

## THE ELEMENTS OF FINANCIAL ECONOMETRICS

Financial econometrics is an interdisciplinary subject that uses statistical methods and economic theory to address a variety of quantitative problems in finance. This compact, master's-level textbook focuses on methodology and includes real financial data illustrations throughout. The mathematical level is purposely kept moderate, allowing the power of the quantitative methods to be understood without too much technical detail. Wherever possible, the authors indicate where to find the relevant R codes to implement the various methods.

This book grew out of the course at Princeton University which is one of the world's flagship programs in computational finance and financial engineering. It will therefore be useful for those with an economics and finance background who are looking to sharpen their quantitative skills, and also for those with strong quantitative skills who want to learn how to apply them to finance.

# THE ELEMENTS OF FINANCIAL ECONOMETRICS

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University Printing House, Cambridge CB2 8BS, United Kingdom  
One Liberty Plaza, 20th Floor, New York, NY 10006, USA  
477 Williamstown Road, Port Melbourne, VIC 3207, Australia  
4843/24, 2nd Floor, Ansari Road, Daryaganj, Delhi-110002, India  
79 Anson Road, #06-04/06, Singapore 079906

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It furthers the University's mission by disseminating knowledge in the pursuit of  
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[www.cambridge.org](http://www.cambridge.org)  
Information on this title: [www.cambridge.org/9781107191174](http://www.cambridge.org/9781107191174)  
DOI: 10.1017/9781108120616

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First published 2017

Printed in the United Kingdom by Clays, St Ives plc

*A catalogue record for this publication is available from the British Library.*

ISBN 978-1-107-19117-4 Hardback

Additional resources for this publication at [www.cambridge.org/fanyao](http://www.cambridge.org/fanyao)

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## Preface

This is an introductory textbook for financial econometrics at the master's level. Readers are assumed to have some background in calculus, linear algebra, statistics, and probability, all at undergraduate level. Knowledge in economics and finance is beneficial but not essential.

This book grew out of the lecture notes for the “Financial Econometrics” course taught by Jianqing Fan for Master in Finance students at Princeton University since 2003 and for Master in Financial Engineering students at Fudan University since 2011. The audiences are always very broad with diverse backgrounds in mathematics, physics, computer science, economics, or finance. Those talented students wish to learn fundamentals of financial econometrics in order to work in the financial industry or to pursue a Ph.D. degree in related fields. The challenge is to give all of them sufficient financial econometrics knowledge upon the completion of the class. This textbook is written with the aim to achieve such an ambitious goal.

We trust that the book will be of interest to those coming to the area for the first time, to readers who already have an economics and finance background but would like to further sharpen their quantitative skills, and also to those who have strong quantitative skills and would like to learn how to apply them to finance. Application-oriented analysts will also find this book useful, as it focuses on methodology and includes numerous case studies with real data sets. We purposely keep the level of mathematics moderate, as the power of the quantitative methods can be understood without sophisticated technical details. We also avoid the cook book style of writing, as a good understanding of statistical and econometric principles in finance enables readers to apply the knowledge far beyond the problems stated in the book. Numerical illustration with real financial data is included throughout the book. We also indicate, whenever possible, where to find the relevant R codes to implement the various methods. Due to the nature of the subject,

it is inevitable that we occasionally step into more sophisticated techniques which rely on more advanced mathematics; such sections are marked with “\*” and can be ignored by beginners. Most technical arguments are collected in a “Complements” section at the end of some chapters, but key ideas are left within the main body of the text.

What is financial econometrics? Broadly speaking, it is an interdisciplinary subject that uses statistical methods and economic theory to address a variety of quantitative problems in finance. These include building financial models, testing financial economics theory, simulating financial systems, volatility estimation, risk management, capital asset pricing, derivative pricing, portfolio allocation, proprietary trading, portfolio and derivative hedging, among others. Financial econometrics is an active field of integration of finance, economics, probability, statistics, and applied mathematics. Financial activities generate many new problems and products, economics provides useful theoretical foundation and guidance, and quantitative methods such as statistics, probability and applied mathematics are essential tools for solving quantitative problems in finance. Professionals in finance now routinely use sophisticated statistical techniques and modern computation power in portfolio management, proprietary trading, derivative pricing, financial consulting, securities regulation, and risk management.

When the class was first taught in 2003, there were very few books on financial econometrics. The books that had a strong impact on our preparation of lecture notes were Campbell et al. (1997) and Fan and Yao (2003). With one semester of teaching, we can only cover the important elements of financial econometrics. We use this name as the title of the book, as it also reflects the modular aspect of the book. This allows us to expand this textbook to cover other fundamental materials in the future. For example, “Simulation methods in finance” and “Econometrics of continuous time finance” are taught at Princeton, but are not included in this book nor in Fudan’s financial engineering class due to time constraints. Another important topic that we wish to cover is the analysis of high-frequency financial data.

The book consists of two integrated parts: The first four chapters are on time series aspects of financial econometrics while the last five chapters focus on cross-sectional aspects. The introduction in Chapter 1 sets the scene for the book: using two financial price time series we illustrate the stylized features in financial returns. The efficient markets hypothesis is deliberated together with statistical tests for random walks and white noise. A compact view of linear time series models is given in Chapter 2, including ARMA models, random walks, and inference with trends. We also include

a brief introduction to the exponential smoothing based forecasting techniques for trends and momentum, which are widely used in the financial industry. Chapter 3 introduces various heteroscedastic volatility models. A compact introduction to state space models including techniques such as Kalman filter and particle filters is included as an appendix. Chapter 4 contains some selective topics in multivariate time series analysis. Within the context of vector autoregressive models, we also introduce topics such as Granger causality, impulse response functions, and cointegration, as they play important roles in economics and finance.

The second part begins with Chapter 5, which introduces portfolio theory and derives the celebrated capital asset model. We also introduce statistical techniques to test such a celebrated model and provide extensive empirical studies. Chapter 6 extends the capital asset pricing model to a multi-factor pricing model. The applications of the factor models and econometrics tests on the validity of such pricing models are introduced. In addition, principal component analysis and factor analysis are briefly discussed. Chapter 7 touches on several practical aspects of portfolio allocation and risk management. The highlights of this chapter include risk assessments of large portfolios, portfolio allocation under gross-exposure constraints, and large volatility matrix estimation using factor models and covariance regularization. Chapter 8 derives the capital asset pricing model from a consumption, investment, and saving point of view. This gives students a different perspective on where the financial prices come from and a chance to appreciate how this differs from pricing financial derivatives. Chapter 9 calculates the prices implied by the models of returns. It gives us an idea of what the fundamental price of a stock is and how the prices are related to the dividend payments and short-term interest rates.

Many people have been of great help to our work on this book. Early drafts of this book have been taught to roughly five hundred students and there are also our enthusiastic readers. In particular, we are grateful to Yingying Fan, Yue Niu, Jingjing Zhang, Feng Yang, Weijie Gu, Xin Tong, Wei Dai, Jiawei Yao, Xiaofeng Shi, and Weichen Wang for their gracious assistance teaching the Financial Econometrics class at Princeton. René Carmona provided us with his course outlines on Financial Econometrics which helped us in the selection of the topics of the course. Many treatments of ARIMA models are inspired by the lecture notes of George Tiao. Alex Furger and Michael Lachans spent a great amount of their precious time proof-reading the final version of the book. We are very grateful for their contributions and generosity. We would like to thank Yacine Ait-Sahalia and Yazhen Wang for stimulating discussions on the topic and Shaojun Guo for formatting the

references of the book. We are indebted to Jiaan Yan for his encouragement and support to publish this book in China and to Xiongwen Lu for inviting us to teach the course in Fudan University. We would also like to thank Yuzuo Chen for providing various editorial assistance.

Jianqing Fan's research was generously supported by the National Science Foundation and National Institutes of Health of the USA, and the Academy of Mathematics and System Science and the National Center for Mathematics and Interdisciplinary Sciences, Chinese Academy of Sciences.