# Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Preface</strong></td>
<td>xi</td>
</tr>
<tr>
<td><strong>Organization and Chapter Summaries</strong></td>
<td>xv</td>
</tr>
<tr>
<td><strong>Notation</strong></td>
<td>xxiii</td>
</tr>
<tr>
<td>Standard Notation</td>
<td>xxiii</td>
</tr>
<tr>
<td>Specific Notation Used Extensively</td>
<td>xxiv</td>
</tr>
<tr>
<td><strong>Acknowledgments</strong></td>
<td>xxv</td>
</tr>
<tr>
<td><strong>1 The Main Themes: Approximate Decision and Sublinear Complexity</strong></td>
<td>1</td>
</tr>
<tr>
<td>1.1 Introduction</td>
<td>1</td>
</tr>
<tr>
<td>1.1.1 Property Testing at a Glance</td>
<td>2</td>
</tr>
<tr>
<td>1.1.2 On the Potential Benefits of Property Testers</td>
<td>3</td>
</tr>
<tr>
<td>1.1.3 On the Flavor of Property Testing Research</td>
<td>4</td>
</tr>
<tr>
<td>1.1.4 Organization and Some Notations</td>
<td>5</td>
</tr>
<tr>
<td>1.2 Approximate Decisions</td>
<td>6</td>
</tr>
<tr>
<td>1.2.1 A Detour: Approximate Search Problems</td>
<td>6</td>
</tr>
<tr>
<td>1.2.2 Property Testing: Approximate Decision Problems</td>
<td>6</td>
</tr>
<tr>
<td>1.2.3 Property Testing: Sublinear Complexity</td>
<td>7</td>
</tr>
<tr>
<td>1.2.4 Symmetries and Invariants</td>
<td>10</td>
</tr>
<tr>
<td>1.2.5 Objects and Representation</td>
<td>13</td>
</tr>
<tr>
<td>1.3 Notions, Definitions, Goals, and Basic Observations</td>
<td>14</td>
</tr>
<tr>
<td>1.3.1 Basics</td>
<td>15</td>
</tr>
<tr>
<td>1.3.2 Ramifications</td>
<td>18</td>
</tr>
<tr>
<td>1.3.3 Proximity-Oblivious Testers (POTs)</td>
<td>19</td>
</tr>
<tr>
<td>1.3.4 The Algebra of Property Testing</td>
<td>22</td>
</tr>
<tr>
<td>1.3.5 Testing via Learning</td>
<td>26</td>
</tr>
<tr>
<td>1.4 Historical Notes</td>
<td>28</td>
</tr>
<tr>
<td>1.5 Suggested Reading and Exercises</td>
<td>30</td>
</tr>
<tr>
<td>Basic Exercises</td>
<td>31</td>
</tr>
<tr>
<td>Additional Exercises</td>
<td>36</td>
</tr>
<tr>
<td>1.6 Digest: The Most Important Points</td>
<td>38</td>
</tr>
<tr>
<td><strong>2 Testing Linearity (Group Homomorphism)</strong></td>
<td>40</td>
</tr>
<tr>
<td>2.1 Preliminaries</td>
<td>40</td>
</tr>
<tr>
<td>2.2 The Tester</td>
<td>40</td>
</tr>
<tr>
<td>2.3 Chapter Notes</td>
<td>45</td>
</tr>
</tbody>
</table>

© in this web service Cambridge University Press
CONTENTS

3 Low-Degree Tests 49
  3.1 A Brief Introduction 49
  3.2 A Kind of Intuition (which may be skipped) 50
    3.2.1 The Univariate Case 50
    3.2.2 The Multivariate Case 51
    3.2.3 Linking the Above Intuition to the Actual Proof 52
  3.3 Background 53
  3.4 The Tester 57
    3.4.1 Analysis of the Tester 58
    3.4.2 Digest (or an Abstraction) 64
  3.5 Chapter Notes 65
    Exercises 67

4 Testing Monotonicity 69
  4.1 Introduction 69
  4.2 Boolean Functions on the Boolean Hypercube 70
    4.2.1 The Edge Test 70
    4.2.2 Path Tests 75
  4.3 Multivalued Functions on the Discrete Line 77
    4.3.1 A Tester Based on Binary Search 77
    4.3.2 Other Testers 81
  4.4 Multivalued Functions on the Hypergrid 82
    4.4.1 Dimension Reduction (Proof of Lemma 4.13) 84
    4.4.2 Range Reduction (Overview of the Proof of Lemma 4.14) 85
  4.5 Chapter Notes 87
    4.5.1 History and Credits 87
    4.5.2 Related Problems 88
    4.5.3 Exercises 89

5 Testing Dictatorships, Juntas, and Monomials 92
  5.1 Introduction 92
  5.2 Testing Dictatorship via Self-correction 93
    5.2.1 The Tester 94
    5.2.2 Testing Monomials 97
    5.2.3 The Self-correction Paradigm: An Abstraction 101
  5.3 Testing Juntas 105
  5.4 Chapter Notes 112
    Basic Exercises 112
    Additional Exercises 115

6 Testing by Implicit Sampling 120
  6.1 Introduction 120
  6.2 Testing Subsets of k-Juntas 121
  6.3 Extension to Properties Approximated by Subsets of k-Juntas 128
  6.4 Chapter Notes 131
    On Testing Problems Associated with Sets of Boolean Functions 131
    Exercises 132
## CONTENTS

7 Lower Bounds Techniques

- 7.1 Introduction
- 7.2 Indistinguishability of Distributions
  - 7.2.1 The Actual Method
  - 7.2.2 Illustrating the Application of the Method
  - 7.2.3 Further Reflections
- 7.3 Reduction from Communication Complexity
  - 7.3.1 Communication Complexity
  - 7.3.2 The Methodology
  - 7.3.3 Illustrating the Application of the Methodology
- 7.4 Reduction among Testing Problems
- 7.5 Lower Bounds for Restricted Testers
  - 7.5.1 One-Sided Error Testers
  - 7.5.2 Nonadaptive Testers
- 7.6 Chapter Notes

8 Testing Graph Properties in the Dense Graph Model

- 8.1 The General Context: Introduction to Testing Graph Properties
  - 8.1.1 Basic Background
  - 8.1.2 Three Models of Testing Graph Properties
- 8.2 The Dense Graph Model: Some Basics
  - 8.2.1 The Actual Definition
  - 8.2.2 Abuses of the Model: Trivial and Sparse Properties
  - 8.2.3 Testing Degree Regularity
  - 8.2.4 Digest: Levin’s Economical Work Investment Strategy
- 8.3 Graph Partition Problems
  - 8.3.1 Testing Bipartiteness
  - 8.3.2 The Actual Definition and the General Result
- 8.4 Connection to Szemerédi’s Regularity Lemma
  - 8.4.1 The Regularity Lemma
  - 8.4.2 Subgraph Freeness
  - 8.4.3 The Structure of Properties That Have Size-Oblivious Testers
- 8.5 A Taxonomy of the Known Results
  - 8.5.1 Testability in $q(\epsilon)$ Queries, for any Function $q$
  - 8.5.2 Testability in $\text{poly}(1/\epsilon)$ Queries
  - 8.5.3 Testability in $\tilde{O}(1/\epsilon)$ Queries
  - 8.5.4 Additional Issues
- 8.6 Chapter Notes

9 Testing Graph Properties in the Bounded-Degree Graph Model

- 9.1 The Bounded-Degree Model: Definitions and Issues
- 9.2 Testing by a Local Search
  - 9.2.1 Testing Subgraph Freeness

---

© in this web service Cambridge University Press

www.cambridge.org
## CONTENTS

9.2.2 Testing Degree Regularity 219  
9.2.3 Testing Connectivity 221  
9.2.4 Testing \( t \)-Connectivity (Overview and One Detail) 223  
9.2.5 Testing Cycle-Freeness (with Two-Sided Error) 227  

9.3 Lower Bounds 230  
9.3.1 Bipartiteness 230  
9.3.2 Applications to Other Properties 232  
9.3.3 Linear Lower Bounds 235  

9.4 Testing by Random Walks 236  
9.4.1 Testing Bipartiteness 236  
9.4.2 One-Sided Error Tester for Cycle-Freeness 243  

9.5 Testing by Implementing and Utilizing Partition Oracles 248  
9.5.1 The Simpler Implementation 252  
9.5.2 The Better Implementation 257  

9.6 A Taxonomy of the Known Results 259  
9.6.1 Testability in \( q(\epsilon) \) Queries, for Any Function \( q \) 259  
9.6.2 Testability in \( \tilde{O}(k^{1/2}) \cdot \text{poly}(1/\epsilon) \) Queries 260  
9.6.3 Additional Issues 261  

9.7 Chapter Notes 262  
9.7.1 Historical Perspective and Credits 262  
9.7.2 Directed Graphs 263  
9.7.3 Exercises 264  

10 Testing Graph Properties in the General Graph Model 271  
10.1 The General Graph Model: Definitions and Issues 272  
10.1.1 Perspective: Comparison to the Two Previous Models 272  
10.1.2 The Actual Definition 273  

10.2 On Obtaining Testers for the Current Model 275  
10.2.1 An Explicit Adaptation: The Case of Connectivity 276  
10.2.2 Using a Reduction: The Case of Bipartiteness 277  

10.3 Estimating the Average Degree and Selecting Random Edges 281  
10.3.1 Lower Bounds 282  
10.3.2 Algorithms 283  

10.4 Using Adjacency Queries: The Case of Bipartiteness 293  

10.5 Chapter Notes 296  
10.5.1 Gaps between the General Graph Model and the Bounded-Degree Model 296  
10.5.2 History and Credits 298  
10.5.3 Reflections 298  
10.5.4 Exercises 300  

11 Testing Properties of Distributions 304  
11.1 The Model 304  
11.1.1 Testing Properties of Single Distributions 305  
11.1.2 Testing Properties of Pairs of Distributions 307  
11.1.3 Label-invariant Properties 308  
11.1.4 Organization 309
## CONTENTS

11.2 Testing Equality to a Fixed Distribution 309
   11.2.1 The Collision Probability Tester and Its Analysis 310
   11.2.2 The General Case (Treated by a Reduction to Testing Uniformity) 313
   11.2.3 A Lower Bound 319

11.3 Testing Equality between Two Unknown Distributions 320
   11.3.1 Detour: Poisson Distributions 321
   11.3.2 The Actual Algorithm and Its Analysis 324
   11.3.3 Applications: Reduction to the Case of Small Norms 330

11.4 On the Complexity of Testing Properties of Distributions 335

11.5 Chapter Notes 340
   11.5.1 History and Credits 340
   11.5.2 Exercises 341

12 Ramifications and Related Topics 348
   12.1 Tolerant Testing and Distance Approximation 348
   12.2 Additional Promises on the Input 351
   12.3 Sample-Based Testers 352
   12.4 Testing with Respect to Other Distance Measures 353
   12.5 Local Computation Algorithms 355
      12.5.1 Definitions 356
      12.5.2 Finding Huge Structures 358
      12.5.3 Local Reconstruction 360
   12.6 Noninteractive Proofs of Proximity (MAPs) 361
   12.7 Chapter Notes 365
      12.7.1 Historical Notes 365
      12.7.2 Massively Parameterized Properties 366
      12.7.3 Exercises 366

13 Locally Testable Codes and Proofs 370
   13.1 Introduction 371
   13.2 Definitions 372
      13.2.1 Codeword Testers 372
      13.2.2 Proof Testers 374
      13.2.3 Ramifications and Relation to Property Testing 376
      13.2.4 On Relating Locally Testable Codes and Proofs 382
   13.3 Results and Ideas 383
      13.3.1 The Mere Existence of Locally Testable Codes and Proofs 384
      13.3.2 Locally Testable Codes and Proofs of Polynomial Length 387
      13.3.3 Locally Testable Codes and Proofs of Nearly Linear Length 397
   13.4 Chapter Notes 403
      13.4.1 Historical Notes 403
      13.4.2 On Obtaining Superfast testers 404
      13.4.3 The Alternative Regime: LTCs of Linear Length 405
      13.4.4 Locally Decodable Codes 407
      13.4.5 Exercises 408

© in this web service Cambridge University Press
## CONTENTS

**Appendix A: Probabilistic Preliminaries**

- **A.1** Notational Conventions 411
- **A.2** Some Basic Notions and Facts 412
- **A.3** Basic Facts Regarding Expectation and Variance 413
- **A.4** Three Inequalities 415
  - A.4.1 Markov’s Inequality 415
  - A.4.2 Chebyshev’s Inequality 416
  - A.4.3 Chernoff Bound 419
  - A.4.4 Pairwise Independent versus Totally Independent Sampling 422

**Appendix B: A Mini-Compendium of General Results**

**Appendix C: An Index of Specific Results**

References 429
Index 443