

Contents

	<i>Preface and Acknowledgments</i>	<i>page xi</i>
	<i>Notation</i>	<i>xiii</i>
1	Introduction	1
	1.1 Motivation	1
	1.2 System Models	3
	1.3 Uncertainty Models	7
	1.3.1 Static Systems	7
	1.3.2 Dynamical Systems	8
	1.4 Application Examples	9
	1.4.1 Power Flow Analysis under Active Power Injection Uncertainty	9
	1.4.2 Analysis of Inertia-less AC Microgrids under Power Injection Uncertainty	10
	1.4.3 Reliability Analysis of Static Systems	11
	1.5 Book Road Map	12
	1.6 Notes and References	15
2	Preliminaries	16
	2.1 Probability and Stochastic Processes	16
	2.1.1 Probability Spaces	16
	2.1.2 Random Variables	17
	2.1.3 Jointly Distributed Random Variables	21
	2.1.4 Random Vectors	27
	2.1.5 Stochastic Processes	29
	2.2 Set Theory	35
	2.2.1 Basic Notions and Notation	35
	2.2.2 Sets in Euclidean Space	36
	2.3 Linear Dynamical Systems	46
	2.3.1 Discrete-Time Systems	46
	2.3.2 Continuous-Time Systems	48
	2.4 Notes and References	53

3	Static Systems: Probabilistic Input Uncertainty	54
3.1	Introduction	54
3.2	Moment Characterization	56
3.2.1	Linear Setting	56
3.2.2	Nonlinear Setting	58
3.3	Distribution Characterization	62
3.3.1	Linear Setting	62
3.3.2	Nonlinear Setting	68
3.4	Performance Characterization	74
3.4.1	Known Input Moments	74
3.4.2	Known Input Probability Density Function	76
3.5	Application to Power Flow Analysis	77
3.5.1	Power Flow Model	78
3.5.2	Power Flow Vector Distribution	79
3.6	Notes and References	90
4	Static Systems: Probabilistic Structural Uncertainty	91
4.1	Introduction	91
4.2	System Stochastic Model	92
4.3	Markov Process Characterization	93
4.3.1	Discrete-Time Case	94
4.3.2	Continuous-Time Case	97
4.4	Performance Characterization	100
4.5	Application to Reliability and Availability Analysis	101
4.5.1	Multi-Component System Input-to-State Characterization	101
4.5.2	Systems with Non-Repairable Components	109
4.5.3	Systems with Repairable Components	118
4.5.4	Reduced-Order Models	124
4.6	Notes and References	129
5	Discrete-Time Systems: Probabilistic Input Uncertainty	130
5.1	Introduction	130
5.2	Discrete-Time Linear Systems	131
5.2.1	Characterization of First and Second Moments	131
5.2.2	Probability Distribution	138
5.3	Discrete-Time Nonlinear Systems	144
5.3.1	Characterization of First and Second Moments	145
5.3.2	Probability Distribution	150
5.4	Analysis of Microgrids under Power Injection Uncertainty	151
5.4.1	System Model	151
5.4.2	Average Frequency Error Statistical Characterization	156
5.4.3	Phase Angle Statistical Characterization	161
5.5	Notes and References	165

6	Continuous-Time Systems: Probabilistic Input Uncertainty	166
6.1	Introduction	166
6.2	Continuous-Time Linear Systems	168
6.2.1	Characterization of First and Second Moments	171
6.2.2	Gaussian Systems	181
6.3	Continuous-Time Nonlinear Systems	187
6.3.1	Moments	188
6.3.2	Probability Distribution	189
6.4	Analysis of Microgrids under Sensor Measurement Uncertainty	192
6.4.1	System Model	192
6.4.2	Average Frequency Error Statistical Characterization	197
6.5	Notes and References	201
7	Static Systems: Set-Theoretic Input Uncertainty	202
7.1	Introduction	202
7.2	Ellipsoid-Based Input Set Description	204
7.2.1	Linear Setting	204
7.2.2	Nonlinear Setting	212
7.3	Zonotope-Based Input Set Description	218
7.3.1	Linear Setting	219
7.3.2	Nonlinear Setting	221
7.4	Performance Requirements Verification	225
7.5	Application to Power Flow Analysis	226
7.5.1	Power Flow Model	227
7.5.2	Ellipsoidal-Based Description of Possible Extraneous Power Injection Values	228
7.5.3	Zonotope-Based Description of Possible Extraneous Power Injection Values	233
7.6	Notes and References	236
8	Discrete-Time Systems: Set-Theoretic Input Uncertainty	237
8.1	Introduction	237
8.2	Discrete-Time Linear Systems	238
8.2.1	Ellipsoidal-Based Input Description	239
8.2.2	Choice of Parameter γ_k	242
8.2.3	Deterministic Inputs	258
8.3	Discrete-Time Nonlinear Systems	259
8.4	Performance Requirements Verification	261
8.5	Analysis of Microgrids under Power Injection Uncertainty	263
8.5.1	System Model	263
8.5.2	Characterization of the Set Containing the Average Frequency Error	265
8.5.3	Characterization of Set Containing the Bus Phase Angles	270
8.6	Notes and References	272

x	Contents	
9	Continuous-Time Systems: Set-Theoretic Input Uncertainty	273
9.1	Introduction	273
9.2	Continuous-Time Linear Systems	275
9.2.1	Choice of Parameter $\beta(t)$	278
9.2.2	Deterministic Inputs	289
9.3	Continuous-Time Nonlinear Systems	290
9.4	Performance Requirements Verification	293
9.5	Case Studies	297
9.5.1	Buck Converter	297
9.5.2	Three-Bus Power System	300
9.6	Notes and References	306
Appendix A	Mathematical Background	307
Appendix B	Power Flow Modeling	320
	<i>References</i>	330
	<i>Index</i>	334