

## Index

The following index combines people, topics, techniques, notations, acronyms, and examples. Persons are cited by last name and one or two initials. References to figures are in **boldface**. Entries pointing to footnotes go as page number followed by “n” and then footnote number, e.g., “352n2”. Multiple crossreferences are separated by semicolons; within a cross-reference, a comma indicates subordination (e.g., see A, B means “look up B under main index term A”).

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 $\sim$ , “distributed as,” 55  
 $\approx$ , “approximately equal to,” 60  
 $\chi_k^2$ , the chisquare distribution on  $k$  degrees  
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 $\chi$  distribution, square root of a chisquare,  
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 ${}_nC_k$ , “enn choose kay,” number of ways of  
 choosing  $k$  items out of  $n$ , 61  
 $\therefore$ , “controlling for,” in report of a regression  
 coefficient, 150  
 $\equiv$ , “defined as” or “reduces to,” 88  
 $e$ , the base of natural logarithms, 2.71828  
 $H$ , the centering matrix, 288  
 $k!$ , “ $k$  factorial,” product of the integers 1  
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*see also* Stirling's formula  
 $^{-1}$ , inverse (of a matrix), 107, 158, **4.2**  
 $|$  , “given,” the notation for conditional  
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 $|\cdot|$ , determinant (of a matrix), 297  
 $|lm_1 - lm_2|$ , distance between two  
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 $J$ , projection matrix for shape coordinates,  
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 $J$ , Jacobian of the Wishart change of  
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 $\int$ , the integral symbol  
 $\bar{\phantom{x}}$ , overbar, meaning “average,” 52

- $\partial$ , “del,” the partial derivative operator, 47n6  
 $'$ , “prime,” matrix transpose, 255  
 $Q$ , bending energy matrix for the semilandmark algorithm, 444  
 $R$ ,  $R^2$ , multiple correlation and its square alternatives for a multiple regression, 158  
 +, in an `lm` command, additive (linear) term  
 \*, in an `lm` command, interaction term (two additive predictors along with their product)  
 $tr$ , trace (sum of diagonal entries of a matrix), 297  
 $UDV'$ , singular-value decomposition, 266  
 $X$ , data matrix, 255  
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