

## Solid State Physics

---

By identifying unifying concepts across solid state physics, this text covers theory in an accessible way to provide graduate students with the basis for making quantitative calculations and an intuitive understanding of effects. Each chapter focuses on a different set of theoretical tools, using examples from specific systems and demonstrating practical applications to real experimental topics. Advanced theoretical methods including group theory, many-body theory, and phase transitions are introduced in an accessible way, and the quasiparticle concept is developed early, with discussion of the properties and interactions of electrons and holes, excitons, phonons, photons, and polaritons. New to this edition are sections on graphene, surface states, photoemission spectroscopy, two-dimensional spectroscopy, transistor device physics, thermoelectricity, metamaterials, spintronics, exciton-polaritons, and flux quantization in superconductors. Exercises are provided to help put knowledge into practice, with a solutions manual for instructors available online, and appendices review the basic math methods used in the book. A complete set of the symmetry tables used in group theory (presented in Chapter 6) is available at [www.cambridge.org/snoke](http://www.cambridge.org/snoke).

David W. Snoke is a Professor at the University of Pittsburgh where he leads a research group studying quantum many-body effects in semiconductor systems. In 2007, his group was one of the first to observe Bose-Einstein condensation of polaritons. He is a Fellow of the American Physical Society.

Cambridge University Press  
978-1-107-19198-3 — Solid State Physics  
2nd Edition  
Frontmatter  
[More Information](#)

---

# Solid State Physics

## Essential Concepts

---

Second Edition

DAVID W. SNOKE  
University of Pittsburgh



CAMBRIDGE  
UNIVERSITY PRESS

CAMBRIDGE  
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom  
One Liberty Plaza, 20th Floor, New York, NY 10006, USA  
477 Williamstown Road, Port Melbourne, VIC 3207, Australia  
314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India  
79 Anson Road, #06–04/06, Singapore 079906

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning, and research at the highest international levels of excellence.

[www.cambridge.org](http://www.cambridge.org)

Information on this title: [www.cambridge.org/9781107191983](http://www.cambridge.org/9781107191983)

DOI: 10.1017/9781108123815

© David Snoke 2020

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 2020

Printed in the United Kingdom by TJ International Ltd, Padstow Cornwall

*A catalogue record for this publication is available from the British Library.*

ISBN 978-1-107-19198-3 Hardback

Additional resources for this publication at [www.cambridge.org/snoke](http://www.cambridge.org/snoke)

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this publication and does not guarantee that any content on such websites is, or will remain, accurate or appropriate.

There is beauty even in the solids.

*I tell you, if these were silent, even the rocks would cry out!*

– Luke 19:40

*For his invisible attributes, namely, his eternal power and  
divine nature, have been clearly perceived, ever since the  
creation of the world, in the things that have been made.*

– Romans 1:20



# Contents

<i>Preface</i>	<i>page xv</i>
<b>1 Electron Bands</b>	<b>1</b>
1.1 Where Do Bands Come From? Why Solid State Physics Requires a New Way of Thinking	1
1.1.1 Energy Splitting Due to Wave Function Overlap	2
1.1.2 The LCAO Approximation	7
1.1.3 General Remarks on Bands	9
1.2 The Kronig–Penney Model	10
1.3 Bloch’s Theorem	16
1.4 Bravais Lattices and Reciprocal Space	18
1.5 X-ray Scattering	27
1.6 General Properties of Bloch Functions	31
1.7 Boundary Conditions in a Finite Crystal	35
1.8 Density of States	38
1.8.1 Density of States at Critical Points	39
1.8.2 Disorder and Density of States	41
1.9 Electron Band Calculations in Three Dimensions	44
1.9.1 How to Read a Band Diagram	44
1.9.2 The Tight-Binding Approximation and Wannier Functions	47
1.9.3 The Nearly Free Electron Approximation	52
1.9.4 $k \cdot p$ Theory	55
1.9.5 Other Methods of Calculating Band Structure	60
1.10 Angle-Resolved Photoemission Spectroscopy	61
1.11 Why Are Bands Often Completely Full or Empty? Bands and Molecular Bonds	65
1.11.1 Molecular Bonds	65
1.11.2 Classes of Electronic Structure	68
1.11.3 $sp^3$ Bonding	69
1.11.4 Dangling Bonds and Defect States	72
1.12 Surface States	74
1.13 Spin in Electron Bands	79
1.13.1 Split-off Bands	80
1.13.2 Spin–Orbit Effects on the $k$ -Dependence of Bands	82
References	85

<b>2</b>	<b>Electronic Quasiparticles</b>	86
2.1	Quasiparticles	86
2.2	Effective Mass	88
2.3	Excitons	91
2.4	Metals and the Fermi Gas	95
2.4.1	Isotropic Fermi Gas at $T = 0$	97
2.4.2	Fermi Gas at Finite Temperature	99
2.5	Basic Behavior of Semiconductors	101
2.5.1	Equilibrium Populations of Electrons and Holes	102
2.5.2	Semiconductor Doping	104
2.5.3	Equilibrium Populations in Doped Semiconductors	106
2.5.4	The Mott Transition	108
2.6	Band Bending at Interfaces	110
2.6.1	Metal-to-Metal Interfaces	110
2.6.2	Doped Semiconductor Junctions	112
2.6.3	Metal–Semiconductor Junctions	115
2.6.4	Junctions of Undoped Semiconductors	118
2.7	Transistors	119
2.7.1	Bipolar Transistors	119
2.7.2	Field Effect Transistors	123
2.8	Quantum Confinement	128
2.8.1	Density of States in Quantum-Confined Systems	130
2.8.2	Superlattices and Bloch Oscillations	132
2.8.3	The Two-Dimensional Electron Gas	137
2.8.4	One-Dimensional Electron Transport	137
2.8.5	Quantum Dots and Coulomb Blockade	139
2.9	Landau Levels and Quasiparticles in Magnetic Field	142
2.9.1	Quantum Mechanical Calculation of Landau Levels	144
2.9.2	De Haas–Van Alphen and Shubnikov–De Haas Oscillations	147
2.9.3	The Integer Quantum Hall Effect	148
2.9.4	The Fractional Quantum Hall Effect and Higher-Order Quasiparticles	153
	References	156
<b>3</b>	<b>Classical Waves in Anisotropic Media</b>	157
3.1	The Coupled Harmonic Oscillator Model	157
3.1.1	Harmonic Approximation of the Interatomic Potential	158
3.1.2	Linear-Chain Model	159
3.1.3	Vibrational Modes in Higher Dimensions	163
3.2	Neutron Scattering	168
3.3	Phase Velocity and Group Velocity in Anisotropic Media	169
3.4	Acoustic Waves in Anisotropic Crystals	171
3.4.1	Stress and Strain Definitions: Elastic Constants	172
3.4.2	The Christoffel Wave Equation	178

3.4.3	Acoustic Wave Focusing	180
3.5	Electromagnetic Waves in Anisotropic Crystals	182
3.5.1	Maxwell's Equations in an Anisotropic Crystal	182
3.5.2	Uniaxial Crystals	185
3.5.3	The Index Ellipsoid	190
3.6	Electro-optics	193
3.7	Piezoelectric Materials	196
3.8	Reflection and Transmission at Interfaces	200
3.8.1	Optical Fresnel Equations	200
3.8.2	Acoustic Fresnel Equations	203
3.8.3	Surface Acoustic Waves	206
3.9	Photonic Crystals and Periodic Structures	207
	References	210
<b>4</b>	<b>Quantized Waves</b>	<b>212</b>
4.1	The Quantized Harmonic Oscillator	212
4.2	Phonons	215
4.3	Photons	220
4.4	Coherent States	224
4.5	Spatial Field Operators	229
4.6	Electron Fermi Field Operators	232
4.7	First-Order Time-Dependent Perturbation Theory: Fermi's Golden Rule	234
4.8	The Quantum Boltzmann Equation	239
4.8.1	Equilibrium Distributions of Quantum Particles	244
4.8.2	The H-Theorem and the Second Law	247
4.9	Energy Density of Solids	250
4.9.1	Density of States of Phonons and Photons	251
4.9.2	Planck Energy Density	252
4.9.3	Heat Capacity of Phonons	253
4.9.4	Electron Heat Capacity: Sommerfeld Expansion	256
4.10	Thermal Motion of Atoms	258
	References	262
<b>5</b>	<b>Interactions of Quasiparticles</b>	<b>263</b>
5.1	Electron–Phonon Interactions	264
5.1.1	Deformation Potential Scattering	264
5.1.2	Piezoelectric Scattering	268
5.1.3	Fröhlich Scattering	270
5.1.4	Average Electron–Phonon Scattering Time	271
5.2	Electron–Photon Interactions	273
5.2.1	Optical Transitions Between Semiconductor Bands	274
5.2.2	Multipole Expansion	277

5.3	Interactions with Defects: Rayleigh Scattering	280
5.4	Phonon–Phonon Interactions	287
5.4.1	Thermal Expansion	290
5.4.2	Crystal Phase Transitions	292
5.5	Electron–Electron Interactions	294
5.5.1	Semiclassical Estimation of Screening Length	297
5.5.2	Average Electron–Electron Scattering Time	300
5.6	The Relaxation-Time Approximation and the Diffusion Equation	302
5.7	Thermal Conductivity	306
5.8	Electrical Conductivity	308
5.9	Thermoelectricity: Drift and Diffusion of a Fermi Gas	313
5.10	Magnetoresistance	318
5.11	The Boltzmann Transport Equation	319
5.12	Drift of Defects and Dislocations: Plasticity	322
	References	325
<b>6</b>	<b>Group Theory</b>	327
6.1	Definition of a Group	327
6.2	Representations	329
6.3	Character Tables	333
6.4	Equating Physical States with the Basis States of Representations	336
6.5	Reducing Representations	340
6.6	Multiplication Rules for Outer Products	346
6.7	Review of Types of Operators	351
6.8	Effects of Lowering Symmetry	352
6.9	Spin and Time Reversal Symmetry	355
6.10	Allowed and Forbidden Transitions	359
6.10.1	Second-Order Transitions	361
6.10.2	Quadrupole Transitions	362
6.11	Perturbation Methods	366
6.11.1	Group Theory in $k \cdot p$ Theory	366
6.11.2	Method of Invariants	370
	References	374
<b>7</b>	<b>The Complex Susceptibility</b>	375
7.1	A Microscopic View of the Dielectric Constant	375
7.1.1	Fresnel Equations for the Complex Dielectric Function	380
7.1.2	Fano Resonances	382
7.2	Kramers–Kronig Relations	383
7.3	Negative Index of Refraction: Metamaterials	388
7.4	The Quantum Dipole Oscillator	391
7.5	Polaritons	399

7.5.1	Phonon-Polaritons	399
7.5.2	Exciton-Polaritons	402
7.5.3	Quantum Mechanical Formulation of Polaritons	404
7.6	Nonlinear Optics and Photon–Photon Interactions	411
7.6.1	Second-Harmonic Generation and Three-Wave Mixing	411
7.6.2	Higher-Order Effects	415
7.7	Acousto-Optics and Photon–Phonon Interactions	417
7.8	Raman Scattering	421
	References	425
<b>8</b>	<b>Many-Body Perturbation Theory</b>	<b>426</b>
8.1	Higher-Order Time-Dependent Perturbation Theory	426
8.2	Polarons	433
8.3	Shift of Bands with Temperature	435
8.4	Line Broadening	436
8.5	Diagram Rules for Rayleigh–Schrödinger Perturbation Theory	441
8.6	Feynman Perturbation Theory	446
8.7	Diagram Rules for Feynman Perturbation Theory	454
8.8	Self-Energy	457
8.9	Physical Meaning of the Green’s Functions	461
8.10	Finite Temperature Diagrams	467
8.11	Screening and Plasmons	471
8.11.1	Plasmons	475
8.11.2	The Conductor–Insulator Transition and Screening	479
8.12	Ground State Energy of the Fermi Sea: Density Functional Theory	482
8.13	The Imaginary-Time Method for Finite Temperature	486
8.14	Symmetrized Green’s Functions	494
8.15	Matsubara Calculations for the Electron Gas	498
	References	504
<b>9</b>	<b>Coherence and Correlation</b>	<b>506</b>
9.1	Density Matrix Formalism	507
9.2	Magnetic Resonance: The Bloch Equations	510
9.3	Optical Bloch Equations	520
9.4	Quantum Coherent Effects	523
9.5	Correlation Functions and Noise	531
9.6	Correlations in Quantum Mechanics	536
9.7	Particle–Particle Correlation	540
9.8	The Fluctuation–Dissipation Theorem	543
9.9	Current Fluctuations and the Nyquist Formula	548
9.10	The Kubo Formula and Many-Body Theory of Conductivity	550
9.11	Mesoscopic Effects	555
	References	562

<b>10 Spin and Magnetic Systems</b>	564
10.1 Overview of Magnetic Properties	564
10.2 Landé g-factor in Solids	568
10.3 The Ising Model	570
10.3.1 Spontaneous Symmetry Breaking	571
10.3.2 External Magnetic Field: Hysteresis	575
10.4 Critical Exponents and Fluctuations	577
10.5 Renormalization Group Methods	584
10.6 Spin Waves and Goldstone Bosons	588
10.7 Domains and Domain Walls	592
10.8 Spin–Spin Interaction	595
10.8.1 Ferromagnetic Instability	597
10.8.2 Localized States and RKKY Exchange Interaction	601
10.8.3 Electron–Hole Exchange	607
10.9 Spin Flip and Spin Dephasing	612
References	617
<b>11 Spontaneous Coherence in Matter</b>	618
11.1 Theory of the Ideal Bose Gas	620
11.2 The Bogoliubov Model	623
11.3 The Stability of the Condensate: Analogy with Ferromagnets	626
11.4 Bose Liquid Hydrodynamics	631
11.5 Superfluids versus Condensates	634
11.6 Constructing Bosons from Fermions	638
11.7 Cooper Pairing	641
11.8 BCS Wave Function	644
11.9 Excitation Spectrum of a Superconductor	648
11.9.1 Density of States and Tunneling Spectroscopy	652
11.9.2 Temperature Dependence of the Gap	656
11.10 Magnetic Effects of Superconductors	658
11.10.1 Critical Field	660
11.10.2 Flux Quantization	663
11.10.3 Type I and Type II Superconductors	665
11.11 Josephson Junctions	669
11.12 Spontaneous Optical Coherence: Lasing as a Phase Transition	674
11.13 Excitonic Condensation	677
11.13.1 Microcavity Polaritons	679
11.13.2 Other Quasiparticle Condensates	684
References	685
<b>Appendix A Review of Bra-Ket Notation</b>	687
<b>Appendix B Review of Fourier Series and Fourier Transforms</b>	689
<b>Appendix C Delta-Function Identities</b>	692

---

<b>Appendix D</b>	<b>Quantum Single Harmonic Oscillator</b>	695
<b>Appendix E</b>	<b>Second-Order Perturbation Theory</b>	698
<b>Appendix F</b>	<b>Relativistic Derivation of Spin Physics</b>	704
	<i>Index</i>	710



## Preface

Imagine teaching a physics course on classical mechanics in which the syllabus is organized around a survey of every type of solid shape and every type of mechanical device. Or imagine teaching thermodynamics by surveying all of the phenomenology of steam engines, rockets, heating systems, and such things. Not only would that be tedious, much of the beauty of the unifying theories would be lost. Or imagine teaching a course on electrodynamics which begins with a lengthy discussion of all the faltering attempts to describe electricity and magnetism before Maxwell. Thankfully, we don't do this in most courses in physics. Instead, we present the main elements of the unifying theories, and use a few of the specific applied and historical cases as examples of working out the theory.

Yet in solid state physics courses, many educators seem to feel a need to survey every type of solid and every significant development in phenomenology. Students are left with the impression that solid state physics has no unifying, elegant theories and is just a grab bag of various effects. Nothing could be further from the truth. There are many unifying concepts in solid state physics. But any book on solid state physics that focuses on unifying concepts must leave out some of the many specialized topics that crowd books on the subject.

This book centers on essential theoretical concepts in all types of solid state physics, using examples from specific systems with real units and numbers. Each chapter focuses on a different set of theoretical tools. "Solid state" physics is particularly intended here, because "condensed matter" physics includes liquids and gases, and this book does not include in-depth discussions of those states. These are covered amply, for example, by Chaikin and Lubensky.<sup>1</sup>

Some books attempt to survey the phenomenology of the entire field, but solid state physics is now too large for any book to do a meaningful survey of all the important effects. The survey approach is also generally unsatisfying for the student. Teaching condensed matter physics by surveying the properties of various materials loses the essential beauty of the topic. On the other hand, some books on condensed matter physics deal only with "toy models," never giving the skills to calculate real-world numbers.

Researchers in the field seem to be split in regard to the importance of the advanced topics of group theory and many-body theory. Some solid state physicists say that all of solid state physics starts with group theory, while others dismiss it entirely – I would guess that well over half of academic researchers in the field have never studied group theory at all. As I discuss in Chapter 1, the existence of electron bands does not depend crucially on

<sup>1</sup> P.M. Chaikin and T.C. Lubensky, *Principles of Condensed Matter Physics* (Cambridge University Press, 2000).

symmetry properties, although the symmetry theory provides a wide variety of tools to use for systems that approximate certain symmetries.

In the same way, there is a divide on many-body theory. Experimentalists tend to avoid the subject altogether, while theorists start with it. This leads to an “impedance mismatch” when experimentalists and theorists talk to each other. In Chapter 8 of this book, I introduce the elements of many-body theory which will allow experimentalists to cross this divide without taking years of theoretical courses, and which will serve as an introduction to students planning to go deeper into these methods. It may be a surprise to some people that there are actually several different diagrammatic approaches, including the Rayleigh–Schrödinger theory common in optics circles, the Feynmann diagrammatic method, and the Matsubara imaginary-time method. All three are surveyed in Chapter 8, with a discussion of their connections.

While group theory and many-body theory may come across as high-level topics to some, others may be surprised to see “engineering” topics such as semiconductor devices, stress and strain matrices, and optics included. While some experimentalists skip group theory and many-body theory in their education, too many theorists skip these basic topics in their training. Understanding the details of these methods is crucial for understanding many of the experiments on fundamental phenomena, as well as applications in the modern world.

In this book, I have tried to focus on unifying and fundamental theories. This raises the question: Does solid state physics really involve fundamental physics? Are there really any important questions at stake? Many physics students think that astrophysics and particle physics address fundamental questions, but solid state physics doesn’t. Perhaps this is because of the way we teach it. Astrophysics and particle physics courses tend to focus much more on unifying, grand questions, especially at the introductory level, while solid state physics courses often focus on a grab bag of various phenomena. If we can get past the listing of material properties, solid state physics does deal with fascinating questions.

One deep philosophical issue is the question of “reductionism” versus “emergent behavior.” Since the time of Aristotle and Democritus, philosophers have debated whether matter can be reduced to “basic building blocks” or if it is infinitely divisible. For the past two centuries, many scientists have tended to assume that Democritus was right – that all matter is built from a few indivisible building blocks, and once we understand these, we can deduce all other behavior of matter from the laws of these underlying building blocks. In the past few decades, many solid state physicists, such as Robert Laughlin, have vociferously rejected this view.<sup>2</sup> They would argue that possibly every quantum particle is divisible, but it doesn’t matter for our understanding of the essential properties of things.

At one time, people thought atoms were indivisible, but it was found they are made of subatomic particles. Then people thought subatomic particles were indivisible, but it was found that at least some of them are made of smaller particles such as quarks. Are quarks indivisible? Many physicists believe there is at least one level lower. As the distance scale gets smaller, the energy cost gets higher. This debate came to a head in the 1980s when the high-energy physics community proposed to spend billions of dollars on the

<sup>2</sup> R. Laughlin, *A Different Universe* (Basic Books, 2005).

Superconducting Supercollider in Texas, far more than the total budget of all other physics in the USA, and some solid state physicists such as Rustum Roy opposed it. In the anti-reductionist view, it is pointless to keep searching for one final list of all particles and forces.

Those who hold to the anti-reductionist view often point to the concept of “renormalization” in condensed matter physics. This is a very general concept. Essentially, it means that we can redefine a system at a higher level, ignoring the component parts from which it is made. Then we can work entirely at the higher level, ignoring the underlying complexities. The properties at this higher level depend only on a few basic properties of the system, which could arise from any number of different microscopic properties.

There are two versions of this. The first is many-body renormalization, introduced in Chapter 2 of this book and developed further in Chapter 8. In this theory, the ground state of a system is defined as the “vacuum,” and excitations out of this state are “quasiparticles” with properties very different from the particles making up the underlying ground state. These quasiparticles then become the new particles of interest, and can themselves make up a new vacuum ground state with additional excitations. As discussed in Chapter 11, this process can be continued to any number of higher levels.

A second type of renormalization is that of renormalization groups, introduced in Chapter 10. In this approach, the essential properties of a system can be described using subsets of the whole, in which properties are averaged. From this a whole field of theory on **universality** has been developed, in which certain properties of systems can be predicted based on just a few attributes of the underlying system, without reference to the microscopic details.

Another deep topic that comes up in solid state physics is the foundations of statistical mechanics. There was enormous controversy at the founding of the field, and much of this controversy was simply swept under the rug in later years, and there is still philosophical debate.<sup>3</sup> The fundamental questions of statistical mechanics arise especially when we deal with nonequilibrium systems, a major topic of solid state physics. In Chapter 4, I present the quantum mechanical basis of irreversible behavior, which involves the concept of “dephasing” which arises in later chapters, especially Chapter 9.

This connects to another important philosophical question, the “measurement” problem of quantum mechanics, that is, what leads to “collapse” of the wave function and what constitutes a measurement. In both quantum statistical mechanics and quantum collapse, we have irreversible behavior arising from an underlying system which is essentially reversible. Is there a connection? The essential paradoxes of quantum mechanics all arise in the context of condensed matter, and going to subatomic particles does not help at all in the resolution of the paradoxes, nor raise new paradoxes.

One of the deepest issues of our day is the question of emergent phenomena. Is life as we know it essentially a generalization of condensed matter physics, in which structure arises entirely from simple interactions at the microscopic level, or do we need entirely new ways of thinking when approaching biophysics, with concepts such as feedback, systems

<sup>3</sup> See, e.g., Harvey Brown, “One and for all: the curious role of probability in the Past Hypothesis,” in *The Quantum Foundations of Statistical Mechanics*, D. Bedingham, O. Maroney, and C. Timpson (eds.) (Oxford University Press, 2017).

engineering, and transmission and processing of information?<sup>4</sup> Phase transitions are often viewed as examples of order coming out of disorder, through the process known as spontaneous symmetry breaking (introduced in Chapters 10 and 11 of this book). The effects that come about in solid state physics due to phase transitions can be dramatic, but we are a long way from extrapolating these to an explanation of the origin of life.

This book does not survey the rapidly evolving field of topological effects in condensed matter physics, except briefly at the end of Chapters 2 and 9. We have yet to create a canon of the truly essential phenomena, though it is already possible to list the various topology classes.<sup>5</sup> A discussion of surface states, which arise in many examples of topological effects, is presented at the end of Chapter 1.

Many people contributed to improving this book. I would like to thank in particular Dan Boyanovsky, David Citrin, Hrvoje Petek, Chris Smallwood, and Zoltan Vörös for critical reading of parts of this manuscript. I would also like to thank my wife Sandra for many years of warm support and encouragement.

David Snoke  
Pittsburgh, 2019

<sup>4</sup> See, e.g., A.D. Lander, “A calculus of purpose,” *PLoS Biology* 2, e164 (2004).

<sup>5</sup> See A.P. Schnyder, S. Ryu, A. Furusaki, and A.W.W. Ludwig, “Classification of topological insulators and superconductors in three spatial dimensions,” *Physical Reviews B* 78, 195125 (2008), and references therein.