*. Thus, the absolute magnitude is the apparent magnitude minus the distance modulus, and is easily calculated from (1.7) if the distance d to the star is known.

1.1.6 Bolometric Magnitudes

The *bolometric magnitude* is the magnitude that a star would have if the detector could collect the entire spectrum of emitted radiant energy. Realistic detectors cannot do this because of inherent detector limitations and losses in the atmosphere and interstellar medium, so it is necessary to apply a *bolometric correction* to raw magnitudes; this correction is designed to add back flux that is absorbed in the atmosphere or otherwise not detected. Then the absolute bolometric magnitude is

$$M_{\text{bol}} = M_{\text{v}} + \text{BC}, \tag{1.8}$$

where BC is the *bolometric correction*. (*Note*: Some authors define instead $M_{\text{bol}} = M_{\text{v}} - \text{BC}$, so be mindful of the sign for BC.) The bolometric correction is large for very hot and very cool stars because they output a substantial portion of their radiation at UV and IR wavelengths, respectively, and these wavelengths are absorbed strongly in the atmosphere. Even above the atmosphere there may be significant corrections for absorption in the interstellar medium.

Example 1.2 Because the Sun emits small amounts of UV and IR radiation relative to visible light, its bolometric correction is small. The Sun has an absolute bolometric magnitude of $M_{\text{bol}}^{\odot} = 4.74$, corresponding to a luminosity of $L_\odot = 3.828 \times 10^{33}\,\text{erg s}^{-1}$ (this luminosity will be estimated from observations below).

For calculations we often find it convenient to write for an arbitrary star

$$M_{\text{bol}} - M_{\text{bol}}^{\odot} = -2.5 \log \frac{L}{L_\odot}, \tag{1.9}$$

which corresponds to expressing the absolute bolometric magnitude M_{bol} and luminosity L for an arbitrary star in units of the corresponding quantities for the Sun.

1.2 Stars as Blackbody Radiators

A temperature can be defined for an object that is in thermodynamical equilibrium. In particular, we may introduce a temperature self-consistently for a star if it is a *blackbody radiator*. Stars are often assumed to be blackbody radiators. They generally are not perfectly so, but this is a sufficiently good approximation to be a very useful starting point.

1.2.1 Radiation Laws

A blackbody radiator has a radiation field that is isotropic, homogeneous, randomly polarized, and independent of the walls of the container. If a body satisfies these conditions, several important *radiation laws* apply.

Planck law: The *Planck radiation law* defines the intensity of emitted radiation for a blackbody. The Planck function $B_\lambda(T)$ giving the power emitted per unit surface area of a blackbody per unit wavelength into unit solid angle is given by

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda kT} - 1}, \tag{1.10}$$

where λ is the wavelength, T is the temperature, h is Planck's constant, c is the speed of light, and k is Boltzmann's constant. Blackbody spectra for several temperatures are illustrated in Fig. 1.2, where the total area under the curve is seen to grow rapidly with temperature and the distribution exhibits a single peak that shifts to shorter wavelengths as the temperature increases. Two other important laws governing this behavior may be derived from the Planck law, the *Stefan–Boltzmann law* and the *Wien displacement law* (see Example 1.3 and Problem 1.4). The first governs the total energy radiated at all wavelengths and the second governs the wavelength at which the peak intensity is emitted.

Stefan–Boltzmann law: The law of Stefan and Boltzmann says that the total energy E radiated per unit time per unit surface area at all wavelengths varies as the fourth power of the temperature,

$$E = \tfrac{1}{4}acT^4 = \sigma T^4, \tag{1.11}$$

where a is the *radiation density constant* and σ is the *Stefan–Boltzmann constant*. Multiplication by the surface area then gives the luminosity. Thus for a spherical blackbody $L = 4\pi R^2 \sigma T^4$, where R is the radius.

Wien law: The Wien displacement law states that for a blackbody radiator the maximum in the radiation distribution as a function of wavelength occurs at

$$\lambda_{\text{max}} = 2.90 \times 10^7 \left(\frac{\text{K}}{T}\right) \text{Å}, \tag{1.12}$$

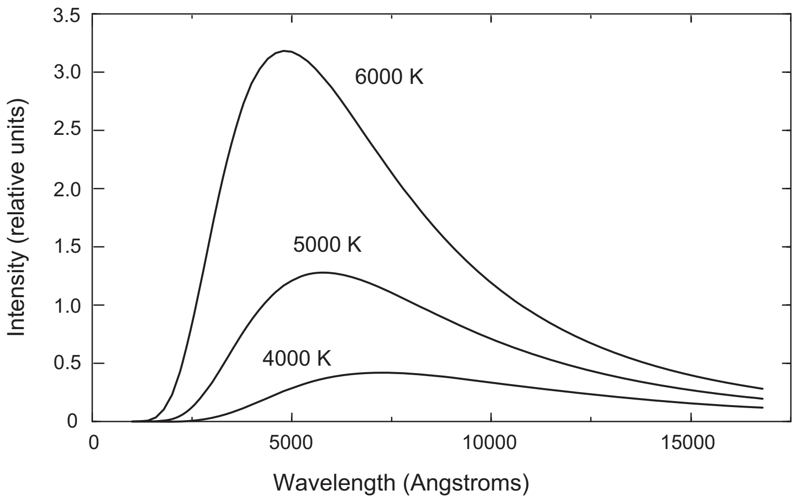

Fig. 1.2 Planck distribution for several temperatures.

where 1 angstrom (Å) $= 10^{-8}$ cm. The Stefan–Boltzmann law explains the increase in total luminosity with temperature seen in Fig. 1.2, while the Wien law accounts for the shift of these distributions to shorter wavelengths as the temperature increases.

Example 1.3 The Stefan–Boltzmann and Wien laws follow from the more general Planck law. We may illustrate by outlining the derivation of the Stefan–Boltzmann law (you are asked to provide the details in Problem 1.4). The total energy flux emitted by a blackbody at temperature T can be expressed as an integral over Eq. (1.10), and using the substitutions

$$u = hc/\lambda kT \qquad \lambda = hc/ukT \qquad d\lambda = \frac{-hc}{u^2 kT}\, du,$$

the integral can be evaluated to give the Stefan–Boltzmann law, $E = \sigma T^4$, where σ is the Stefan–Boltzmann constant.

The Wien law results from differentiating Eq. (1.10) to find the maximum, as you are asked to show in Problem 1.13.

1.2.2 Effective Temperatures

If a star is assumed to be a blackbody radiator, we may use the Stefan–Boltzmann law to define an *effective surface temperature* T_e through the relation

$$L = 4\pi\sigma R^2 T_e^4. \tag{1.13}$$

That is, the effective temperature T_e is the temperature that a perfect blackbody of radius R would need in order to radiate the observed luminosity of the star. This is an integral condition that requires the total luminosity of the star and the fictitious blackbody that

Box 1.3 Luminosity and Effective Temperature of the Sun

The preceding discussion may be illustrated by determining the surface flux, total luminosity, and effective surface temperature of the Sun – a star for which the required quantities are known rather well. The total radiant flux of the Sun on our upper atmosphere (the *solar constant*) has a value of 1.36×10^6 erg cm^{-2}s^{-1}, and the Sun subtends an average angle of 32 minutes of arc in our sky at an average distance of 1 AU $\sim 1.496 \times 10^8$ km. Then from geometry the radius of the Sun is given by

$$R_\odot = \tan(16') \text{ AU} = 0.00465 \text{ AU} = 6.96 \times 10^{10} \text{ cm},$$

and if the distance to the Sun is r, the flux at the solar surface is related to the flux on our upper atmosphere by the inverse square intensity law,

$$\text{Flux (solar surface)} = \left(\frac{r}{R_\odot}\right)^2 \times \text{Flux (Earth)} = 6.28 \times 10^{10} \text{ erg cm}^{-2}\text{s}^{-1}.$$

The total solar luminosity follows from integrating this flux over the surface area of the Sun,

$$L_\odot = 4\pi R_\odot^2 \times \text{Flux (solar surface)} \simeq 3.82 \times 10^{33} \text{ erg s}^{-1},$$

and this allows an effective surface temperature for the Sun to be calculated as

$$T_\text{e}^\odot = \left(\frac{L_\odot}{4\pi\sigma R_\odot^2}\right)^{1/4} = 5770 \text{ K}.$$

A more careful analysis yields the standard value of $T_\text{e}^\odot \sim 5777$ K. Using this temperature in the Wien law (1.12) indicates that the Sun's spectrum peaks at about 5020 Å, which is the yellow–green part of the spectrum. More generally, R is known for only a few nearby bright stars; for others, R must be estimated in some model-dependent way, and an estimate of the distance to the star is required in order to determine the luminosity.

approximates it to be equivalent, but does not constrain whether the detailed wavelength distribution of emitted radiation for the star and the fictitious blackbody are equivalent. Box 1.3 illustrates the use of Eq. (1.13) to determine an effective temperature for the Sun.

1.2.3 Stellar Radii from Effective Temperatures

The (limited) direct ways available to determine the physical sizes of stars will be discussed later in this chapter. However, if a star of known luminosity is assumed to be a spherical blackbody and the effective temperature T_e is estimated from the spectrum (see Chapter 2), Eq. (1.13) can be used to solve for a radius. For calculations, it is often useful to relate the radius R and effective temperature T_e of a star to corresponding quantities for the Sun through

$$\frac{R}{R_\odot} = \left(\frac{T_\text{e}^\odot}{T_\text{e}}\right)^2 \left(\frac{L}{L_\odot}\right)^{1/2}, \tag{1.14}$$