

## Fundamental Equations

Schrödinger equation:

$$i\hbar \frac{\partial \Psi}{\partial t} = H\Psi$$

Time-independent Schrödinger equation:

$$H\psi = E\psi, \quad \Psi = \psi e^{-iEt/\hbar}$$

Hamiltonian operator:

$$H = -\frac{\hbar^2}{2m} \nabla^2 + V$$

Momentum operator:

$$\mathbf{p} = -i\hbar \nabla$$

Time dependence of an expectation value:

$$\frac{d\langle Q \rangle}{dt} = \frac{i}{\hbar} \langle [H, Q] \rangle + \left\langle \frac{\partial Q}{\partial t} \right\rangle$$

Generalized uncertainty principle:

$$\sigma_A \sigma_B \geq \left| \frac{1}{2i} \langle [A, B] \rangle \right|$$

Heisenberg uncertainty principle:

$$\sigma_x \sigma_p \geq \hbar/2$$

Canonical commutator:

$$[x, p] = i\hbar$$

Angular momentum:

$$[L_x, L_y] = i\hbar L_z, \quad [L_y, L_z] = i\hbar L_x, \quad [L_z, L_x] = i\hbar L_y$$

Pauli matrices:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Fundamental Constants

Planck’s constant:	$\hbar$	$=$	$1.05457 \times 10^{-34}$	J s
Speed of light:	$c$	$=$	$2.99792 \times 10^8$	m/s
Mass of electron:	$m_e$	$=$	$9.10938 \times 10^{-31}$	kg
Mass of proton:	$m_p$	$=$	$1.67262 \times 10^{-27}$	kg
Charge of proton:	$e$	$=$	$1.60218 \times 10^{-19}$	C
Charge of electron:	$-e$	$=$	$-1.60218 \times 10^{-19}$	C
Permittivity of space:	$\epsilon_0$	$=$	$8.85419 \times 10^{-12}$	C <sup>2</sup> /J m
Boltzmann constant:	$k_B$	$=$	$1.38065 \times 10^{-23}$	J/K

Hydrogen Atom

Fine structure constant:	$\alpha$	$=$	$\frac{e^2}{4\pi\epsilon_0\hbar c}$	$=$	1/137.036
Bohr radius:	$a$	$=$	$\frac{4\pi\epsilon_0\hbar^2}{m_e e^2} = \frac{\hbar}{\alpha m_e c}$	$=$	$5.29177 \times 10^{-11}$ m
Bohr energies:	$E_n$	$=$	$-\frac{m_e e^4}{2(4\pi\epsilon_0)^2 \hbar^2 n^2}$	$=$	$\frac{E_1}{n^2}$ ( $n = 1, 2, 3, \dots$ )
Binding energy:	$-E_1$	$=$	$\frac{\hbar^2}{2m_e a^2} = \frac{\alpha^2 m_e c^2}{2}$	$=$	13.6057 eV
Ground state:	$\psi_0$	$=$	$\frac{1}{\sqrt{\pi a^3}} e^{-r/a}$		
Rydberg formula:	$\frac{1}{\lambda}$	$=$	$R \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$		
Rydberg constant:	$R$	$=$	$-\frac{E_1}{2\pi\hbar c}$	$=$	$1.09737 \times 10^7$ /m

## Introduction to Quantum Mechanics

Third edition

Changes and additions to the new edition of this classic textbook include:

- A new chapter on Symmetries and Conservation Laws
- New problems and examples
- Improved explanations
- More numerical problems to be worked on a computer
- New applications to solid state physics
- Consolidated treatment of time-dependent potentials

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# Contents

<i>Preface</i>	<i>page xi</i>
----------------	----------------

<b>I</b>	<b>THEORY</b>	<b>1</b>
<b>1</b>	<b>The Wave Function</b>	<b>3</b>
1.1	The Schrödinger Equation	3
1.2	The Statistical Interpretation	3
1.3	Probability	8
1.3.1	Discrete Variables	8
1.3.2	Continuous Variables	11
1.4	Normalization	14
1.5	Momentum	16
1.6	The Uncertainty Principle	19
	Further Problems on Chapter 1	20
<b>2</b>	<b>Time-Independent Schrödinger Equation</b>	<b>25</b>
2.1	Stationary States	25
2.2	The Infinite Square Well	31
2.3	The Harmonic Oscillator	39
2.3.1	Algebraic Method	40
2.3.2	Analytic Method	48
2.4	The Free Particle	55
2.5	The Delta-Function Potential	61
2.5.1	Bound States and Scattering States	61
2.5.2	The Delta-Function Well	63
2.6	The Finite Square Well	70
	Further Problems on Chapter 2	76
<b>3</b>	<b>Formalism</b>	<b>91</b>
3.1	Hilbert Space	91
3.2	Observables	94
3.2.1	Hermitian Operators	94
3.2.2	Determinate States	96

vi	Contents	
	3.3 Eigenfunctions of a Hermitian Operator	97
	3.3.1 Discrete Spectra	98
	3.3.2 Continuous Spectra	99
	3.4 Generalized Statistical Interpretation	102
	3.5 The Uncertainty Principle	105
	3.5.1 Proof of the Generalized Uncertainty Principle	105
	3.5.2 The Minimum-Uncertainty Wave Packet	108
	3.5.3 The Energy-Time Uncertainty Principle	109
	3.6 Vectors and Operators	113
	3.6.1 Bases in Hilbert Space	113
	3.6.2 Dirac Notation	117
	3.6.3 Changing Bases in Dirac Notation	121
	Further Problems on Chapter 3	124
	<b>4 Quantum Mechanics in Three Dimensions</b>	<b>131</b>
	4.1 The Schrödinger Equation	131
	4.1.1 Spherical Coordinates	132
	4.1.2 The Angular Equation	134
	4.1.3 The Radial Equation	138
	4.2 The Hydrogen Atom	143
	4.2.1 The Radial Wave Function	144
	4.2.2 The Spectrum of Hydrogen	155
	4.3 Angular Momentum	157
	4.3.1 Eigenvalues	157
	4.3.2 Eigenfunctions	162
	4.4 Spin	165
	4.4.1 Spin 1/2	167
	4.4.2 Electron in a Magnetic Field	172
	4.4.3 Addition of Angular Momenta	176
	4.5 Electromagnetic Interactions	181
	4.5.1 Minimal Coupling	181
	4.5.2 The Aharonov–Bohm Effect	182
	Further Problems on Chapter 4	187
	<b>5 Identical Particles</b>	<b>198</b>
	5.1 Two-Particle Systems	198
	5.1.1 Bosons and Fermions	201
	5.1.2 Exchange Forces	203

	Contents	vii
5.1.3 Spin	206	
5.1.4 Generalized Symmetrization Principle	207	
5.2 Atoms	209	
5.2.1 Helium	210	
5.2.2 The Periodic Table	213	
5.3 Solids	216	
5.3.1 The Free Electron Gas	216	
5.3.2 Band Structure	220	
Further Problems on Chapter 5	225	
<b>6 Symmetries &amp; Conservation Laws</b>	<b>232</b>	
6.1 Introduction	232	
6.1.1 Transformations in Space	232	
6.2 The Translation Operator	235	
6.2.1 How Operators Transform	235	
6.2.2 Translational Symmetry	238	
6.3 Conservation Laws	242	
6.4 Parity	243	
6.4.1 Parity in One Dimension	243	
6.4.2 Parity in Three Dimensions	244	
6.4.3 Parity Selection Rules	246	
6.5 Rotational Symmetry	248	
6.5.1 Rotations About the $z$ Axis	248	
6.5.2 Rotations in Three Dimensions	249	
6.6 Degeneracy	252	
6.7 Rotational Selection Rules	255	
6.7.1 Selection Rules for Scalar Operators	255	
6.7.2 Selection Rules for Vector Operators	258	
6.8 Translations in Time	262	
6.8.1 The Heisenberg Picture	264	
6.8.2 Time-Translation Invariance	266	
Further Problems on Chapter 6	268	
<b>II APPLICATIONS</b>	<b>277</b>	
<b>7 Time-Independent Perturbation Theory</b>	<b>279</b>	
7.1 Nondegenerate Perturbation Theory	279	
7.1.1 General Formulation	279	
7.1.2 First-Order Theory	280	



viii	Contents	
	7.1.3 Second-Order Energies	284
	7.2 Degenerate Perturbation Theory	286
	7.2.1 Two-Fold Degeneracy	286
	7.2.2 “Good” States	291
	7.2.3 Higher-Order Degeneracy	294
	7.3 The Fine Structure of Hydrogen	295
	7.3.1 The Relativistic Correction	296
	7.3.2 Spin-Orbit Coupling	299
	7.4 The Zeeman Effect	304
	7.4.1 Weak-Field Zeeman Effect	305
	7.4.2 Strong-Field Zeeman Effect	307
	7.4.3 Intermediate-Field Zeeman Effect	309
	7.5 Hyperfine Splitting in Hydrogen	311
	Further Problems on Chapter 7	313
	<b>8 The Variational Principle</b>	327
	8.1 Theory	327
	8.2 The Ground State of Helium	332
	8.3 The Hydrogen Molecule Ion	337
	8.4 The Hydrogen Molecule	341
	Further Problems on Chapter 8	346
	<b>9 The WKB Approximation</b>	354
	9.1 The “Classical” Region	354
	9.2 Tunneling	358
	9.3 The Connection Formulas	362
	Further Problems on Chapter 9	371
	<b>10 Scattering</b>	376
	10.1 Introduction	376
	10.1.1 Classical Scattering Theory	376
	10.1.2 Quantum Scattering Theory	379
	10.2 Partial Wave Analysis	380
	10.2.1 Formalism	380
	10.2.2 Strategy	383
	10.3 Phase Shifts	385
	10.4 The Born Approximation	388

10.4.1	Integral Form of the Schrödinger Equation	388
10.4.2	The First Born Approximation	391
10.4.3	The Born Series	395
	Further Problems on Chapter 10	397
<b>11</b>	<b>Quantum Dynamics</b>	<b>402</b>
11.1	Two-Level Systems	403
11.1.1	The Perturbed System	403
11.1.2	Time-Dependent Perturbation Theory	405
11.1.3	Sinusoidal Perturbations	408
11.2	Emission and Absorption of Radiation	411
11.2.1	Electromagnetic Waves	411
11.2.2	Absorption, Stimulated Emission, and Spontaneous Emission	412
11.2.3	Incoherent Perturbations	413
11.3	Spontaneous Emission	416
11.3.1	Einstein's <i>A</i> and <i>B</i> Coefficients	416
11.3.2	The Lifetime of an Excited State	418
11.3.3	Selection Rules	420
11.4	Fermi's Golden Rule	422
11.5	The Adiabatic Approximation	426
11.5.1	Adiabatic Processes	426
11.5.2	The Adiabatic Theorem	428
	Further Problems on Chapter 11	433
<b>12</b>	<b>Afterword</b>	<b>446</b>
12.1	The EPR Paradox	447
12.2	Bell's Theorem	449
12.3	Mixed States and the Density Matrix	455
12.3.1	Pure States	455
12.3.2	Mixed States	456
12.3.3	Subsystems	458
12.4	The No-Clone Theorem	459
12.5	Schrödinger's Cat	461
<b>Appendix</b>	<b>Linear Algebra</b>	<b>464</b>
A.1	Vectors	464
A.2	Inner Products	466

x	Contents	
	A.3 Matrices	468
	A.4 Changing Bases	473
	A.5 Eigenvectors and Eigenvalues	475
	A.6 Hermitian Transformations	482
	<i>Index</i>	486

# Preface

Unlike Newton's mechanics, or Maxwell's electrodynamics, or Einstein's relativity, quantum theory was not created—or even definitively packaged—by one individual, and it retains to this day some of the scars of its exhilarating but traumatic youth. There is no general consensus as to what its fundamental principles are, how it should be taught, or what it really “means.” Every competent physicist can “do” quantum mechanics, but the stories we tell ourselves about what we are doing are as various as the tales of Scheherazade, and almost as implausible. Niels Bohr said, “If you are not confused by quantum physics then you haven't really understood it”; Richard Feynman remarked, “I think I can safely say that nobody understands quantum mechanics.”

The purpose of this book is to teach you how to *do* quantum mechanics. Apart from some essential background in Chapter 1, the deeper quasi-philosophical questions are saved for the end. We do not believe one can intelligently discuss what quantum mechanics *means* until one has a firm sense of what quantum mechanics *does*. But if you absolutely cannot wait, by all means read the Afterword immediately after finishing Chapter 1.

Not only is quantum theory conceptually rich, it is also technically difficult, and exact solutions to all but the most artificial textbook examples are few and far between. It is therefore essential to develop special techniques for attacking more realistic problems. Accordingly, this book is divided into two parts;<sup>1</sup> Part I covers the basic theory, and Part II assembles an arsenal of approximation schemes, with illustrative applications. Although it is important to keep the two parts *logically* separate, it is not necessary to study the material in the order presented here. Some instructors, for example, may wish to treat time-independent perturbation theory right after Chapter 2.

This book is intended for a one-semester or one-year course at the junior or senior level. A one-semester course will have to concentrate mainly on Part I; a full-year course should have room for supplementary material beyond Part II. The reader must be familiar with the rudiments of linear algebra (as summarized in the Appendix), complex numbers, and calculus up through partial derivatives; some acquaintance with Fourier analysis and the Dirac delta function would help. Elementary classical mechanics is essential, of course, and a little electrodynamics would be useful in places. As always, the more physics and math you know the easier it will be, and the more you will get out of your study. But quantum mechanics is not something that flows smoothly and naturally from earlier theories. On the contrary, it represents an abrupt and revolutionary departure from classical ideas, calling forth a wholly new and radically counterintuitive way of thinking about the world. That, indeed, is what makes it such a fascinating subject.

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<sup>1</sup> This structure was inspired by David Park's classic text *Introduction to the Quantum Theory*, 3rd edn, McGraw-Hill, New York (1992).

At first glance, this book may strike you as forbiddingly mathematical. We encounter Legendre, Hermite, and Laguerre polynomials, spherical harmonics, Bessel, Neumann, and Hankel functions, Airy functions, and even the Riemann zeta function—not to mention Fourier transforms, Hilbert spaces, hermitian operators, and Clebsch–Gordan coefficients. Is all this baggage really necessary? Perhaps not, but physics is like carpentry: Using the right tool makes the job *easier*, not more difficult, and teaching quantum mechanics without the appropriate mathematical equipment is like having a tooth extracted with a pair of pliers—it’s possible, but painful. (On the other hand, it can be tedious and diverting if the instructor feels obliged to give elaborate lessons on the proper use of each tool. Our instinct is to hand the students shovels and tell them to start digging. They may develop blisters at first, but we still think this is the most efficient and exciting way to learn.) At any rate, we can assure you that there is no *deep* mathematics in this book, and if you run into something unfamiliar, and you don’t find our explanation adequate, by all means *ask* someone about it, or look it up. There are many good books on mathematical methods—we particularly recommend Mary Boas, *Mathematical Methods in the Physical Sciences*, 3rd edn, Wiley, New York (2006), or George B. Arfken, Hans J. Weber, and Frank E. Harris, *Mathematical Methods for Physicists*, 7th edn, Academic Press, Waltham (2013). But whatever you do, don’t let the mathematics—which, for us, is only a *tool*—obscure the physics.

Several readers have noted that there are fewer worked examples in this book than is customary, and that some important material is relegated to the problems. This is no accident. We don’t believe you can learn quantum mechanics without doing many exercises for yourself. Instructors should of course go over as many problems in class as time allows, but students should be warned that this is not a subject about which *anyone* has natural intuitions—you’re developing a whole new set of muscles here, and there is simply no substitute for calisthenics. Mark Semon suggested that we offer a “Michelin Guide” to the problems, with varying numbers of stars to indicate the level of difficulty and importance. This seemed like a good idea (though, like the quality of a restaurant, the significance of a problem is partly a matter of taste); we have adopted the following rating scheme:

- \* an *essential* problem that every reader should study;
- \*\* a somewhat more difficult or peripheral problem;
- \*\*\* an unusually challenging problem, that may take over an hour.

(No stars at all means fast food: OK if you’re hungry, but not very nourishing.) Most of the one-star problems appear at the end of the relevant section; most of the three-star problems are at the end of the chapter. If a computer is required, we put a mouse in the margin. A solution manual is available (to instructors only) from the publisher.

In preparing this third edition we have tried to retain as much as possible the spirit of the first and second. Although there are now two authors, we still use the singular (“I”) in addressing the reader—it feels more intimate, and after all only one of us can speak at a time (“we” in the text means you, the reader, and I, the author, working together). Schroeter brings the fresh perspective of a solid state theorist, and he is largely responsible for the new chapter on symmetries. We have added a number of problems, clarified many explanations, and revised the Afterword. But we were determined not to allow the book to grow fat, and for that reason we have eliminated the chapter on the adiabatic approximation (significant insights from that chapter have been incorporated into Chapter 11), and removed material from Chapter 5 on statistical mechanics (which properly belongs in a book on thermal physics). It goes without

saying that instructors are welcome to cover such other topics as they see fit, but we want the textbook itself to represent the essential core of the subject.

We have benefitted from the comments and advice of many colleagues, who read the original manuscript, pointed out weaknesses (or errors) in the first two editions, suggested improvements in the presentation, and supplied interesting problems. We especially thank P. K. Aravind (Worcester Polytech), Greg Benesh (Baylor), James Bernhard (Puget Sound), Burt Brody (Bard), Ash Carter (Drew), Edward Chang (Massachusetts), Peter Collings (Swarthmore), Richard Crandall (Reed), Jeff Dunham (Middlebury), Greg Elliott (Puget Sound), John Essick (Reed), Gregg Franklin (Carnegie Mellon), Joel Franklin (Reed), Henry Greenside (Duke), Paul Haines (Dartmouth), J. R. Huddle (Navy), Larry Hunter (Amherst), David Kaplan (Washington), Don Koks (Adelaide), Peter Leung (Portland State), Tony Liss (Illinois), Jeffery Mallow (Chicago Loyola), James McTavish (Liverpool), James Nearing (Miami), Dick Palas, Johnny Powell (Reed), Krishna Rajagopal (MIT), Brian Raue (Florida International), Robert Reynolds (Reed), Keith Riles (Michigan), Klaus Schmidt-Rohr (Brandeis), Kenny Scott (London), Dan Schroeder (Weber State), Mark Semon (Bates), Herschel Snodgrass (Lewis and Clark), John Taylor (Colorado), Stavros Theodorakis (Cyprus), A. S. Tremsin (Berkeley), Dan Velleman (Amherst), Nicholas Wheeler (Reed), Scott Willenbrock (Illinois), William Wootters (Williams), and Jens Zorn (Michigan).