

Finite Elements for Engineers with Ansys Applications

The finite element method (FEM) is indispensable in the modeling and simulation of various engineering and physical systems, including structural analysis, stress, strain, fluid mechanics, heat transfer, dynamics, eigenproblems, design optimization, sound propagation, electromagnetics, and coupled field problems.

Incorporating theory, development of the method, and the use of FEM in the commercial sector, this textbook integrates basic theory with real-life, design-oriented problems using Ansys, the most commonly used computational software in the field.

For students as well as practicing engineers and designers, each chapter is highly illustrated and presented in a step-by-step manner. Fundamental concepts are presented in detail with reference to easy to understand worked examples that clearly introduce the method before progressing to more advanced mathematical content.

Included are step-by-step solutions for project type problems using modeling software, special chapters for modeling and the use of Ansys and Workbench programs, and extensive sets of problems and projects round out each chapter.

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Preface

The finite element method (FEM) is broadly defined as a numerical approach for approximating the governing equations of any continuous system. The method has evolved into an indispensable technology in the modeling and simulation of various engineering and physical systems. The advent and advancement of digital computers have enabled the application of the method in all fields of engineering. The FEM has a number of distinctive features and advantages which make it superior to other numerical approaches. It is adaptable for solving problems with general geometry and boundary conditions and may easily incorporate nonlinear effects, different material models, thermal effects and coupled phenomena. Essentially, it may be applied to any physical problem described by the equations of calculus, e.g. differential, integral, or variational equations. Additionally, FEM computer programs have become common tools in the hands of engineers, designers and analysts.

This text is intended to target users of the FEM for design and analysis from both academic and industrial environments. There is a large number of finite element texts in the market. On one hand, many of the available texts rigorously discuss the theoretical aspects and developments of the method and on the other hand some texts concentrate mainly on the practical use of the method and commercial programs. This text strives to strike a balance between the two approaches so that it will be appealing to senior undergraduate students and beginner graduate students as well as practicing engineers and designers. Before indulging in mathematical treatments, the text highlights the basic concepts of the FEM with minimal mathematical involvement. From the decades of teaching experience of the author, this has proved to help students understand the theory and become confident in using programs. The philosophy is to let the reader master the basic concepts before digging into rigorous mathematical development. The conceptual and theoretical chapters in the text are full of simple hand-calculation and Matlab problems. Each of these chapters concludes with a detailed project that provides step-by-step examples using one of the major FE commercial programs (Ansys or Workbench). However, the first three chapters are followed by a special application chapter with a brief introduction to Ansys and Workbench as well as some project-type problems. Hence, this book integrates the basic theory with real-life, design-oriented problems which involve practicing and using some of the most common computational software program in the field. Material is presented in a step-by-step manner in each chapter and fundamental concepts are presented in

detail with reference to simple and easy-to-understand problems, assuming that the reader has no previous knowledge of the method. This approach improves and enhances readability, reduces frustration and increases retention. At the end of each section, there is synopsis box outlining the main concepts, important equations and conclusions of the section. At the end of each chapter, there is a large selection of hand-calculation, Matlab, Ansys/Workbench and design problems that have been collected over decades of teaching. Most of the problems are solved in a supplementary solution manual.

The book is organized into seven chapters. The first four chapters are devoted to discussions of various FEM concepts along with the practical use of the Ansys and Workbench programs. Chapter 1 outlines the general steps of finite element analysis (FEA) through simple geometric problems, provides conceptual definitions to various terms used in the analysis and gives a brief history of the method and its various applications. The basic concepts, the phases of using the method, the underlying assumptions, simple element stiffness developments and the transformation and imposition of simple boundary conditions are discussed in Chapter 2 without going into deep mathematical treatments. Chapter 3 provides practical guidelines for the modeling phase in FEA. This chapter highlights geometric modeling, problem classification, element choice, symmetry and boundary conditions, mesh intensity and lumped loading. Chapter 4 gives a brief introduction to the use of the Ansys and Workbench programs along with step-by-step examples for using them in the analysis of two- and three-dimensional linear elastic problems. This chapter may be considered as an application of the concepts in Chapter 3 and, as a logical consequence, the problem sets for these two chapters are given after Chapter 4. Chapter 5 marks the start of the rigorous mathematical treatment of the finite element method. This chapter examines two basic methods of formulation in finite elements: the virtual work principle and the weighted residual Galerkin method. Examples of the use of each method in linear static, heat transfer and field problems are provided. Chapter 6 introduces the first rigorous applications to linear static analysis problems. This chapter starts by highlighting the general formulation for linear static analysis problems and then details various element applications for beam, plane stress/strain, axisymmetric and three-dimensional solid elements. Following the element development, a section on thermal stress analysis is provided, which is followed by a detailed discussion of isoparametric elements. A final section on handling linear multi-point constraint equations is provided; this is followed by an extensive problem set, with projects. The final chapter discusses conduction heat transfer analysis, and it starts with a brief introduction to three-dimensional transient conduction analysis in solids with convection and radiation boundary conditions. Then, the chapter discusses various methods for solving the transient equation and provides detailed matrix equations for the analysis along with sample element developments. The chapter ends with

detailed analysis problems and an Ansys/WB project. The treatment in this chapter is rather condensed and brief; it is meant to provide a quick review of the basic equations in the field of application with minimal mathematical treatment. Finally, some of the necessary background is provided in Appendices A and B. These include: Matrices and Systems of Linear Equations (Appendix A), and Vectors and Tensors (Appendix B).

Gratitude is owed to many friends, colleagues and researchers who have contributed through discussions, ideas and suggestions to improve the text and bring it to its current format. My journey with the field has spanned 45 years, starting at Cairo University where I was introduced to the principles of mechanics by my uncle Abdul-Aziz Gadala and to FEA by the late Professor Salah Bayoumi. The final part of my journey was at the University of British Columbia (UBC) in Vancouver, where I spent over 25 years teaching design and FEA. Between the two stops, I passed by many stations including McMaster University, Atomic Energy of Canada, Ontario Hydro Research, Engineering Mechanics Research Corporation in Michigan and the University of Michigan. During this journey, I have collaborated with numerous industrial partners in Canada, Europe, the United States and the Middle East. Undoubtedly, this collaboration has contributed significantly to this work and elevated it to the current format. Without such collaboration and the fruitful ideas and help of my own students at UBC this book could not have been written.

Finally, special thanks to my wife Omnia and my children, Drs. Marwa, Mariam, and Ibrahim, for their immense support and forbearance with the numerous occasions when the task of writing has deprived them of invaluable family time together.

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