

Index

The following index combines people, topics, techniques, notations, acronyms, and examples. Persons are cited by last name and one or two initials. References to figures are in **boldface**. Entries pointing to footnotes go as page number followed by “n” and then footnote number, e.g., “352n2”. Multiple crossreferences are separated by semicolons; within a cross-reference, a comma indicates subordination (e.g., see A, B means “look up B under main index term A”).

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 ϵ , epsilon, error in a measurement
 η , eta, error in an equation
 μ , mu, a mean or average
 π , pi, 3.14159...
 ϕ , phi
 Π , stretched upper-case Pi, “product”
 σ , sigma, standard deviation (if one subscript) or covariance (if two)
 Σ , upper-case sigma, sum
 Ω , upper-case omega, joint probability
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mathematical notations
 \sim , “distributed as,” 55
 \approx , “approximately equal to,” 60
 χ_k^2 , the chisquare distribution on k degrees
 of freedom (sum of k squared
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 χ distribution, square root of a chisquare,
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 ${}_nC_k$, “enn choose kay,” number of ways of
 choosing k items out of n , 61
 \cdot , “controlling for,” in report of a regression
 coefficient, 150
 \equiv , “defined as” or “reduces to,” 88
 e , the base of natural logarithms, 2.71828
 H , the centering matrix, 288
 $k!$, “ k factorial,” product of the integers 1
 through k , 60
see also Stirling's formula
 $^{-1}$, inverse (of a matrix), 107, 158, **4.2**
 $|$, “given,” the notation for conditional
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 $|\cdot|$, determinant (of a matrix), 297
 $|lm_1 - lm_2|$, distance between two
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 J , projection matrix for shape coordinates,
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 J , Jacobian of the Wishart change of
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 \int , the integral symbol
 $\bar{}$, overbar, meaning “average,” 52

- ∂ , “del,” the partial derivative operator, 47n6
 $'$, “prime,” matrix transpose, 255
 Q , bending energy matrix for the semilandmark algorithm, 444
 R , R^2 , multiple correlation and its square alternatives for a multiple regression, 158
 +, in an `lm` command, additive (linear) term
 *, in an `lm` command, interaction term (two additive predictors along with their product)
 tr , trace (sum of diagonal entries of a matrix), 297
 UDV' , singular-value decomposition, 266
 X , data matrix, 255
 $X'X/n$, covariance matrix, 255
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