

A Student's Guide to Atomic Physics

This concise and accessible book provides a detailed introduction to the fundamental principles of atomic physics at an undergraduate level. Concepts are explained in an intuitive way, and the book assumes only a basic knowledge of quantum mechanics and electromagnetism. With a compact format specifically designed for students, the first part of the book covers the key principles of the subject, including quantum theory of the hydrogen atom, radiative transitions, the shell model of multi-electron atoms, spin–orbit coupling, and the effects of external fields. The second part provides an introduction to four key applications of atomic physics: lasers, cold atoms, solid-state spectroscopy, and astrophysics. This highly pedagogical text includes worked examples and end-of-chapter problems to allow students to test their knowledge, as well as numerous diagrams of key concepts, making it perfect for undergraduate students looking for a succinct primer on the concepts and applications of atomic physics.

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MARK FOX
University of Sheffield



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Preface

Undergraduate students come across the concepts of atomic physics at various stages during their degree programs. For example, the Bohr model is a central part of introductory courses on quantum physics, while the hydrogen atom is a key element in a first course on quantum mechanics. After that, the more advanced topics could either be a component of a second, broad quantum physics module, or a stand-alone unit. This book is designed for the latter approach, without necessarily excluding its usefulness for the former, where it might be used, for example, in conjunction with a text on nuclear physics.

The book evolved from a detailed set of lecture notes prepared for a third-year module at the University of Sheffield. The notes were prepared to respond to the lack of a short text at the right level. The subject material was either scattered across various chapters of large quantum physics texts, or was included in introductory sections of more advanced texts. Neither case was particularly suited to the needs of the students.

The range of topics included within the book aims to cover the core curriculum on atomic physics set out by the Institute of Physics, and might be useful either to second- or third-year students within the United Kingdom, depending on how a particular university subdivides the syllabus. For readers outside the United Kingdom, the text is pitched at intermediate-level students. It assumes basic familiarity with the techniques of quantum mechanics, but does not have the depth required for masters-level courses.


The course notes have been freely available on the Internet for several years, and I was approached by several publishers who thought they could form the basis for a textbook. Having already written two textbooks, I was well aware of the extra effort required to turn a set of lecture notes into a book and resisted the approaches I received. However, I then discovered the Cambridge Student's Guide series, and realized that it is the right place for the material.

Its inclusion within the series makes it clear that the book does not claim to be an authoritative reference work, but rather an intermediate-level text aimed at explaining the basic concepts to undergraduate students.

The text is divided into two parts:

- Part I: Fundamental Principles (Chapters 1–8)
- Part II: Applications of Atomic Physics (Chapters 9–12)

The first part should be useful for undergraduate students at most universities, as it covers the core concepts of university-level atomic physics. The second part will find varied use, depending on how a particular university organizes its course. Chapter 9 covers most of the basic ideas required for the laser-physics component of Institute of Physics (IOP) curriculum. Chapter 10 gives a brief introduction to the techniques of laser cooling that underpin a large sector of modern atomic physics research. Chapter 11 reflects the author's own background in semiconductor physics and solid-state lasers. The final chapter arose from the suggestions of the manuscript reviewers, and its writing involved a fascinating learning experience for the author.

Texts within the Cambridge Student's Guide series are deliberately kept short. For this reason, some nonessential material that was in the first draft of the manuscript has been moved to an online supplement. The sections where additional notes are available online are identified by the  symbol in the margin. Another key feature of the series is the inclusion of worked examples and exercises. Solutions to the exercises are available from the online resources.

I am very grateful to numerous people who have helped in various ways to bring the book to fruition. First, I would like to thank the generations of students at the University of Sheffield who have taken the course and provided feedback on the notes. I am also grateful to my colleagues at the University of Sheffield, on whom I have bounced ideas and with whom I have clarified concepts. Among these, I would like to single out Professor Paul Crowther, who provided invaluable help with Chapter 12. My knowledge of astrophysics was very limited before I wrote the chapter, and his critical reading of the manuscript has both greatly improved it and also ironed out deficiencies in my understanding. I would also like to thank people around the world who provided feedback on the Internet version of the notes, especially Dr. André Xuereb, from the University of Malta, for his comments on the 2013 version.

Second, I would like to thank the people who taught me atomic physics at the University of Oxford, especially my tutor, Professor Roger Cashmore, and my lecturers, Dr. Alan Corney and Dr. Kem Woodgate. I regard this book as an

introduction to their excellent texts, which are both still in print and included in the References. The structure of Part I broadly follows a set of lecture notes by Professors Paul Ewart and Derek Stacey at the University of Oxford, although the final ordering of material departs a little from their plan. Professor Stacey also provided comments on Part I of the manuscript, which have helped to iron out some potentially confusing statements.

Next, I would like to thank Dr. Nicholas Gibbons at Cambridge University Press for introducing me to the *Student's Guide* series and supporting the project. I am especially grateful to him for finding a very helpful set of reviewers at the syndicate approval stage. These anonymous reviewers provided numerous helpful suggestions. In particular, the final chapter is included on their suggestion, while much of Chapter 1 is a response to one of the reviewers. This reviewer pointed out that my original notes took several basic concepts for granted, and this prompted me to rewrite the first three sections to provide fundamental definitions.

Finally, I would like to thank Dr. John Pantazis, from Amptek, Inc., for providing the data in Figure 4.6(a), and Róisín Munnelly at Cambridge University Press for her role as Content Manager. Her patience in seeing the project through to completion is much appreciated.

Symbols

The list gives the main symbols used in the text, excluding some that are used infrequently and are defined *in situ*. In some cases, it is necessary to use the same symbol to represent different quantities. Whenever this occurs, it should be obvious from the context which meaning is intended.

a_0	Bohr radius
a_H	Bohr radius of hydrogen
A	area
A_{ij}	Einstein A coefficient
\mathbf{B}	magnetic field (flux density)
B_{ij}	Einstein B coefficient
d	distance
e	magnitude of electron charge
E	energy
\mathcal{E}	electric field
\mathbf{F}	force, total angular momentum
$g(E)$	density of states at energy E
$g(\nu)$	spectral line-shape function
g	degeneracy
g_J	Landé g -factor
g_N	nuclear g -factor
g_s	electron spin g -factor
h	Planck's constant
\hbar	$h/2\pi$
\hat{H}	Hamiltonian
H'	perturbation
i	electrical current

Symbols

xv

I	moment of inertia, optical intensity, nuclear spin
\mathbf{I}	nuclear angular momentum
I_z	z component of nuclear angular momentum
\mathbf{j}	angular momentum (single electron)
J	exchange constant
\mathbf{J}	total angular momentum
\mathbf{l}	orbital angular momentum (single electron)
l_z	z component of orbital angular momentum (single electron)
L	orbital angular momentum
m	mass, magnetic quantum number
m^*	effective mass
m_H	mass of hydrogen atom
M_{ij}	matrix element
n	refractive index
N	number of atoms per unit volume
\mathbf{p}	electric dipole moment, linear momentum
P	power, pressure
q	charge
r	radius
\mathbf{r}	position vector
R	reflectivity
\mathbb{R}	pumping rate per unit volume
R_H	Rydberg energy of hydrogen
s	spin angular momentum (single electron)
s_z	z component of spin angular momentum (single electron)
\mathbf{S}	spin angular momentum
t	time
T	temperature
u	initial velocity
$u(\nu)$	spectral energy density at frequency ν
\mathbf{v}	velocity
V	voltage, potential energy, volume
W_{ij}	transition rate
x	position coordinate
$\hat{\mathbf{x}}$	unit vector along the x -axis
y	position coordinate
$\hat{\mathbf{y}}$	unit vector along the y -axis
Y_{l,m_l}	spherical harmonic function
z	position coordinate, Doppler redshift

$\hat{\mathbf{z}}$	unit vector along the z -axis
Z	atomic number
α	fine-structure constant, absorption coefficient, polarizability
γ	gyromagnetic ratio, gain coefficient
$\mathbf{\Gamma}$	torque
δ	frequency detuning
$\delta_{k,k'}$	Kronecker delta function
$\delta(x)$	Dirac delta function
ϵ_r	relative permittivity
θ	polar angle
λ	wavelength
λ_{deB}	de Broglie wavelength
$\boldsymbol{\mu}$	magnetic dipole moment
ν	frequency
$\bar{\nu}$	wave number
τ	lifetime
τ_c	collision time
ϕ	azimuthal angle
ψ, Ψ	wave function
ω	angular frequency

Quantum Numbers

In atomic physics, lower- and uppercase letters refer to individual electrons or whole atoms respectively.

F	hyperfine total angular momentum
I	nuclear spin
j, J	total electron angular momentum
l, L	orbital angular momentum
m	magnetic
M_F	z component of hyperfine angular momentum
M_I	z component of nuclear spin
m_j, M_J	z component of electron total angular momentum
m_l, M_L	z component of orbital angular momentum
m_s, M_S	z component of spin angular momentum
n	principal
s, S	spin

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