

Contents

<i>Preface</i>	<i>page xi</i>
1 <i>p</i>-Adic Analysis: Essential Ideas and Results	1
1.1 The Field of <i>p</i> -Adic Numbers	1
1.2 Topology of \mathbb{Q}_p^N	2
1.3 The Bruhat–Schwartz Space and the Fourier Transform	3
1.4 Distributions	4
1.5 Some Function Spaces	6
2 Ultrametric Geometry: Cluster Networks and Buildings	8
2.1 Introduction	8
2.2 Clustering, Trees, and Ultrametric Spaces	9
2.3 Family of Metrics and Multiclustering	12
2.4 Affine Bruhat–Tits Buildings and Cluster Networks	13
2.5 Groups Acting on Trees and the Vladimirov Operator	17
3 <i>p</i>-Adic Wavelets	20
3.1 Introduction	20
3.2 Basis of <i>p</i> -Adic Wavelets	26
3.3 Coherent States	28
3.4 Orbits of Mean-Zero Test Functions as Wavelet Frames	30
3.5 Multidimensional Wavelets and Representation Theory	33
3.6 Wavelets with Matrix Dilations	34
3.7 Wavelet Transform of Distributions	40
3.8 Relation to the Haar Basis on the Real Line	41
3.9 <i>p</i> -Adic Multiresolution Analysis	43
3.10 <i>p</i> -Adic One-Dimensional Haar Wavelet Bases	45
3.11 <i>p</i> -Adic Scaling Functions	48
3.12 Multiresolution Frames of Wavelets	49
3.13 Multidimensional Multiresolution Wavelet Bases	51

3.14	p -Adic Shannon–Kotelnikov Theorem	53
3.15	Spectral Theory of p -Adic Pseudodifferential Operators	54
3.16	Wavelets and Operators for General Ultrametric Spaces	59
4	Ultrametricity in the Theory of Complex Systems	63
4.1	Introduction	63
4.2	p -Adic Parametrization of the Parisi Matrix	65
4.3	Dynamics on Complex Energy Landscapes	67
4.4	Actomyosin Molecular Motor	70
4.5	2-Adic Model of the Genetic Code	73
5	Some Applications of Wavelets and Integral Operators	76
5.1	Pseudodifferential Equations	76
5.2	Non-linear Equations and the Cascade Model of Turbulence	78
5.3	p -Adic Brownian Motion	81
6	p-Adic and Ultrametric Models in Geophysics	83
6.1	Tree-like Structures in Nature	84
6.2	p -Adic Configuration Space for Networks of Capillaries and Balance Equations for Densities of Fluids	85
6.3	Non-linear p -Adic Dynamics	89
7	Recent Development of the Theory of p-Adic Dynamical Systems	94
7.1	Van der Put Series and Coordinate Representations of Dynamical Maps	96
7.2	Recent Results about Measure-Preserving Functions and Ergodic Dynamics	99
7.3	Ergodic Dynamical Systems Based on 1-Lipschitz Functions	105
8	Parabolic-Type Equations, Markov Processes, and Models of Complex Hierarchical Systems	114
8.1	Introduction	114
8.2	Operators W , Parabolic-Type Equations, and Markov Processes	115
8.3	Elliptic Pseudodifferential Operators, Parabolic-Type Equations and Markov Processes	121
8.4	Non-Archimedean Reaction–Ultradiffusion Equations and Complex Hierarchic Systems	123
9	Stochastic Heat Equation Driven by Gaussian Noise	133
9.1	Introduction	133
9.2	p -Adic Parabolic-Type Pseudodifferential Equations	134
9.3	Positive-Definite Distributions and the Bochner–Schwartz Theorem	136
9.4	Stochastic Integrals and Gaussian Noise	138

<i>Contents</i>	ix
9.5 Stochastic Pseudodifferential Equations Driven by a Spatially Homogeneous Noise	148
10 Sobolev-Type Spaces and Pseudodifferential Operators	155
10.1 Introduction	155
10.2 The Spaces \mathcal{H}_∞	156
10.3 A Hörmander–Łojasiewicz-Type Estimation	162
10.4 The Spaces \mathcal{W}_∞	165
10.5 Pseudodifferential Operators on \mathcal{W}_∞	168
10.6 Existence of Fundamental Solutions	170
10.7 Igusa’s Local Zeta Functions and Fundamental Solutions	172
10.8 Local Zeta Functions and Pseudodifferential Operators in \mathcal{H}_∞	175
11 Non-Archimedean White Noise, Pseudodifferential Stochastic Equations, and Massive Euclidean Fields	177
11.1 Introduction	177
11.2 Preliminaries	178
11.3 Pseudodifferential Operators and Green Functions	179
11.4 The Generalized White Noise	187
11.5 Euclidean Random Fields as Convolved Generalized White Noise	190
11.6 The p -Adic Brownian Sheet on \mathbb{Q}_p^N	195
12 Heat Traces and Spectral Zeta Functions for p-Adic Laplacians	198
12.1 Introduction	198
12.2 A Class of p -Adic Laplacians	199
12.3 Lizorkin Spaces, Eigenvalues, and Eigenfunctions for A_β Operators	201
12.4 Heat Traces and p -Adic Heat Equations on the Unit Ball	203
12.5 Analytic Continuation of Spectral Zeta Functions	210
<i>References</i>	214
<i>Index</i>	236