

Large-Scale System Analysis under Uncertainty

Discover a comprehensive set of tools and techniques for analyzing the impact of uncertainty on large-scale engineered systems. Providing accessible yet rigorous coverage, this book showcases the theory through detailed case studies drawn from electric power application problems, including the impact of integration of renewable-based power generation in bulk power systems, the impact of corrupted measurement and communication devices in microgrid closed-loop controls, and the impact of components failures on the reliability of power supply systems. The case studies also serve as a guide on how to tackle similar problems that appear in other engineering application domains, including automotive and aerospace engineering.

This is essential reading for academic researchers and graduate students in power systems engineering, and dynamic systems and control engineering.

Alejandro D. Domínguez-García is a Professor in the Department of Electrical and Computer Engineering at the University of Illinois at Urbana-Champaign, where he also holds Research Professor appointments with the Coordinated Science Laboratory and the Information Trust Institute.

“Uncertainty and reliability are of higher and higher priority as marginal costs go down and the number of components goes up. This book is comprehensive and rigorous, full of derivations and nuanced examples – an excellent reference for anyone seeking solid foundations.”

Josh Taylor, University of Toronto

“Professor Domínguez-García’s book should be required reading for all graduate students in electric power systems. It fills a critical gap by focusing on the dynamical systems and control engineering foundations that underpin power system analysis and control, rather than focusing on the application domain problems themselves.”

Johanna Mathieu, University of Michigan

“With climate change disruptions being a new normal, electric vehicles taking over our roads, and renewable energies being an existential matter for all of us, there is a considerable need for a reference book on power system analysis under uncertainties. This book is advantageous to both students and practitioners working in the planning and operation of the electric grid.”

Reza Argandeh, Western Norway University of Applied Sciences

Large-Scale System Analysis under Uncertainty

With Electric Power Applications

ALEJANDRO D. DOMÍNGUEZ-GARCÍA
University of Illinois at Urbana-Champaign

Cambridge University Press
978-1-107-19208-9 — Large-Scale System Analysis Under Uncertainty
With Electric Power Applications
Frontmatter
[More Information](#)

CAMBRIDGE UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India

103 Penang Road, #05–06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning, and research at the highest international levels of excellence.

www.cambridge.org

Information on this title: www.cambridge.org/9781107192089

DOI: 10.1017/9781108123853

© Cambridge University Press 2022

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 2022

A catalogue record for this publication is available from the British Library.

Library of Congress Cataloging-in-Publication Data

Names: Domínguez-García, Alejandro D., 1977– author.

Title: Large-scale system analysis under uncertainty : with electric power applications / Alejandro D. Domínguez-García, Department of Electrical and Computer Engineering, University of Illinois at Urbana-Champaign.

Description: Cambridge, United Kingdom ; New York, NY, USA :

Cambridge University Press, 2022. | Includes bibliographical references and index.

Identifiers: LCCN 2021024851 (print) | LCCN 2021024852 (ebook) |

ISBN 9781107192089 (hardback) | ISBN 9781108123853 (epub)

Subjects: LCSH: Systems engineering–Risk assessment. | Electric power systems–Risk assessment. | Uncertainty.

Classification: LCC TA169.55.R57 D66 2022 (print) |

LCC TA169.55.R57 (ebook) | DDC 621.31–dc23

LC record available at <https://lcn.loc.gov/2021024851>

LC ebook record available at <https://lcn.loc.gov/2021024852>

ISBN 978-1-107-19208-9 Hardback

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this publication and does not guarantee that any content on such websites is, or will remain, accurate or appropriate.

Cambridge University Press
978-1-107-19208-9 — Large-Scale System Analysis Under Uncertainty
With Electric Power Applications
Frontmatter
[More Information](#)

**In memory of my father,
Ángel Domínguez Casás (1930 – 2020),
who instilled in me a passion for mathematics**

Contents

	<i>Preface and Acknowledgments</i>	<i>page xi</i>
	<i>Notation</i>	<i>xiii</i>
1	Introduction	1
	1.1 Motivation	1
	1.2 System Models	3
	1.3 Uncertainty Models	7
	1.3.1 Static Systems	7
	1.3.2 Dynamical Systems	8
	1.4 Application Examples	9
	1.4.1 Power Flow Analysis under Active Power Injection Uncertainty	9
	1.4.2 Analysis of Inertia-less AC Microgrids under Power Injection Uncertainty	10
	1.4.3 Reliability Analysis of Static Systems	11
	1.5 Book Road Map	12
	1.6 Notes and References	15
2	Preliminaries	16
	2.1 Probability and Stochastic Processes	16
	2.1.1 Probability Spaces	16
	2.1.2 Random Variables	17
	2.1.3 Jointly Distributed Random Variables	21
	2.1.4 Random Vectors	27
	2.1.5 Stochastic Processes	29
	2.2 Set Theory	35
	2.2.1 Basic Notions and Notation	35
	2.2.2 Sets in Euclidean Space	36
	2.3 Linear Dynamical Systems	46
	2.3.1 Discrete-Time Systems	46
	2.3.2 Continuous-Time Systems	48
	2.4 Notes and References	53

3	Static Systems: Probabilistic Input Uncertainty	54
3.1	Introduction	54
3.2	Moment Characterization	56
3.2.1	Linear Setting	56
3.2.2	Nonlinear Setting	58
3.3	Distribution Characterization	62
3.3.1	Linear Setting	62
3.3.2	Nonlinear Setting	68
3.4	Performance Characterization	74
3.4.1	Known Input Moments	74
3.4.2	Known Input Probability Density Function	76
3.5	Application to Power Flow Analysis	77
3.5.1	Power Flow Model	78
3.5.2	Power Flow Vector Distribution	79
3.6	Notes and References	90
4	Static Systems: Probabilistic Structural Uncertainty	91
4.1	Introduction	91
4.2	System Stochastic Model	92
4.3	Markov Process Characterization	93
4.3.1	Discrete-Time Case	94
4.3.2	Continuous-Time Case	97
4.4	Performance Characterization	100
4.5	Application to Reliability and Availability Analysis	101
4.5.1	Multi-Component System Input-to-State Characterization	101
4.5.2	Systems with Non-Repairable Components	109
4.5.3	Systems with Repairable Components	118
4.5.4	Reduced-Order Models	124
4.6	Notes and References	129
5	Discrete-Time Systems: Probabilistic Input Uncertainty	130
5.1	Introduction	130
5.2	Discrete-Time Linear Systems	131
5.2.1	Characterization of First and Second Moments	131
5.2.2	Probability Distribution	138
5.3	Discrete-Time Nonlinear Systems	144
5.3.1	Characterization of First and Second Moments	145
5.3.2	Probability Distribution	150
5.4	Analysis of Microgrids under Power Injection Uncertainty	151
5.4.1	System Model	151
5.4.2	Average Frequency Error Statistical Characterization	156
5.4.3	Phase Angle Statistical Characterization	161
5.5	Notes and References	165

6	Continuous-Time Systems: Probabilistic Input Uncertainty	166
6.1	Introduction	166
6.2	Continuous-Time Linear Systems	168
6.2.1	Characterization of First and Second Moments	171
6.2.2	Gaussian Systems	181
6.3	Continuous-Time Nonlinear Systems	187
6.3.1	Moments	188
6.3.2	Probability Distribution	189
6.4	Analysis of Microgrids under Sensor Measurement Uncertainty	192
6.4.1	System Model	192
6.4.2	Average Frequency Error Statistical Characterization	197
6.5	Notes and References	201
7	Static Systems: Set-Theoretic Input Uncertainty	202
7.1	Introduction	202
7.2	Ellipsoid-Based Input Set Description	204
7.2.1	Linear Setting	204
7.2.2	Nonlinear Setting	212
7.3	Zonotope-Based Input Set Description	218
7.3.1	Linear Setting	219
7.3.2	Nonlinear Setting	221
7.4	Performance Requirements Verification	225
7.5	Application to Power Flow Analysis	226
7.5.1	Power Flow Model	227
7.5.2	Ellipsoidal-Based Description of Possible Extraneous Power Injection Values	228
7.5.3	Zonotope-Based Description of Possible Extraneous Power Injection Values	233
7.6	Notes and References	236
8	Discrete-Time Systems: Set-Theoretic Input Uncertainty	237
8.1	Introduction	237
8.2	Discrete-Time Linear Systems	238
8.2.1	Ellipsoidal-Based Input Description	239
8.2.2	Choice of Parameter γ_k	242
8.2.3	Deterministic Inputs	258
8.3	Discrete-Time Nonlinear Systems	259
8.4	Performance Requirements Verification	261
8.5	Analysis of Microgrids under Power Injection Uncertainty	263
8.5.1	System Model	263
8.5.2	Characterization of the Set Containing the Average Frequency Error	265
8.5.3	Characterization of Set Containing the Bus Phase Angles	270
8.6	Notes and References	272

x	Contents	
9	Continuous-Time Systems: Set-Theoretic Input Uncertainty	273
9.1	Introduction	273
9.2	Continuous-Time Linear Systems	275
9.2.1	Choice of Parameter $\beta(t)$	278
9.2.2	Deterministic Inputs	289
9.3	Continuous-Time Nonlinear Systems	290
9.4	Performance Requirements Verification	293
9.5	Case Studies	297
9.5.1	Buck Converter	297
9.5.2	Three-Bus Power System	300
9.6	Notes and References	306
Appendix A	Mathematical Background	307
Appendix B	Power Flow Modeling	320
	<i>References</i>	330
	<i>Index</i>	334

Preface and Acknowledgments

This book presents a collection of uncertainty analysis techniques for systems whose behavior can be mathematically represented by a set of algebraic or differential equations describing the relation between certain variables of interest. The techniques included revolve around probabilistic and set-theoretic descriptions of some uncertain phenomena that drive the system response, for example, random load variations in an electric power system, or manifest themselves as changes in the system structure, such as a power line outage caused by a storm. The case studies used throughout the book draw heavily from electric power system applications; however, the techniques presented are general and can be used in other applications, such as aerospace and automotive.

Many of the techniques presented in the book were developed in the area of systems theory and control. These techniques are very powerful and universally applicable; however, it requires a certain level of mathematical sophistication to understand the theory behind them. The goal of the book is to make these techniques accessible to applied researchers and engineers across multiple domains while maintaining a certain level of rigor in the exposition. In doing so, I have tried to make the book as self-contained as possible by including a preliminaries chapter and a mathematical background appendix that reviews most fundamental concepts used throughout the book, namely probability, stochastic processes, set theory, and linear dynamical systems theory. Except for the introductory and preliminaries chapters, the structure of subsequent chapters is always the same. Specifically, the first part of each chapter presents the general theory for a particular analysis technique interspersed with small examples that illustrate its use. The second part of each chapter illustrates the application of the particular technique to the analysis of problems encountered in electric power applications.

The inspiration for this book came from the book by Fred C. Schweppe entitled *Uncertain Dynamic Systems*, which published in 1973. I first became familiar with the book while I was working on my PhD when Professor George Verghese, who was a member of my thesis committee, pointed it out to me because of the material it contained on set-theoretic techniques for uncertainty analysis of linear dynamical systems. This material ended up being very relevant for my graduate research work on reliability and performance analysis of fault-tolerant systems, and it formed the core of one of the chapters of my PhD thesis; thus, I am greatly indebted to Professor Verghese.

Shortly after joining the University of Illinois, I developed a graduate-level course entitled Dynamic System Reliability. Early on in this course, I adopted Schweppe's aforementioned book as a reference. The book had been long out of print, but I was fortunate enough to get permission from the publisher to have the book reprinted locally for exclusive use in the course. As the course material evolved, over numerous offerings, it became apparent that Schweppe's book, while a fantastic reference, was not a perfect match for the course syllabus. As a result, I decided to develop a set of lecture notes that would align better with the course material; those notes eventually became the core material for this book.

Since my arrival at Illinois, I have worked with many undergraduate and graduate students – too many to name them all here – and I am grateful for having had the opportunity to work with a group of such talented individuals. Special thanks go to my former graduate students Stanton Cady, Sairaj Dhople, Christine Chen, Xichen Jiang, Eric Hope, and Madi Zholbaryssov as several of the application examples featured in the book are drawn from our joint research. I would also like to thank the students who have taken my graduate course and provided feedback on early versions of some of the book chapters. My former PhD student, Madi Zholbaryssov, read the whole manuscript very thoroughly and helped in fixing several issues; the end result is better because of him and I am very thankful for it. I am eternally grateful to my friend and long-term collaborator Christoforos Hadjicostis, who read early versions of the manuscript, providing encouragement and critical feedback early in the writing process, and also gave a thorough read to the final manuscript. Finally, I would like to thank my colleagues at the Department of Electrical Engineering at Illinois for providing a stimulating intellectual environment – George Gross, Daniel Liberzon, Pete Sauer, and Venu Veeravalli deserve a special mention for all the mentoring and encouragement they have provided over the years.

This book is dedicated to my father, Ángel, who unexpectedly passed away as I was applying the final touches to the manuscript. He and my mother, Vicenta, provided a nurturing environment when I was growing up and prioritized the education of their six children over all material things. They instilled in me a curiosity for learning and the importance of hard work that led me to pursue an academic career and ultimately resulted in the writing of this book. The final words are for the three loves of my life, my two daughters, Maia and Lia, and my wife, Cristina. Maia and Lia were both born in the six-year span that it took me to complete the book, and they brightened some difficult periods in the writing process. Cristina's appetite for learning, work ethic, and determination were a continuous source of inspiration during the writing process, and a constant reminder of values I hold dear.

Notation

Set Theory

\mathbb{N}	Set of natural numbers
\mathbb{Z}	Set of integer numbers
\mathbb{R}	Set of real numbers
\mathbb{C}	Set of complex numbers
\mathbb{R}^n	Set of n -dimensional real vectors
$\mathbb{R}^{n \times m}$	Set of $(n \times m)$ -dimensional real matrices
$\text{int}(\mathcal{X})$	Interior of the set $\mathcal{X} \in \mathbb{R}^n$
$\text{cl}(\mathcal{X})$	Closure of the set $\mathcal{X} \in \mathbb{R}^n$
$\text{bd}(\mathcal{X}), \partial\mathcal{X}$	Boundary of the set $\mathcal{X} \in \mathbb{R}^n$
$S_{\mathcal{X}}(\cdot)$	Support function of the set $\mathcal{X} \in \mathbb{R}^n$

Vectors and Matrices

$x^\top y, \langle x, y \rangle$	Inner product of two vectors, x, y
$\ x\ _2$	Euclidean norm, or 2-norm, of a vector x
$\mathbf{0}_n$	All-zeros vector of dimension n
$\mathbf{1}_n$	All-ones vector of dimension n
$\mathbf{0}_{n \times m}$	All-zeros matrix of dimensions $n \times m$
I_n	Identity matrix of dimensions $n \times n$
A^\top	Transpose of a real matrix A
A^{-1}	Inverse of a nonsingular matrix A
$\text{diag}(x_1, x_2, \dots, x_n)$	$(n \times n)$ -dimensional diagonal matrix whose (i, i) entry is equal to x_i
$\text{rank}(A)$	Rank of a matrix A
$\text{tr}(A)$	Trace of a square matrix A
$\text{det}(A)$	Determinant of a square matrix A
$\text{vec}(A)$	Column vector obtained by stacking the columns of a matrix A
$\text{real}(\lambda)$	Real part of eigenvector λ
$\sigma(A)$	Spectrum of a square matrix $A \in \mathbb{R}^{n \times n}$, i.e., set of eigenvalues of A
$A \otimes B$	Kronecker product of matrices A and B .

Functions

$f: \mathcal{X} \rightarrow \mathcal{Y}$	A function mapping elements in \mathcal{X} into elements in \mathcal{Y}
$f^{-1}: \mathcal{Y} \rightarrow \mathcal{X}$	Inverse function of $f: \mathcal{X} \rightarrow \mathcal{Y}$
$f, f(\cdot)$	Shorthand notation for $f: \mathcal{X} \rightarrow \mathcal{Y}$
$f: \mathbb{R}^n \rightarrow \mathbb{R}$	A function mapping n -dimensional real vectors into real numbers
$f: \mathbb{R}^n \rightarrow \mathbb{R}^m$	A function mapping n -dimensional real vectors into m -dimensional real vectors
$\nabla f(x)$	Gradient of a real-valued function $f(\cdot)$
$\frac{\partial f(x)}{\partial x}, J_f(x)$	Jacobian of a vector-valued function $f(\cdot)$ at x
$\left. \frac{\partial f(x)}{\partial x} \right _{x=x_0}$	Jacobian of a vector-valued function $f(\cdot)$ at $x = x_0$

Probability and Stochastic Processes

$\Pr(A)$	Probability that event A has occurred
$F_X(\cdot)$	Cumulative distribution function (cdf) of random variable X
$p_X(\cdot)$	Probability mass function (pmf) of a discrete random variable X
$f_X(\cdot)$	Probability density function (pdf) of continuous random variable X
$F_{X,Y}(\cdot, \cdot)$	Joint cdf of random variables X and Y
$p_{X,Y}(\cdot, \cdot)$	Joint pmf of discrete random variables X and Y
$f_{X,Y}(\cdot, \cdot)$	Joint pdf of continuous random variables X and Y
$f_{X Y}(\cdot y)$	Conditional pdf of random variable X given $Y = y$
$E[\cdot]$	Expectation operator
μ_X	Mean of random variable X
$c_{X,Y}$	Covariance of random variables X and Y
$r_{X,Y}$	Correlation of random variables X and Y
σ_X^2	Variance of random variable X
m_X	Mean of random vector X
Σ_X	Covariance matrix of random vector X
S_X	Correlation matrix of random vector X
$C_{X,Y}$	Covariance matrix of random vectors X and Y
$R_{X,Y}$	Correlation matrix of random vectors X and Y
$m_X[\cdot]$	Mean function of discrete-time stochastic vector process X
$C_X[\cdot, \cdot]$	Covariance function of discrete-time stochastic vector process X
$R_X[\cdot, \cdot]$	Correlation function of discrete-time stochastic vector process X
$m_X(\cdot)$	Mean function of continuous-time stochastic vector process X
$C_X(\cdot, \cdot)$	Covariance function of continuous-time stochastic vector process X
$R_X(\cdot, \cdot)$	Correlation function of continuous-time stochastic vector process X
$\delta(\cdot)$	Dirac delta function

Linear Dynamical Systems

- $\Phi_{k,\ell}$ State-transition matrix of a discrete-time state-space model
 $\Phi(t, \tau)$ State-transition matrix of a continuous-time state-space model

Power Networks

- θ_i Phase angle of the phasor associated with bus i 's sinusoidal voltage
 V_i Magnitude of the phasor associated with bus i 's sinusoidal voltage
 p_i Active power injected into a power network at bus i
 q_i Reactive power injected into a power network at bus i
 \bar{Y}_{ik} Shunt admittance of a transmission line linking bus i and bus j
 \bar{Z}_{ik} Series impedance of a transmission line linking bus i and bus j
 X_{ik} Imaginary part of \bar{Z}_{ik}
 \bar{Y} Power network admittance matrix
 G Real part of \bar{Y}
 B Imaginary part of \bar{Y}
 $\bar{y}_{i,k}$ (i, k) entry of the power network admittance matrix
 $g_{i,k}$ Real part of $\bar{y}_{i,k}$
 $b_{i,k}$ Imaginary part of $\bar{y}_{i,k}$
 M Incidence matrix of the graph associated with a power network
 p.u. Per unit