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GALOIS REPRESENTATIONS AND (φ, Γ)-MODULES

Understanding Galois representations is one of the central goals of number theory. Around 1990, Fontaine devised a strategy to compare such p-adic Galois representations with the seemingly much simpler objects of (semi)linear algebra, the so-called etale (φ, Γ)-modules. This book is the first to provide a detailed and self-contained introduction to this theory.

The close connection between the absolute Galois groups of local number fields and those of local function fields in positive characteristic is established using the recent theory of perfectoid fields and the tilting correspondence. The author works in the general framework of the Lubin–Tate extensions of local number fields and provides an introduction to Lubin–Tate formal groups and to the formalism of ramified Witt vectors. This book will allow graduate students to acquire the necessary foundations for solving a research problem in this area while also offering researchers many basic results in one convenient location.

Peter Schneider is a professor in the Mathematical Institute at the University of Münster. His research interests lie within the Langlands program, which relates Galois representations to representations of p-adic reductive groups, as well as within number theory and representation theory. He is the author of *Nonarchimedean Functional Analysis*, *p-Adic Lie Groups*, and *Modular Representation Theory of Finite Groups*, and he is a member of the National German Academy of Science Leopoldina and of the Academia Europaea.

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Galois Representations and (φ, Γ)-Modules

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Preface

To understand Galois representations is one of the central goals of number theory. This book is concerned with the p-adic Galois representations of a p-adic number field. Around 1990 Fontaine devised a strategy to compare such p-adic Galois representations to the seemingly much simpler objects of (semi)linear algebra, the so-called etale (φ, Γ)-modules. We will give a detailed and basically self-contained introduction to this theory. One of its key technical features is the close connection between the absolute Galois groups of local number fields and those of local function fields in positive characteristic. Instead of Fontaine's original method we will use the very recent theory of perfectoid fields and the tilting correspondence to establish this connection. In addition, we will work in the more general framework of Lubin–Tate extensions of local number fields. Therefore the book also contains an introduction to the Lubin–Tate formal groups and to the formalism of ramified Witt vectors.

This book grew out of a masters-level course which I taught at Münster in 2015. I hope that it will allow graduate students to acquire the necessary foundations for solving a research problem in this area, while also giving researchers a reference at hand for many basic results. I want to thank M. Bornmann, M. Kley, and O. Venjakob for reading part of or all the manuscript. My thanks also go to D. Tranah at Cambridge University Press and to S. Parkinson for their help with the final editing.